

**FREQUENCY COMPARISON AT 633 NM WAVELENGTH:  
DETERMINATION OF DIAGONAL ELEMENTS OF MATRIX MEASUREMENTS  
BY USING A MASTER-SLAVE He-Ne LASER SYSTEM**

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An internal-mirror He-Ne laser (slave laser) PLL (phase locked loop) offset locked to an iodine-stabilized He-Ne laser (master laser) was used in a frequency comparison. The frequency discrepancy  $\sigma$  between the master laser and the slave laser was less than 15 Hz (the Allan variance stability measurement with a 10 s gate time). In order to express the frequency difference three different ways were used to process the experimental data: the standard matrix measurement, the diagonal (line-by-line) measurement and the complete matrix measurement. Deviations smaller than 1.5 kHz among the frequency differences of a group of hyperfine components were observed when the results obtained by using all three methods for data processing were compared. The power coefficients proved to get close values for the same group of components, but significant deviations appeared when different groups of components were considered no matter the method we used for data reduction.

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## **1. Introduction**

It was understood by the Comité International des Poids et Mesures (CIPM) that the practical realization of the meter would, from time to time, be *normal* to take account of new measurements and improvements in techniques of laser stabilization. Therefore, in 1992 the CIPM adopted the first revision of the 1983 *mise en pratique* of the new definition of the meter in its Recommendation 3 (CI-92) [1] and, in 1997 a second revision was again adopted in its Recommendation 1 (CI-1997) [2]. At the same time, revised lists of recommended radiations were released. The consistency of the first revision was verified through a series of laser comparisons throughout the world (see for example results regarding bilateral and grouped frequency comparisons carried out during three consecutive years - from 1993 to 1995 - with participation of the lasers from the Bureau International des Poids et Mesures (BIPM) [3]).

Among the radiations included in the lists for the practical realization of the meter the most popular is that corresponding to a He-Ne laser stabilized by saturated absorption in an intracavity cell filled with  $^{127}\text{I}_2$ , using third derivative detection technique [4, 5] at wavelength 633 nm. The reproducibility of these devices is usually much better than 2.5 parts in  $10^{11}$ , if operating conditions tighter than those presently recommended in [2] are used.

For the frequency comparison the laser under study was an iodine-stabilized He-Ne laser while the reference frequency came from a low-power internal-mirror He-Ne laser (the slave laser) PLL offset locked to another iodine-stabilized He-Ne laser (the master laser). For the best frequency reproducibility, some critical operation conditions specified by the CIPM in the Recommendation 1 (CI-1997) were adjusted and then strictly checked on both iodine-stabilized He-Ne lasers just before starting a new set of measurements (we refer to the modulation amplitude and the iodine temperature).

Under these conditions, variation of the output power of the laser under study was realized by rotation of the relative polarization orientation between the gain tube and the iodine absorption cell and the frequency shifts were regarded as essential determinations.

In order to find out the frequency difference between compared lasers we followed the matrix method, while both iodine-stabilized He-Ne lasers were alternately locked to each components of a group. Three different ways in processing the experimental data were used: the standard matrix measurement, the diagonal (line-by-line) measurement and complete matrix measurement.

The experimental work was carried out at the National Research Laboratory of Metrology (NRLM), Tsukuba, Japan during Oct.-Dec., 1997.

## 2. Experimental set-up

The experimental set-up used to measure the frequency difference is presented in Fig. 1. All measurements regarding the frequency comparisons were made by beat-frequency technique, which provides a very sensitive mean of checking the behaviour of the frequency-stabilized lasers. The radiation coming from the low-power internal-mirrors He-Ne laser NRLM/FO1 and that coming from the iodine-stabilized He-Ne laser NRLM/P1 FO1 were optically mixed on the surface of a beam divider (BS1) and directed on the surface of an avalanche photodiode (APD1). The frequency offset between the laser NRLM/FO1 and the laser NRLM/P1 was controlled by a PLL type electronic controller (PLLOFEC).

