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ANALYTICAL TREATMENT OF THE THREE-DIMENSIONAL MODEL OF STIMULATED BRILLOUIN SCATTERING WITH AXIAL SYMMETRIC PUMP WAVE

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Analytical treatment of the three-dimensional SBS model is presented and developed in order to guide the SBS geometry optimisation and the study of the transversal effects (including the phase conjugation fidelity of the Stokes wave). Analytical solutions were found for Gaussian and axial symmetric pump beams, which provide important control parameters for the SBS process and are easier to be used in comparison with the numerical results. The consistency of the analytical solutions obtained for Gaussian non-depleted and depleted pump waves with those obtained for more general pump waves and with the numerical results was ensured. The analytical results fit well the present experimental results. We have also calculated the SBS reflectivity and phase conjugation fidelity. In the practical cases, these important parameters can be written as simple functions of the pump energy can be related in a manner, which is well supported by the numerical simulation and experimental data.

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1. Introduction

SBS can be modelled by a set of three differential equations describing the interaction between the light waves and the acoustical wave yielded in a nonlinear medium [1-10]. Due to the complexity of the nonlinear differential equations of the three-dimensional SBS model, several authors have solved them in different approximations or numerically, giving information about the evolution in space and time of the three interacting waves: pump wave, Stokes wave and acoustic wave.

Ridley et al. [11] studied the three-dimensional SBS process when the pump intensity profile is Gaussian; they have obtained an analytical solution in the case of quasi-steady state and non-depleted pump and numerical results in the case of depleted pump. Suni and Falk [12] and Miller et al. [13] have done numerical simulations of 2D and 3D SBS, in steady state and non-depleted pump approximations, valid near the SBS threshold only. Numerical models for 1D SBS, in depleted steady state, were presented by Tang [14] and by Menzel and Eichler [15]. 3D SBS, in depleted steady state, was numerically studied by Kummrow [16], Moore et al. [17], who developed the light waves in terms of Hermite-Gauss orthonormal functions. Stoddard et al. developed an analytic model for the evaluation of the scattering cross-section, which depends on the aperture, natural and induced non-uniformities [18].

In the study of the transverse effects in SBS, Visnyauskas et al. [19] constructed a simplified three-dimensional model (with cylindrical symmetry) and found out an analytical solution for an initial condition taken as a series of Gauss-Laguerre polynomials. The solution permits the correlation of some transversal and longitudinal effects (for ex. the dependence of Stokes pulse duration on the divergence angle and the dependence of SBS reflectivity on the transversal envelope of the pump intensity).

V. Raab et al. [20] studied the transversal effects in optical phase conjugation in lasers with SBS mirror. The expansion in Gauss-Hermite modes is performed and the transversal problem is linearized for the clarity of the solutions.

V. Babin et al [21] and V. Vlad et al. [22] have built a three-dimensional wave model for SBS and they have analytically treated the SBS process, in the case of slowly varying envelope approximation and steady state regime.

Kuzin, Petrov and Fotiadi [23] and Mashkov and Temkin [24] studied the transversal eigenmodes propagation in different SBS waveguiding structures. Rae et al. used in [25] a numerical model for SBS in optical fibers with non-uniform properties, which in turn induce spatial non-uniformities of the induced acoustic field. Anikeev et al[26], Lehmberg [27], Hu et al.[28], studied numerically 3D steady state SBS, with depleted pump, in optical fibers, where electric fields are expanded in the series of ortho-normal functions corresponding to the propagation modes of the fibers.

Recently, S. Afshaarvahid et al. [29] have presented a transient, three-dimensional model of SBS and have used it to study the phase conjugation in SBS and the mode structure of the Stokes and pump pulse inside the SBS cell. They confirm the experimental observation of pulse-shape dependence of SBS phase-conjugation fidelity presented by Brent Dane et al. [30].

In this paper, we find analytical solutions of SBS with a depleted spatial Gaussian (TEM_{00}) pump beam, in steady state, which lead to high reflectivity and phase conjugation fidelity. Our solutions are compared with the results of Ridley et al.[11] and the numerical the transient, 3D numerical simulations of Afshaarvahid and Munch [29] and a good agreement is found. The reflectivity and fidelity are calculated and the analytical results fit well the experimental and numerical data.

More generally, we present analytical solutions of the SBS three-dimensional model in the case of pump beams with cylindrical symmetry [21, 22], which lead to the previous results in the case of the Gaussian pump beams.

2. SBS with a spatial Gaussian pump beam

SBS with a spatial Gaussian (TEM_{00}) pump beam can be conveniently described using *steady state approximation* (pump duration > (phonon lifetime)$\cdot(g_B I_p L)^{1/2}$; in SBS equations, the time derivatives vanish), which leads (in cylindrical coordinates) to:

$$\frac{\partial E_P(\zeta,r)}{\partial \zeta} - i\frac{\partial^2 E_P(\zeta,r)}{\partial r^2} = -\sigma_B \cdot \left|E_S(\zeta,r)\right|^2 E_P(\zeta,r) \tag{1'}$$

$$\frac{\partial E_S(\zeta,r)}{\partial \zeta} + i\frac{\partial^2 E_S(\zeta,r)}{\partial r^2} = -\sigma_B \left|E_P(\zeta,r)\right|^2 E_S(\zeta,r), \tag{1''}$$

where $E_P(\zeta,r)$ is the complex amplitude of the pump wave, $E_S(\zeta,r)$ is the complex amplitude of the Stokes wave, $\sigma_B = (g_B^e L I_0)\cdot e^{-\alpha\zeta/k} = g_{Bn}\cdot e^{\alpha\zeta/k}$, $I_0 = I_p(0)$, α is the linear absorption coefficient of the SBS material and ($\xi = kx$, $\eta = ky$, $\zeta = kz$, $r = \sqrt{\xi^2+\eta^2}$) are the normalized coordinates. Considering the pump wave with Gaussian space (and time) profile:

$$E_P(\zeta,r) = E_P(\zeta)\cdot \exp\left[-r^2/w_p^2(\zeta)\right], \tag{2}$$

the intensity of this particular, but often used, pump is:

$$\left|E_p\right|^2 = I_p(r) = I_p(0)\cdot\exp\left(2r^2/w_p^2\right) \cong I_p(0)\cdot\left[1-\left(2r^2/w_p^2\right)\right], \tag{3}$$

where w_p is the pump beam width and the parabolic approximation was used (near the origin, where the most significant amplification occurs).