The radiations coming from the laser under study (the iodine-stabilized He-Ne laser NRLM/N4) and that coming from the reference laser (the low-power internal mirror He-Ne laser NRLM/FO1) were optically mixed using a second beam divider (BS2) and further directed on the surface of an avalanche photodiode (APD2). A frequency counter (FC) with a programmable gate time interval displayed the frequency of the frequency beats while a frequency analyser (FA) allowed to monitor the spectral accuracy and the signal-to-noise ratio of the input signal.

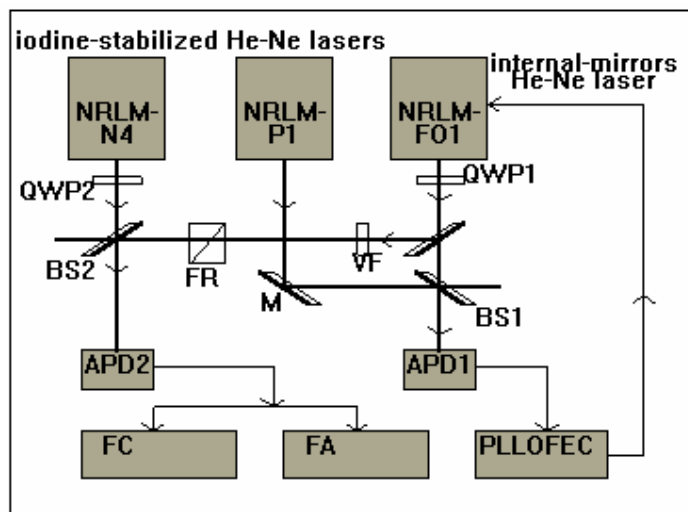


Fig. 1. The set-up used for frequency difference measurements. APD- avalanche photodiode; BS-beam divider; QWP- quarter-wave plate; VF- neutral variable filter; M- mirror; PLL OFEC-PLL offset frequency electronic controller; FC – frequency counter; FA - frequency analyser.

Some optical components as: a variable transmission neutral filter (VF), a Faraday rotator (FR) and two quarter-wave plates (QWP1,2) helped both to improve the quality of the optical beats and discriminate against accidental optical back-coupling effects.

More detailed information on the lasers involved in comparison regarding principal geometrical parameters, operation principles and performances are given elsewhere [6, 7].

### 3. Frequency difference measurements

Although, in principle, the operation of an iodine-stabilized He-Ne lasers at 633 nm wavelength under conditions specified in Recommendation 3 (CI-1997) is a sufficient condition to ensure that their absolute frequencies lie inside the given uncertainty of 2.5 parts in  $10^{11}$ , the frequency comparisons are necessary to test the performances from time-to-time or for example to determine the sharpness of the traceability [8].

An *easier* and *faster* way in performing the frequency difference measurements was only the first step in our experimental work. Then, it became capital to *compare the frequency differences* obtained by processing the experimental data with the standard matrix method and the new proposed methods. The small deviations among the results equally *validate* and *ensure* the continuity in the field of frequency difference measurements.

The participating lasers were switched ON during the first day of the measurements and remained in operation throughout the comparison in order to obtain a good thermal equilibrium in both mechanics and electronics. Before starting a new set of measurements, all operating parameters of the two iodine stabilized He-Ne lasers were adjusted to be at the nominal value: modulation amplitude 6 MHz peak-to-peak, cold finger temperature 15 °C. The one-way intracavity power of laser NRLM/N4 remained a variable parameter. An RF spectrum analyser, a calibrated thermocouple and a power meter were used to measure the values of the above mentioned parameters (the one-way intracavity power resulted from considering the output mirror transmission coefficient : 0.5%).

During the experimental work the two iodine-stabilized He-Ne lasers were successively locked to the saturated absorption hyperfine components of the  $^{127}\text{I}_2$  intracavity systems [3] from **d** to **j**. In order to get reliable measurements, a signal-to-noise ratio of at least (+40 dB) was obtained for the electrical signal reaching the input of the frequency counter.