We assume that the scattered beam will have a similar spatial distribution and look for a solution of SBS equations of the form:

$$E_S(\zeta,r) = E_S(\zeta)\cdot \exp\left[-r^2/w_S^2(\zeta)\right], \tag{4}$$

where w_S is the scattered beam width. Another approximation, usually met in experiments, will be also considered in our calculations: *the week-diffracted (low divergent) wave approximation (large beam radius).*

Introducing (3) and (4) in the Eqs. (1') and (1''), we can arrive to the following equation system:

$$\frac{\partial I_P}{\partial \zeta} - \left(2r^2\right)\cdot I_P\left[\frac{\partial}{\partial \zeta}\left(\frac{1}{W_P}\right) - 4i\left(\frac{1}{W_P}\right)^2 + \frac{2i}{r^2}\left(\frac{1}{W_P}\right) - \frac{\sigma_B}{r^2}\left(1-\frac{2r^2}{W_S}\right)I_S\right] = 0$$

$$\frac{\partial I_S}{\partial \zeta} + \left(2r^2\right)\cdot I_S\left[-\frac{\partial}{\partial \zeta}\left(\frac{1}{W_S}\right) - 4i\left(\frac{1}{W_S}\right)^2 + \frac{2i}{r^2}\left(\frac{1}{W_S}\right) + \frac{\sigma_B}{r^2}\left(1-\frac{2r^2}{W_P}\right)I_P\right] = 0 \qquad (5)$$

where $I_P = \left|E_P\right|^2$, $I_S = \left|E_S\right|^2$, $W_P = w_p^2$ and $W_S = w_S^2$.

If $r \to 0$, the Eqs. (5) lead to:

$$\frac{\partial I_P}{\partial \zeta} = -2\sigma_B \cdot I_P I_S + 4i\left(\frac{1}{W_P}\right)I_P$$

$$\frac{\partial I_S}{\partial \zeta} = -2\sigma_B \cdot I_P I_S - 4i\left(\frac{1}{W_S}\right)I_S \qquad (6)$$

and if $r \to \infty$, (5) become:

$$\frac{\partial}{\partial \zeta}\left(\frac{1}{W_P}\right) - 4i\left(\frac{1}{W_P}\right)^2 + 2\sigma_B \cdot I_S\left(\frac{1}{W_S}\right) = 0$$

$$-\frac{\partial}{\partial \zeta}\left(\frac{1}{W_S}\right) - 4i\left(\frac{1}{W_S}\right)^2 - 2\sigma_B \cdot I_S\left(\frac{1}{W_P}\right) = 0. \qquad (7)$$

For an arbitrary radial coordinate, *r*, the equation system (5) has four prime integrals, i.e. four nonlinear differential coupled equations, which describe both the longitudinal effects, by $I_P(\zeta), I_S(\zeta)$, and the transversal ones, by $\frac{1}{W_P(\zeta)}, \frac{1}{W_S(\zeta)}$:

$$\frac{\partial I_P}{\partial \zeta} = 4i\left(\frac{1}{W_P}\right)\cdot I_P - 2\sigma_B \cdot I_P I_S$$

$$\frac{\partial I_S}{\partial \zeta} = -4i\left(\frac{1}{W_S}\right)\cdot I_S - 2\sigma_B \cdot I_P I_S$$

$$\frac{\partial}{\partial \zeta}\left(\frac{1}{W_P}\right) = 4i\left(\frac{1}{W_P}\right)^2 - 2\sigma_B \cdot \left(\frac{1}{W_S}\right)I_S$$

$$\frac{\partial}{\partial \zeta}\left(\frac{1}{W_S}\right) = -4i\left(\frac{1}{W_S}\right)^2 - 2\sigma_B \cdot \left(\frac{1}{W_P}\right)I_P. \qquad (8)$$

Considering the initial conditions:

$$I_P(\zeta)\Big|_{\zeta=0} = I_{Po}; \quad I_S(\zeta)\Big|_{\zeta=\zeta_c} = I_{Sc}; \quad \frac{1}{W_P(\zeta)}\Big|_{\zeta=0} = \frac{1}{W_{Po}}; \quad \frac{1}{W_S(\zeta)}\Big|_{\zeta=\zeta_c} = \frac{1}{W_{Sc}}, \qquad (9)$$

the singular solution (for $\partial/\partial\zeta = 0$) of the system (8) has the form:

$$I_{Po} = -I_{Sc};$$

$$I_{Sc} = I_{Sc};$$

$$W_{Po} = \left[(1/2i)\sigma_B \cdot I_{Sc}\right]^{-1};$$

$$W_{Sc} = \left[(1/2i)\sigma_B \cdot I_{Sc}\right]^{-1}. \tag{10}$$

In (10), there are three independent equations. Developing the SBS nonlinear equation system in Taylor series, in the vicinity of the initial conditions, one can obtain:

$$\begin{aligned}
&\frac{\partial I_P}{\partial \zeta_1} = z_0 I_S - \left(\frac{1}{W_P}\right); \qquad (\zeta_1 = 4i \cdot I_{Sc}\zeta; \quad z_0 = \frac{\sigma_B}{2i})\\
&\frac{\partial I_S}{\partial \zeta_1} = -z_0 I_P - \left(\frac{1}{W_S}\right);\\
&\frac{\partial}{\partial \zeta_1}\left(\frac{1}{W_P}\right) = -z_0^2 I_S + 2z_0\left(\frac{1}{W_P}\right) - z_0\left(\frac{1}{W_S}\right);\\
&\frac{\partial}{\partial \zeta_1}\left(\frac{1}{W_S}\right) = -z_0^2 I_P + z_0\left(\frac{1}{W_P}\right) - 2z_0\left(\frac{1}{W_S}\right).
\end{aligned} \tag{11}$$

The characteristic determinant of the linearized SBS equation system (11) is zero, thus, the four functions $\left(I_P, I_S, W_P^{-1}, W_S^{-1}\right)$ are not linear independent in the initial conditions (point). We choose the independent functions of this system $\left(I_S, W_P^{-1}, W_S^{-1}\right)$ and the independent equations:

$$\begin{aligned}
&\frac{\partial I_S}{\partial \zeta_1} = -\left(\frac{1}{W_S}\right);\\
&\frac{\partial}{\partial \zeta_1}\left(\frac{1}{W_P}\right) = -z_0^2 I_S + 2z_0\left(\frac{1}{W_P}\right) - z_0\left(\frac{1}{W_S}\right);\\
&\frac{\partial}{\partial \zeta_1}\left(\frac{1}{W_S}\right) = z_0\left(\frac{1}{W_P}\right) - 2z_0\left(\frac{1}{W_S}\right).
\end{aligned} \tag{12}$$

The characteristic equation of the system (12):

$$\lambda^3 - 3z_0^2 \cdot \lambda - z_0^3 = 0 \tag{13}$$

has the solutions:

$$\lambda_1 = -i\sigma_B \cdot \cos\left(\frac{\pi}{9}\right); \ \lambda_2 = +i\sigma_B \cdot \cos\left(\frac{2\pi}{9}\right); \quad \lambda_3 = +i\sigma_B \cdot \cos\left(\frac{4\pi}{9}\right). \tag{14}$$