#### 3.1 Methods used for data processing

The frequency discrepancy  $\sigma$  between the lasers NRLM/FO1 and NRLM/FO1 was measured by Allan variance stability measurements [9]. It proved to be less than 2 parts in  $10^{14}$  or 3 parts in  $10^{14}$  if a gate time  $\tau$  of 100 s or, respectively, 10 s was considered (see the graph presented in Fig. 2).

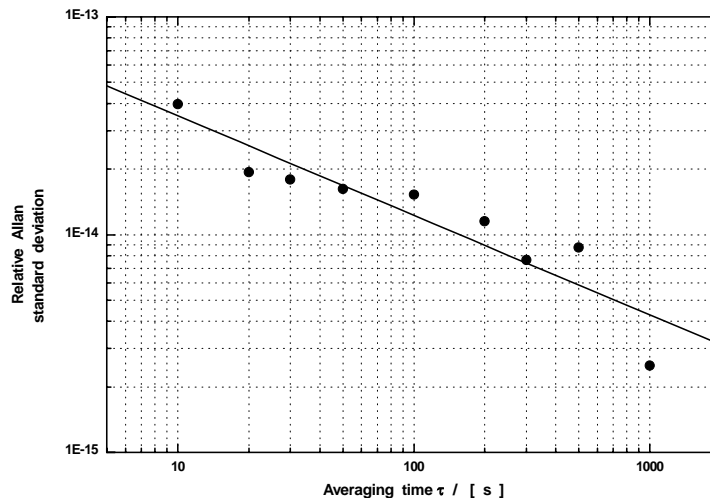


Fig. 2. Frequency stability measurement using Allan variance determinations. Compared lasers are: the iodine - stabilized He-Ne laser NRLM/P1 and the low-power internal mirrors He-Ne laser NRLM/FO1 when operated as master-slave system.

The laser system also proved a fast frequency response. Based on the above results, it could

be assumed the performances of the master laser NRLM/P1 were precisely transferred to the slave laser NRLM/FO1. It could be *assumed* the frequency beats resulting from mixing radiations coming from the lasers NRLM/N4 and NRLM/FO1 represents - up to a small uncertainty of tens of Hz - the frequency difference between the lasers NRLM/N4 and NRLM/P1.

Under the above assumptions and due to its general acceptance, the matrix determinations were considered essential in our frequency comparison, but three different ways were used to process the experimental data - the standard, the diagonal and the complete matrix methods - and results were compared.

- In case the *standard matrix method* would be used for data processing, each frequency difference measurement for a given group of components is provided from a square matrix whose elements were the frequency differences (as a mean of five measurements with an integration time of 10 s) obtained by successively locking the two iodine-stabilized He-Ne lasers NRLM/N4 and NRLM/P1 to all possible combinations of hyperfine components of the group except those for which both lasers would have been stabilized to the same component (a square matrix of a group of  $N$  hyperfine components has  $N^2-N$  elements) [10]. The average of all differences between pairs of frequency difference measurements is taken as the frequency difference between compared lasers when that group of components was considered.

- In the case *the diagonal (line-by-line) method* would be used for data processing, the average of frequency differences obtained by successively locking the two compared lasers on the same component of a group (the diagonal elements of a square matrix represent  $N$  determinations) is *proposed* to be considered as the frequency difference between compared lasers for the given group of components.

- In the case *the complete matrix method* would be used for data processing, the average of all differences between pairs of frequency difference measurements representing different components to which are added those representing diagonal elements of the matrix is *proposed* to be the frequency difference between compared lasers for a given group of components.