In these conditions, the solutions of the SBS equation system (12) take the form:

$$\begin{aligned}
&I_S(\zeta) = c_{21} \cdot e^{\mu_1\sigma_B\zeta} + c_{22} \cdot e^{-\mu_2\sigma_B\zeta} + c_{23} \cdot e^{-\mu_3\sigma_B\zeta} \ ;\\
&\frac{1}{W_P(\zeta)} = c_{31} \cdot e^{\mu_1\sigma_B\zeta} + c_{32} \cdot e^{-\mu_2\sigma_B\zeta} + c_{33} \cdot e^{-\mu_3\sigma_B\zeta};\\
&\frac{1}{W_S(\zeta)} = c_{41} \cdot e^{\mu_1\sigma_B\zeta} + c_{42} \cdot e^{-\mu_2\sigma_B\zeta} + c_{43} \cdot e^{-\mu_3\sigma_B\zeta},
\end{aligned} \tag{15}$$

where the following notations are used: $\mu_1 = 4I_{Sc} \cdot \cos(\pi/9)$, $\mu_2 = 4I_{Sc} \cdot \cos(2\pi/9)$ and $\mu_3 = 4I_{Sc} \cdot \cos(4\pi/9)$.

The evolution of the pump (I_P) can be derived from:

$$\frac{\partial I_P(\zeta)}{\partial \zeta} = z_0 I_S(\zeta) - \frac{1}{W_P(\zeta)} \tag{16}$$

as: $$I_P(\zeta) = \left(\frac{z_0 c_{21} - c_{31}}{\mu_1\sigma_B}\right)e^{\mu_1\sigma_B\zeta} - \left(\frac{z_0 c_{22} - c_{32}}{\mu_2\sigma_B}\right)e^{-\mu_2\sigma_B\zeta} - \left(\frac{z_0 c_{23} - c_{33}}{\mu_3\sigma_B}\right)e^{-\mu_3\sigma_B\zeta}. \tag{17}$$

In order to obtain finite solutions in (15), we impose the following condition to the integration constants:

$$c_{21} = c_{31} = c_{41} = 0 . \tag{18}$$

Using the initial conditions, one can deduce:

$$c_{22} = \frac{(I_{s0}/I_{p0})e^{-\mu_3\sigma_B L} - (I_s(L)/I_{p0})}{e^{-\mu_3\sigma_B L} - e^{-\mu_2\sigma_B L}};\ \ c_{23} = -\frac{(I_{s0}/I_{p0})e^{-\mu_2\sigma_B L} - (I_s(L)/I_{p0})}{e^{-\mu_3\sigma_B L} - e^{-\mu_2\sigma_B L}};$$

$$c_{32} = \frac{(1/W_p(0))e^{-\mu_3\sigma_B L} - (1/W_p(L))}{e^{-\mu_3\sigma_B L} - e^{-\mu_2\sigma_B L}};\ \ c_{33} = -\frac{(1/W_p(0))e^{-\mu_2\sigma_B L} - (1/W_p(L))}{e^{-\mu_3\sigma_B L} - e^{-\mu_2\sigma_B L}};$$

$$c_{42} = \frac{(1/W_S(0))e^{-\mu_3\sigma_B L} - (1/W_S(L))}{e^{-\mu_3\sigma_B L} - e^{-\mu_2\sigma_B L}};\ \ c_{43} = -\frac{(1/W_S(0))e^{-\mu_2\sigma_B L} - (1/W_S(L))}{e^{-\mu_3\sigma_B L} - e^{-\mu_2\sigma_B L}}. \tag{19}$$

Using these results, we can calculate an overall parameter, which is simple to be measured, the SBS (energy) reflectivity:

$$R = \left(\frac{\in_S}{\in_p}\right), \tag{20}$$

where $\in_{p,S}$ are the energies of the pump (subscript *p*) and Stokes pulse (subscript *S*), respectively.

In the case of week diffraction, the intensity of the pump field, with Gaussian spatial and temporal profiles, can be written as:

$$I_p(r,z,t) \approx I_0 e^{-2\frac{r^2}{w_p^2}} . e^{-2\frac{t^2}{t_L^2}} , \tag{21}$$

with $I_0 = I_p(0)$ is the peak intensity (at $r = 0$, $z = 0$), $w_p(z)$ is the pump beam radius and t_L is the laser pulse duration.

The intensity of the Stokes beam may be considered also as product of spatial and temporal factors, when the strength of the nonlinear coupling is small:

$$\begin{aligned} I_S(r,z,t) &\approx I_{S0}(z) e^{-\frac{r^2}{w_S^2(z)}} \cdot e^{-\frac{t^2}{t_S^2}} = e^{-\frac{r^2}{(w_p-\Delta w_S)^2}} \cdot e^{-\frac{t^2}{(t_p-\Delta t_S)^2}} \approx e^{-\frac{r^2}{w_p^2}} \cdot e^{-2\frac{r^2}{w_p^2}\cdot\left(\frac{\Delta w_S}{w_p}\right)} \cdot e^{-\frac{t^2}{t_p^2}} \cdot e^{-2\frac{t^2}{t_p^2}\cdot\left(\frac{\Delta t_S}{t_p}\right)} \\ &= e^{-2\frac{r^2}{w_p^2}\cdot\left(1-\frac{w_S}{w_p}\right)} \cdot e^{-2\frac{t^2}{t_p^2}\cdot\left(1-\frac{t_S}{t_p}\right)} \cdot I_p = f_1(r, w_p, (w_S/w_p)\ldots) \cdot f_2(t, t_p, (t_S/t_p), \ldots) \cdot I_p \end{aligned} \tag{22}$$

where $f_1(r, w_p, (w_S/w_p)\ldots)$ is a function related to the phase conjugation fidelity ($f_1 \to 1,\ w_S(z)/w_p \to 1$), the function $f_2(t, t_p, (t_S/t_p), \ldots)$ is related to the laser pulse compression ($f_2 \to 1,\ t_S/t_p \to 1$). When SBS reaches the steady state (defined by us in the saturation regime [5]), we can consider that the fidelity is close to one and almost constant in a definite plane and that the laser pulse compression is negligible, with $f_2 \approx 1$.