In order to measure the diagonal elements of the matrix, a frequency offset larger than the frequency intervals between compared components had to be set between the lasers NRLM/FO1 and NRLM/P1. Frequency offsets continuously adjustable from 100,000.00 to 400,000.00 kHz were easily available by using an improved PLL offset-locking system (the frequency interval between components **d** and **j** equals 186.681 MHz [1, 2]). Under these circumstances, the diagonal elements of a square matrix of four, three or seven components from **d** to **j** were measured by using the experimental set-up presented in Fig. 1.

### 3.2 Comparative results

In order to compare the results, the following groups of hyperfine components - available on both iodine stabilized He-Ne lasers - were considered: (**d, e, f, g**), (**h, i, j**), (**d, e, f, g, h, i, j**). Note the following indexes were used to present the results: - regarding the method used to process data: S, D, C - for standard, diagonal or complete matrix method, respectively; - regarding the group of hyperfine components: dg, hj, dj - for the group of components mentioned above.

Comparing the frequency differences obtained for a group of hyperfine components when different methods were used for the data processing, relative small deviations were obtained (see the graph in Fig. 3).

More, the frequency differences for different values of one-way intracavity power of the laser NRLM/N4 were calculated by making use of equation resulting when the experimental data (points in Fig. 3) were fitted by a linear curve and the results of computations at the nominal value of 10 mW are given in Table 1. Also, the power coefficients were computed by using the same equation and the results are given in Table 2 (all three methods for data processing were used in order to compute the frequency differences for all groups of components).

More, the frequency differences for different values of one-way intracavity power of the laser NRLM/N4 were calculated by making use of equation resulting when the experimental data - represented by points in Fig. 3 - were fitted by a linear curve. The results of computations made at the nominal value of 10 mW are given in Table 1. Also, the power coefficients were computed by using

the same equation and the results are given in Table 2 when all three methods for data processing were used in order to compute the frequency differences for a group of components.

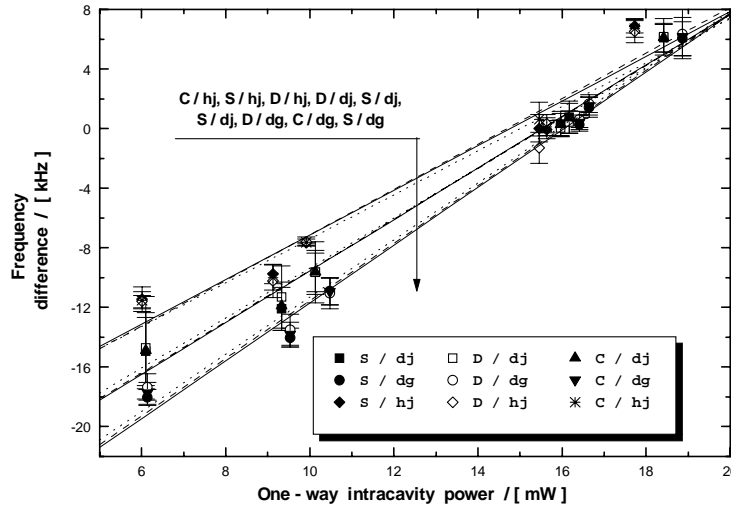


Fig. 3. Frequency differences between the lasers NRLM/ N4 and NRLM/FO1 versus one-way intracavity power of the laser NRLM/ N4 when the frequency offset and the nominal frequency separations given by CCDM [3] were subtracted (the lines are the linear fits on the experimental data). The abbreviations used: S, D and C - stand for standard, diagonal or complete matrix methods; dg, hj and dj - stand for the following groups of hyperfine components (d, e, f, g), (h, i, j) and (d, e, f, g, h, i, j), respectively.

Table 1. Computed frequency differences for an intracavity power of 10 mW (see text for the abbreviations).

Data Processing Method	Power coefficient (kHz/mW) for the group:		
	(d, e, f, g)	(h, i, j)	(d, e, f, g, h, i, j)
S	1.93	1.49	1.72
D	1.91	1.47	1.68
C	1.92	1.52	1.71

Table 2. Power coefficients calculated considering a linear fit on the experimental data given Fig. 3 (see text for the abbreviations).

Intracavity Power (mW)	Method for data processing	Expected frequency difference (kHz) for the group:		
		(d,e, f, g)	(h, i, j)	(d, e, f, g, h, i, j)
10	S	-11.70	-7.14	-9.56
	D	-11.31	-7.38	-9.35
	C	-11.54	-7.13	-9.51

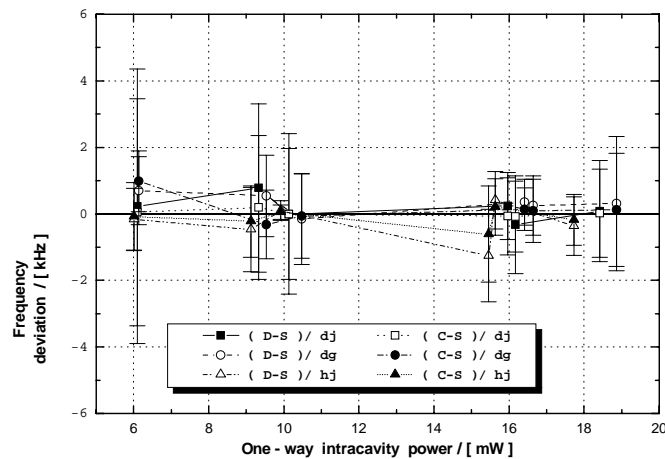


Fig. 4. Deviations of the frequency differences calculated by diagonal or complete matrix methods from the frequency differences measured by standard matrix method (see text).

The deviations resulting when the frequency differences obtained by using the standard matrix method were subtracted from those obtained by using the diagonal method or the complete matrix method were calculated for each group of hyperfine components (following the convention outlined above, the points were marked as D-S or C-S, respectively, on the graphs presented in Fig. 4).

#### 4. Conclusions

The frequency difference between two iodine-stabilized He-Ne lasers is usually measured by using the standard matrix method [10]. The beats resulting from locking the lasers to the same spectral component are not used because there is no information about which laser frequency is higher. A master-slave laser system was used to overcome this limitation.

The frequency of the slave laser NRLM/FO1 was *normal* on the frequency of the master laser NRLM/P1 and a frequency discrepancy  $\sigma$  of less than 15 Hz resulted from considering the Allan variance stability measurements for 10 s gate time. Also, frequency offsets continuously adjustable from 100 to 400 MHz were easily available by using an improved PLL offset-locking system (the frequency of the slave laser was higher than the frequency of the master laser).

Under these circumstances, it was assumed that all performances regarding frequency stability and repeatability of the iodine-stabilized He-Ne laser NRLM/P1 were transferred to- and well preserved by the low-power internal mirrors He-Ne laser. The frequency beats resulting when both iodine-stabilized He-Ne lasers NRLM/N4 and NRLM/P1 locked to the same hyperfine component from **d** to **j** were counted (as far as our knowledge goes it is for the first time to report direct line-by-line measurements). Absolute values smaller than 1.5 kHz resulted when frequency differences computed by using any of the three methods for data processing were compared. Therefore, it was proved that *very small differences are expected* to appear if different methods of data reduction were used (note that 1.5 kHz is from far less than 12.5 kHz, the reproducibility expected from these lasers [1, 2]).

Some advantages of using the diagonal method in computing the frequency differences are easily understandable as for example: - short time is required to complete a measurement (only  $N$  determinations are necessary to evaluate the frequency difference, see chapter 3.1) and, - it gives the opportunity to study the influence of laser operation parameters when both lasers are frequency-locked on the same components.

Another remarks regard the power coefficients: very small differences were noticed among values of the power coefficients for a group of components, but significant differences appeared

among the values obtained when considering different groups. In fact, this *could explain at some extent* the deviations appearing among the frequency differences between two iodine stabilized He-Ne lasers when different groups of components are considered for computations (besides the data reported in this paper, see also the frequency differences reported in the reference [11]).

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