The corresponding energies and the SBS reflectivity may be written as:

$$\in_p = I_0 \int_o^{w_p} e^{-2r^2/w_p^2} \rho d\rho \cdot \int_0^{t_p} e^{-2t^2/t_p^2} dt \approx 1.19\pi^{3/2} I_0 w_p^2 \cdot t_p = 6.637 I_0 w_p^2 \cdot t_p;$$

$$\in_S = 6.637 I_{S0} w_S^2 \cdot t_S;$$

$$R = \frac{\in_S}{\in_p} = \frac{I_{S0}(z) w_S^2(z)}{I_0 w_p^2} \cdot \frac{t_S}{t_p}. \tag{23}$$

In (23), we could identify the first factor to a transversal component of the SBS reflectivity, R_{trans}. Thus, it is important to remark that the SBS reflectivity shows a different dependence on the pump energy, in its rapid rising part, if there is pulse compression or if there is no pulse compression. In the steady state, where the pulse compression is always close to unity, the dependence of *R* on the pump energy is almost saturated and essentially determined by the transverse features of the Stokes wave.

In the case of negligible pump pulse compression, which is expected for long pump pulses, the SBS reflectivity can be further evaluated by:

$$R = \left| \frac{I_S(\zeta)\cdot W_S(\zeta)}{I_P(\zeta)\cdot W_P(\zeta)} \right|. \tag{24}$$

After the calculation of the integration constants, we can further obtain:

$$R = R_0 \frac{ch[2.185 I_S(L)\sigma_B kz + \ln\phi_{1R}] + R_{01}}{ch[2.185 I_S(L)\sigma_B kz + \ln\phi_{2R}] + R_{02}}, \tag{25}$$

where:

$$R_0 = \sigma_B \cdot \left(\frac{c_{32}}{c_{42}}\right)\sqrt{\frac{c_{33}c_{42}}{c_{32}c_{43}}}\left[\frac{\sigma_B^2 + 4(c_{33}/c_{23})^2}{\sigma_B^2 + 4(c_{32}/c_{22})^2}\right]^{-1/4}\left[\sigma_B^2 + (c_{32}/c_{22})^2\right]^{-1/2};$$

$$\varphi_{3H} = \sqrt{\frac{c_{43}c_{32}}{c_{42}c_{33}}} = 1;\ \varphi_{2R} = \sqrt{\frac{c_{43}c_{23}}{c_{42}c_{22}}}\cdot\sqrt[4]{\frac{\sigma_B^2 + 4(c_{33}/c_{23})^2}{\sigma_B^2 + 4(c_{32}/c_{22})^2}};$$

$$R_{01} = \frac{1}{2}\cdot\frac{c_{22}c_{33} - c_{23}c_{32}}{\sqrt{c_{22}c_{23}c_{32}c_{33}}};$$

$$R_{02} = \frac{1}{2}\cdot\left[\frac{c_{23}}{c_{22}}\cdot\sqrt{\frac{\sigma_B^2 + 4(c_{33}/c_{23})^2}{\sigma_B^2 + 4(c_{32}/c_{22})^2}} - \frac{c_{43}}{c_{42}}\right]\left[\sqrt{\frac{c_{43}c_{23}}{c_{42}c_{22}}}\cdot\sqrt[4]{\frac{\sigma_B^2 + 4(c_{33}/c_{23})^2}{\sigma_B^2 + 4(c_{32}/c_{22})^2}}\right]^{-1}.$$

In the small SBS signal or reflectivity (the pump non-depletion) regime, one can introduce: $\sigma_B = g_B I_0 L << 1; \varphi_{1R} = 1; \varphi_{2R} \approx 1; R_{01} = 0; R_{02} \approx 0$ and the reflectivity (at $z = L$) takes the simple form:

$$R = \sigma_B \frac{I_S(0) - I_S(L)}{w_S^{-2}(0) - w_S^{-2}(L)}, \tag{26}$$

which shows the expected linear dependence on the pump intensity ($I_0 \propto \in_p$). We remark that, for pump short pulse duration (shorter than the phonon lifetime), the pulse compression could occur, which multiplies the reflectivity from (26) by a factor $1/\sqrt{I_0}$ and leads to the parabolic dependence: $R \propto \sqrt{I_0}$.

In the steady-state (saturation), we can find again $\varphi_{1R} \approx 1;\ \varphi_{2R} \approx 1;\ R_{01} = 0;\ R_{02} \approx 0$ and the reflectivity can be expressed as:

$$R \approx \frac{w_S^2(L)}{w_p^2(L)}\left[1 - \frac{1}{2\sigma_B^2}\frac{w_p^4(0)}{w_p^4(L)}\right]. \tag{27}$$

The numerical calculations of Ridley et al [11] show that the Stokes beam is narrowed due to the higher gain at the centre of the pump beam and ensure the test inequality $R < 1$ for (27).

Our analytical results were also checked experimentally using a Nd:YAG laser (oscillator-amplifier system), the measuring and coupling systems and the carbon disulphide (in a glass cell) as nonlinear material (Fig. 1).

The oscillator consists of a Nd:YAG laser, Q-switched operated using a Pockels cell with KDP crystal. The output energy from the oscillator was maximum 0.4mJ (maximum voltage of power supply), in a beam near diffraction limited and with reduced spectral width and in a pulse with a time duration of 60ns. The amplified pulse was focused with convergent lenses into a cell containing CS_2 as nonlinear medium. An optical isolator (a Glan prism and a quarter-wave plate) is introduced between the amplifier and the cell. The laser was operated at a repetition rate of 1Hz. In the experiment, we have used a laser beam diameter of 5mm and a lens with focal length of 100mm, which lead to a beam waist of 0.006mm (in the focal plane).

In order to measure the SBS reflectivity, a wedge or a mirror is placed into the beam to get a reference beam for the pump energy and the backscattered Stokes energy. The energy of the beams is measured using pyroelectric detectors RJP-735 in combination with a monitor RJ-7200 (Laser Precision) and the pulse duration using a fast photodiode and a Tektronix oscilloscope TDS 520 type.

In Fig. 2, the dependence of the SBS reflectivity, *R*, in function of the pump energy is shown. The reflectivity increases with the energy of the incident light pulse, saturating at approximately 90%. The SBS threshold energy is estimated at 0.4mJ, by extrapolating the experimental curve of the SBS reflectivity. The experimental data are obtained up to the energy corresponding to the breakdown energy in the carbon disulphide at usual chemical purity. One can notice that a good agreement between the calculated reflectivity and the experimental data is obtained for both the small signal (reflectivity) and the saturation regimes, only.

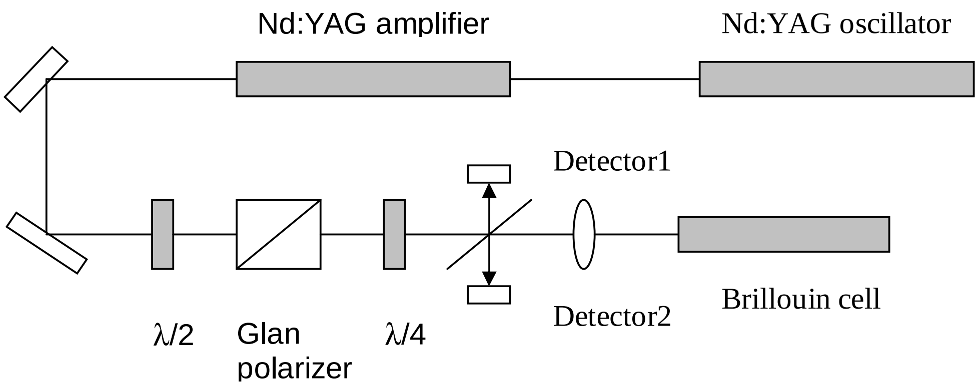

Fig.1. Experimental setup for the measurement of SBS reflectivity.

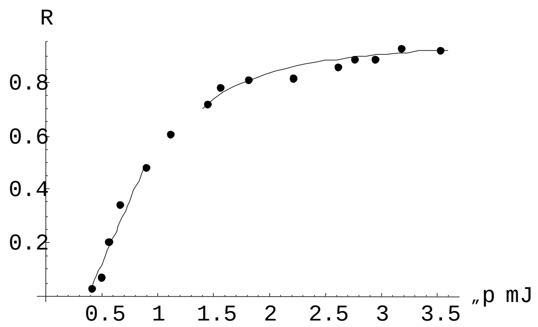

Fig. 2. The dependence of the SBS reflectivity, R, in function of the Gaussian pump energy (including pump depletion). In the small signal region, the linear dependence derived in Eq. (26) is drawn with continuous line and in the saturation region, the analytical dependence from Eq. (27) is used to fit the experimental data (marked by points).

The fidelity of the phase conjugated Stokes wave can be derived from the definition given by Zeldovich et al [1]:

$$H(\zeta)=\frac{\left|\iint_{\Sigma} E_P(\zeta,r)\cdot E_S(\zeta,r)dr^2\right|^2}{\iint_{\Sigma}\left|E_P(\zeta,r)\right|^2\cdot dr^2\cdot\iint_{\Sigma}\left|E_S(\zeta,r)\right|^2\cdot dr^2}. \qquad (28)$$

With our analytical solution from (15), it is possible to calculate:

$$H(z)=H_0\frac{ch[2.185 I_S(L)\sigma_B kz+\ln\phi_{1H}]+ch(\ln\phi_{3H})}{ch[2.185 I_S(L)\sigma_B kz+\ln\phi_{2H}]+1}. \qquad (29)$$

In (29), we have denoted:

$$H_0 = \left| 4 \cdot \frac{\sqrt{(c_{32}c_{42})\cdot(c_{33}c_{43})}}{(c_{32}+c_{42})\cdot(c_{33}+c_{43})} \right|; \varphi_{1H} = \sqrt{\frac{c_{43}c_{33}}{c_{42}c_{32}}}\ ;\ \varphi_{2H} = \left|\frac{c_{33}+c_{43}}{c_{32}+c_{42}}\right|; \varphi_{3H} = \sqrt{\frac{c_{43}c_{32}}{c_{42}c_{33}}}\ .$$

We can evaluate the fidelity in the small SBS signal (pump non-depletion), $\sigma_B = g_B I_0 L << 1$; $\varphi_{1H} = \varphi_{2H} = \varphi_{3H} = 1$) region and for $z = L$, as:

$$H = 4\left| \frac{[w_p^{-2}(0) - w_p^{-2}(L)][w_S^{-2}(0) - w_S^{-2}(L)]}{\{[w_p^{-2}(0) - w_p^{-2}(L)] + [w_S^{-2}(0) - w_S^{-2}(L)]\}^2} \right|, \tag{30}$$

which is smaller than 1, for $w_S < w_p$ and could reach 1, when $[w_p^{-2}(0) - w_p^{-2}(L)] = [w_S^{-2}(0) - w_S^{-2}(L)]$.

In the steady state (saturation), we have found: $\varphi_{1H} = \varphi_{2H} = \varphi_{3H} = 1$ and the fidelity from (29) takes the form:

$$H = \frac{4w_p^2(L)w_S^2(L)}{\left[w_p^2(L) + w_S^2(L)\right]^2} = \frac{4w_S^2(L)/w_p^2(L)}{\left[1 + w_S^2(L)/w_p^2(L)\right]^2}\ . \tag{31}$$

Taking into account the relation between the beam size ratio and the reflectivity from (27), we can further find the simple relation:

$$H = \frac{4R}{(1+R)^2};\ \ R < 1\,, \tag{32}$$

which shows that fidelity grows to 1 faster than reflectivity and tends to 1, as $R \to 1$. This dependence is shown in Fig. 3.

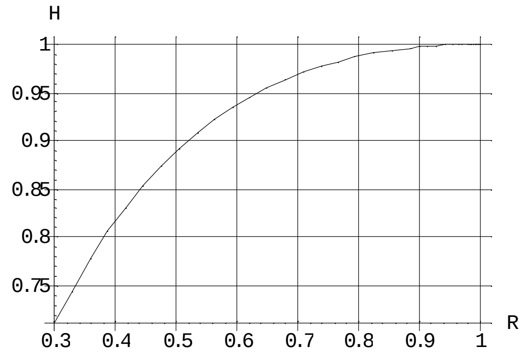

Fig. 3. The dependence of the SBS fidelity, H, on the SBS reflectivity, R, for Gaussian pump waves (including pump depletion).

Our analytical results are in agreement with the best simultaneous experimental results for SBS reflectivity and phase-conjugation fidelity, obtained by Brent Dane et al.[30], with a single-frequency TEM_{oon} Q-switched Nd:YLF laser (λ = 1053nm, t_p = 15ns), in liquid carbon tetrachloride (τ ~ 1ns) and gaseous N_2 at 90atm (τ ~15ns), which are denoted in the following table, R_{exp} and H_{exp}, respectively.

Table 1.

$\in_p\ [mJ]$	10	20	30	40	50	60	70	80
R_{exp}	0	0.3	0.45	0.54	0.6	0.63	0.67	0.7
H_{exp}	0	0.73	0.85	0.9	0.93	0.94	0.96	0.97
H_t	0	0.71	0.86	0.91	0.94	0.95	0.96	0.97
R_t	0	-	0.4	0.55	0.62	0.66	0.68	0.7

In the Table 1, the fidelity, H_t, is calculated with the Eq.(32) using the values R_{exp} and R_t is calculated with our Eq.(27) for the saturation regime. The theoretical data compare favourably to the experimental ones, H_{exp}.

One remark that, *once the SBS reflectivity is measured, the fidelity could be estimated* by Eq.(32), at least when the pump beam duration is much larger than the phonon lifetime (and the fidelity fluctuations are small).

Thus, we could predict the fidelity in our experiment devoted to the measurement of SBS reflectivity. Using the reflectivity data versus pump energy from Fig. 3, we have calculated the fidelity, H_t, with Eq.(32).

Table 2.

$\in_p$ $[mJ]$	0.55	0.66	0.88	1.1	1.44	1.55	1.8	2.2	2.6	2.94	3.52
R_{exp}	0.2	0.34	0.48	0.61	0.72	0.78	0.81	0.82	0.86	0.89	0.92
H_t	0.56	0.76	0.88	0.94	0.97	0.98	0.99	0.99	0.99	0.966	0.998

In this experiment, the fidelity could reach very high values due to the high SBS reflectivity.

This result is also in a good agreement with the numerical calculations of Afshaarvahid and Munch [29]. For standard experimental conditions, with a pump pulse of 30ns, w_p = 0.5cm and energy 140mJ and for a SBS material with the refractive index n = 1 and phonon lifetime 0.85ns, the calculated overall reflectivity reaches 78% (at saturation) and the fidelity, 94%. Our Eq.(32) and the above reflectivity lead to H = 0.98.

Using the numerical calculated data from Fig.7 of [29], denoted by R_{sim} and H_{sim}, respectively, we have checked again our analytical relation between SBS reflectivity and fidelity from (32). The predictions for the fidelity are denoted by H_t and are shown in Table 3.

Table 3.

$\in_p / \in_{pth}$	2	3	6	8	10	15	20	30
R_{sim}	0.35	0.5	0.7	0.75	0.8	0.86	0.88	0.95
H_{sim}	0.77	0.85	0.92	0.94	0.95	0.97	0.97	0.98
H_t	0.77	0.84	0.96	0.98	0.99	0.99	0.99	0.99

One could remark that the agreement of our prediction for fidelity (Eq. 32) holds well with the numerical calculations even in the non-stationary SBS, for Gaussian pump waves.

3. Analytical solution of the 3D SBS model with a spatial axial symmetric pump

In order to emphasise the transverse effects in SBS, we can write the wave equation, which describes the evolution of the electrical field of the light beam inside of the nonlinear medium, as:

$$\left(\frac{n}{c}\right)^2 \frac{\partial^2 \vec{E}}{\partial t^2} - \frac{\partial^2 \vec{E}}{\partial x^2} - \frac{\partial^2 \vec{E}}{\partial y^2} - \frac{\partial^2 \vec{E}}{\partial z^2} + \frac{n}{c}\frac{\partial}{\partial t}\left(\alpha \cdot \vec{E} + \frac{4\pi}{nc}\frac{\partial \vec{P}^{NL}}{\partial t}\right) = 0 , \qquad (33)$$

where n is the refraction index, c – the speed of light in vacuum, $\vec{E}$ - the amplitude of the optical field, α - the linear optical losses and $\vec{P}^{NL}$ - the nonlinear polarization of the medium. The initial conditions for the electrical field and its derivative are:

$$\vec{E}(t; x, y, z)\big|_{t=0} = 0$$

$$\frac{\partial \vec{E}}{\partial t}(t; x, y, z)\big|_{t=0} = 0 \cdot \qquad (34)$$

We can define the function:

$$F(\vec{r};t)=\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}-\left(\frac{n}{c}\right)\cdot\frac{\partial}{\partial t}\left[\alpha\cdot E+\left(\frac{4\pi}{nc}\right)\frac{\partial P^{NL}}{\partial t}\right]. \quad (35)$$

For regulated distributions, the general solution of Eq. (33) with initial conditions (34) can be written in the form:

$$E(x,y,z,t)=\frac{c}{n}\int_0^t d\tau\cdot\int_{z-\frac{c}{n}(t-\tau)}^{z+\frac{c}{n}(t-\tau)} F(x,y,\xi,\tau)d\xi \quad (36)$$

with: $$F(x,y,z,t)=-\left(\frac{n}{c}\right)\cdot\frac{\partial}{\partial t}\left[\alpha\cdot E+\left(\frac{4\pi}{nc}\right)\frac{\partial P^{NL}}{\partial t}-\frac{c}{n}\cdot\int dt\left\{\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}\right\}\right]. \quad (37)$$

We can introduce the "longitudinal characteristics" (coordinates) of the scattering process as:

$$\xi_L=\left(\frac{c}{n}\right)\cdot t+z;\quad \xi_S=\left(\frac{c}{n}\right)\cdot t-z, \quad (38)$$

with the meaning of the directions along which the pump and the Stokes fields are propagating, respectively. With transformations (38), the solution (36) becomes:

$$E(x,y;\xi_L,\xi_S)=\frac{c}{2n}\cdot\int_0^{\frac{n}{2c}(\xi_L+\xi_S)} d\tau\cdot\int_{-\xi_S+\frac{c\tau}{n}}^{\xi_L-\frac{c\tau}{n}} F(x,y;\xi,\tau)d\xi \quad (39)$$

The derivatives of the light electrical field along the pump and scattered (Stokes) characteristics (ξ_L,ξ_S) are respectively:

$$\frac{\partial E_L(x,y,\xi_L,\xi_S)}{\partial\xi_L}=\frac{c}{2n}\cdot\int_0^{\frac{n}{2c}(\xi_L+\xi_S)} F\left[\left(\xi_L-\frac{c\tau}{n}\right),\tau\right]d\tau \quad \frac{\partial E_S(x,y,\xi_L,\xi_S)}{\partial\xi_S}=\frac{c}{2n}\cdot\int_0^{\frac{n}{2c}(\xi_L+\xi_S)} F\left[\left(-\xi_S+\frac{c\tau}{n}\right),\tau\right]d\tau \quad (40)$$

and can be developed in the following manner:

$$\frac{\partial E_L}{\partial\xi_L}=-\frac{\alpha}{4}\cdot E_L-\frac{\pi}{n^2}\cdot\frac{\partial}{\partial\xi_L}P^{NL}+\frac{D_L}{\pi}\cdot\xi_L\hat{\Delta}_\perp E_L$$

$$\frac{\partial E_S}{\partial\xi_S}=-\frac{\alpha}{4}\cdot E_S-\frac{\pi}{n^2}\cdot\frac{\partial}{\partial\xi_S}P^{NL}+\frac{D_S}{\pi}\cdot\xi_S\hat{\Delta}_\perp E_S, \quad (41)$$

where $\hat{\Delta}_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the transverse Laplace operator and $(D_L,D_S)$are the integration constants. Eqs. (41) are obtained from the wave equations (hyperbolic type ones) by integration of the solution along the characteristics. They may be thought as deriving from Fokker-Planck type equations by a special averaging. For Gaussian processes, one can take $D_L=D_S$.

In order to simplify the SBS equations, we can normalize the amplitude of the pump, Stokes and acoustic fields as:

$$E_L^{'}=\sqrt{\frac{cn}{8\pi I_0}}\cdot E_L,\quad E_S^{'}=\sqrt{\frac{cn}{8\pi I_0}}\cdot E_S,\quad E_{ac}^{'}=\frac{\pi\gamma^e}{n^2}\cdot\left(\frac{\Delta\rho}{\rho_0}\right) \quad (42)$$

with I_0 – the maximum pump intensity, γ^e – the electrostrictive coefficient, ρ_0 - the density of the propagation medium and $\Delta\rho$ - the density variation due to the interaction with the laser (optical) field. Furthermore, we introduce the phases of the pump, Stokes and acoustic fields:

$$\varphi_L=\omega_L t+k_L\cdot z;\quad \varphi_S=\omega_S t-k_S\cdot z \text{ and } \varphi=\omega t+k\cdot z, \quad (43)$$

with ω_i - the angular frequency of the corresponding waves and k_i – the corresponding wave vectors ($i = L,S$).

The general SBS equation system, including transverse effects, takes the form along the three characteristics ($\varphi_L,\varphi_S,\varphi$):

$$\frac{\partial E_L^{'}}{\partial \varphi_L} = -\frac{\alpha'}{2}\cdot E_L^{'} - \frac{\partial}{\partial \varphi_L}\left(E_S^{'}\cdot E_{ac}^{'}\right) + D_{L\cdot}\cdot\varphi_L\cdot\hat{\Delta}_\perp\left(E_L^{'}\right)$$
$$\frac{\partial E_S^{'}}{\partial \varphi_S} = -\frac{\alpha'}{2}\cdot E_S^{'} + \frac{\partial}{\partial \varphi_S}\left(E_L^{'}\cdot E_{ac}^{'}\right) + D_{L\cdot}\cdot\varphi_S\cdot\hat{\Delta}_\perp\left(E_S^{'}\right) \tag{44}$$
$$\frac{\partial E_{ac}^{'}}{\partial \varphi} = -2A\cdot E_{ac}^{'} + g_{Bn}\cdot(E_L^{'}\cdot E_S^{'}) + D_L\cdot\varphi\cdot\hat{\Delta}_\perp\left(E_{ac}^{'}\right),$$

where $\alpha' = \alpha/2k_L = \alpha/2k_S = \alpha/k$ are the normalized losses, $A = 2\omega/\Gamma_B$ is the gain of the acoustic field, $g_{Bn} = g_B^e\cdot L_B\cdot I_0$ is the normalized Brillouin gain, $g_B^e = \dfrac{\omega_L^2\cdot\left(\gamma^e\right)^2}{c^3\cdot n\cdot\rho_0\cdot \mathrm{v}\cdot\Gamma_B}$ is the Brillouin gain, $L_B = 4\pi\mathrm{v}/\omega$ - the minimum interaction length and v - the hypersound velocity. The third equation of system (44) describes the evolution of the acoustical field $E_{ac}{'}$ along the characteristic line φ. It is deduced in the same way as the equations for the optical fields $E_L{'}$, $E_S{'}$, but from the Navier-Stokes equation.

Using the method developed by Yariv [2], we can project the characteristics $(d\varphi_{L,S})$ on the characteristic $(d\varphi)$:

$$d\varphi_L \approx \left(\frac{\omega_L}{\omega}\right)d\varphi,\quad d\varphi_S \approx \left(\frac{\omega_S}{\omega}\right)d\varphi. \tag{45}$$

We shall consider the SBS geometry with *cylindrical (axial) symmetry*, when the transverse Laplace operator becomes: $\hat{\Delta}_\perp = \partial^2/\partial r^2 + \frac{1}{2}\frac{\partial}{\partial r}$, where by r is denoted the normalized, radial coordinate. We introduce the absorption in the fields as:

$$E_L^{'} = E_L^{''}\cdot e^{-\alpha''\cdot\varphi} \text{ and } E_S^{'} = E_S^{''}\cdot e^{-\alpha''\varphi}, \tag{46}$$

in order to simplify the equation writing to:

$$\frac{\partial E_L^{''}}{\partial\varphi} = (2A+\alpha'')E_{ac}^{'}E_s^{''} - g_{Bn}\cdot\cdot e^{-2\alpha''\varphi}\,E_L^{''}E_S^{''2} - \left(E_S^{''} + E_{ac}^{'}E_L^{''}\right)D_L\cdot\varphi\frac{\partial^2 E_{ac}^{'}}{\partial r^2} + $$
$$+ D_L\cdot\varphi\frac{\partial^2 E_{ac}^{'}}{\partial r^2} - E_{ac}^{'}\cdot D_L\cdot\varphi\frac{\partial^2 E_L^{''}}{\partial r^2}$$
$$\frac{\partial E_S^{''}}{\partial\varphi} = -(2A+\alpha'')E_{ac}^{'}E_L^{''} + g_{Bn}\cdot e^{-2\alpha''\varphi}E_L^{''2}E_S^{''} - \left(E_{ac}^{'}E_S^{''} - E_L^{''}\right)D_L\varphi\frac{\partial^2 E_{ac}^{'}}{\partial r^2} + \tag{47}$$
$$+ D_L\varphi\frac{\partial^2 E_S^{''}}{\partial r^2} + E_{ac}^{'}D_L\varphi\frac{\partial^2 E_L^{''}}{\partial r^2}$$
$$\frac{\partial E_{ac}^{'}}{\partial\varphi} = -2AE_{ac}^{'} + g_{Bn}e^{-2\alpha''\varphi}\,E_L^{''}E_S^{''} + D_L\varphi\frac{\partial^2 E_{ac}^{'}}{\partial r^2},$$

where $\alpha'' \approx \alpha'(\omega_L/2\omega)$. For weak acoustical field, $E_{ac}^{'} << 1$, the SBS equation system (47) can be written in the form :

$$\frac{\partial E_L^{''}(\varphi,r)}{\partial\varphi} = D_L\cdot\varphi\frac{\partial^2 E_L^{''}(\varphi,r)}{\partial r^2} - \left[g_{Bn}\cdot e^{-2\alpha''\varphi}\right]\cdot E_L^{''}\cdot E_S^{''2}$$
$$\frac{\partial E_S^{''}(\varphi,r)}{\partial\varphi} = D_L\cdot\varphi\cdot\frac{\partial^2 E_S^{''}(\varphi,r)}{\partial r^2} + \left[g_{Bn}\cdot e^{-2\alpha''\varphi}\right]\cdot E_L^{''2}\cdot E_S^{''}. \tag{48}$$

Integrating Eqs. (48) along the acoustical characteristic direction $\omega_S t - k_S z = 0$, one can obtain the equations of evolution for the transversal pump and Stokes field components (denoted $E_{L_T}^{''}$ and $E_{S_T}^{''}$) :

$$D_L \cdot \varphi \cdot \frac{\partial^2 E''_{L_T}}{\partial r^2} = +g_{Bn} \cdot e^{-2\alpha''\varphi} \cdot E''^{\,2}_{S_T} \cdot E''_{L_T}$$

$$D_L \cdot \varphi \cdot \frac{\partial^2 E''_{S_T}}{\partial r^2} = -g_{Bn} \cdot e^{-2\alpha''\varphi} \cdot E''^{\,2}_{L_T} \cdot E''_{S_T} \tag{49}$$

or, more compactly:

$$\frac{\partial^2 E''_{L_T}}{\partial r^2} = +S \cdot E''^{\,2}_{S_T} \cdot E''_{L_T}$$

$$\frac{\partial^2 E''_{S_T}}{\partial r^2} = -S \cdot E''^{\,2}_{L_T} \cdot E''_{S_T}\ , \tag{50}$$

where $S = g_{Bn} \cdot \left[\exp\left(-\alpha \frac{\omega_L \cdot}{k \cdot \omega} \varphi \right) \right] / \varphi$.

The integration along the longitudinal characteristics (φ_L, φ_S) leads to the decoupling of the longitudinal derivative in (48). The equations (49) and (50) have as boundary conditions the longitudinal solutions, found by us in the 1D SBS model [5]. Considering the transversal effects as a "correction" of the longitudinal ones, as was also done by Menzel and Eichler [15], we could obtain the transversal solutions of the SBS process.

This equation system can be reduced to the differential equation system of the first order:

$$\frac{\partial E''_{L_T}}{\partial r} = \sqrt[4]{-S \frac{c_1}{c_2}} E''^{\frac{1-c_2}{2}}_{L_T} E''^{\frac{1+c_1}{2}}_{S_T}$$

$$\frac{\partial E''_{S_T}}{\partial r} = \sqrt[4]{-S \frac{c_2}{c_1}} E''^{\frac{1+c_2}{2}}_{L_T} E''^{\frac{1-c_1}{2}}_{S_T}\ , \tag{51}$$

where c_1 and c_2 are the prime integrals of the system (50), which describe the strength of the nonlinear coupling between the two fields. The selection of these prime integrals could provide the general solutions for the axial-symmetric pump and Stokes waves.

Particularly, taking a weak coupling of the fields by the selection of the first integer numbers for the prime integrals:

$$c_1 = -1; \quad c_2 = +1, \tag{52}$$

Eqs. (51) become:

$$\frac{\partial E''_{L_T}}{\partial r} = -\sqrt[4]{S}$$

$$\frac{\partial E''_{S_T}}{\partial r} = +\sqrt[4]{S} \cdot E''_{L_T} \cdot E''_{S_T} \tag{53}$$

which have as parameter the Brillouin gain, S , only.

Taking into account the correction of the axial losses, we can find the wave intensities:

$$I_{L_T}(r,z,t) = \left| \sqrt{I_{L_0}(t)} - \sqrt[4]{S \cdot I_0^2} \cdot r \right|^2 \cdot exp\left[-\alpha \cdot \frac{\omega_L}{k \cdot \omega} (\omega t + kz) \right]; \qquad r \le \frac{\sqrt{I_{L_0}/I_0}}{\sqrt[4]{S}};$$

$$I_{S_T}(r,z,t) = I_{S_0} \cdot exp\left[\frac{I_{L_0}(t)}{2I_0} - \left(\frac{r - \sqrt{I_{L_0}(t)} / \sqrt[4]{S \cdot I_0^2}}{\sqrt{2} / \sqrt[4]{S}} \right)^2 \right] \cdot exp\left[-\alpha \cdot \frac{\omega_L}{k \cdot \omega} (\omega t + kz) \right]. \tag{54}$$

We can remark that the solution (54) is valid for any temporal pump distribution, $I_{L_0}(t)$ and the selected integration constants could describe the non-stationary (small reflectivity) Gaussian case only.

In order to check the solution (54), we can calculate again the SBS reflectivity:

$$R = (\in_S / \in_L) = f(\in_L)\ , \tag{55}$$

where $\in_{L,S}$ are the energies of the pump (subscript L) and Stokes pulse (subscript S), respectively. Assuming that there is no laser pulse compression, the SBS reflectivity

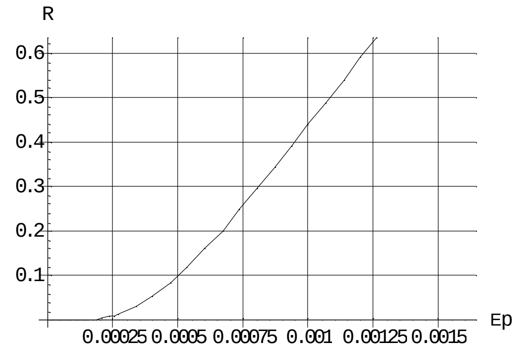

Fig. 4. The SBS reflectivity vs. normalized pump energy, in the small signal regime, for the simple axial-symmetric pump beam from Eq.(54) and $R_0 = 1$.

at small pump intensity, deduced from the solutions (54), is:

$$R_{trans}(\in_L) \approx R_0 \frac{erf\left[0.37 \in_L^{-1/4}\right]}{\in_L^{1/2}} \approx \frac{2}{\pi} R_0 \frac{\exp\left[0.139 \in_L^{-1/2}\right]}{\in_L^{1/2}} , \tag{56}$$

when

$$0.37 \in_L^{-1/4} << 1. \tag{57}$$

This small signal reflectivity is shown in Fig.4, as a function of the pump energy and is in agreement with the previous results with Gaussian beams and with the experimental data.

4. Conclusions

Analytical three-dimensional SBS models were presented and developed. These models are important for the SBS geometry optimisation and the study of the transversal effects (including the phase conjugation fidelity of the Stokes wave).

Analytical solutions were found, which provide important control parameters for the SBS process and are easier to be used in comparison with the numerical results. The consistency of the analytical solutions obtained for Gaussian non-depleted and depleted pump waves with those obtained for more general pump waves and with the numerical results was ensured. The analytical results fit well the present experimental results.

In order to check the theoretical models, we have also calculated the SBS reflectivity and phase conjugation fidelity. In the practical cases, these important parameters can be written as simple functions of the pump energy. Moreover, we have proved that they can be related in a manner, which is well supported by the numerical simulation and experimental data.

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