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IDENTIFICATION OF NONLINEARTIES IN ANELASTIC POLYCRYSTALLINE MATERIALS USING VOLTERRA - FOURIER TRANSFORM

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A nonlinear rheological model built on the basis of structural properties of polycrystalline solids is presented. A Fourier - Volterra series method of model parameters identification from the experimental data is proposed.

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1. Introduction

For many applications the polycrystalline solids are satisfactorily described by linear constitutive equations [1]. On the other hand, the relaxation of the residual stress existing in the mechanically soldered or casted metallic structures can be facilitated by vibrations applied to the mechanical structure. Linear models are not able to explain such phenomena, due to the fact that the superposition principle states that the relaxation response and the response to vibrations of the solid are independent in the case of a linear anelastic solid. Several nonlinear rheologic models and phenomena where already published [2,3].

In the last years, a highly sensitive technique based on the Fourier transform was introduced in order to permit high quality investigations of the experimental data [4]. In case of our nonlinear constitutive equation, elegant solutions, with good simply physical significance, can be obtained by Volterra - Fourier series [5,6]. For this reason we will generalize the standard Fourier transform rheology method.

In this paper a general nonlinear rheological model explaining the relaxation facilitated by vibrations is proposed; a method based on the Volterra - Fourier transform for model parameter identification from experimental data is discussed.

2. The nonlinear model

From the structural studies on the polycrystalline solids it results that if a constant stress is applied on the grain boundaries, the rate of the relaxation process is [7]:

$$v = \frac{1}{\tau} = \frac{\alpha\sigma + \beta}{\tau_0} \tag{1}$$

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where τ is the nonlinear relaxation constant, α and β are constants of material, and τ_0 is the time constant in the linear model approximation. From the experiments results that α is a small parameter, and $\beta \approx 1$.

If in the standard linear anelastic solid is considered that τ is given by (1), will result the nonlinear constitutive equation [2]:

$$\frac{d}{dt}(\sigma - E_r\varepsilon) + \frac{1}{\tau_0}[\alpha(\sigma - E_r\varepsilon) + \beta](\sigma - E_r\varepsilon) = \delta E\frac{du}{dt} \tag{2}$$

If the new variable

$$x = \sigma - E_r\varepsilon \qquad\qquad x(0) = \delta E\varepsilon_0$$

is introduced, the constitutive equation (2) takes the form:

$$\frac{dx}{dt} + \frac{\beta}{\tau_0}x + \frac{\alpha}{\tau_0}x^2 = \delta E\frac{du}{dt} \quad . \tag{3}$$

This model can be written in a more general form:

$$\frac{dx}{dt} + \frac{\beta}{\tau_0}x + \frac{1}{\tau_0}\sum_{k=2}^{n}\alpha_k x^k = \delta E\frac{du}{dt} \tag{4}$$

which represents a nonlinear model with memory.

3. Volterra series method

There are methods expressing the response of the nonlinear systems in terms of generalized transfer functions. For a nonlinear system described by equations of type (4) the solution $x(t)$ can be expressed in terms of Volterra series as [5,6]:

$$x(t) = \int_{-\infty}^{\infty} h_1(\tau_1)u(\tau_1 - t)d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)u(\tau_1 - t)u(\tau_2 - t)d\tau_1 d\tau_2 + \ldots$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} h_n(\tau_1,\tau_2,\ldots\tau_n)u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_n)d\tau_1 d\tau_2\ldots d\tau_n + \ldots \tag{5}$$

where $h_n(\tau_1, \tau_2, \ldots, \tau_n)$ is the n^{th} order ($n = 1, 2, 3, \ldots$) transfer function, and u represents the input excitation. In our case $u(t) = \delta E\, d\varepsilon(t)/dt$. The form of this kind of nonlinear transfer function depend on the due form of excitation function $u(t)$.

The n-dimensional Fourier transform of the n^{th} order transfer function is:

$$H_n(\omega_1, \omega_2, \ldots, \omega_n) =$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} h_n(\tau_1,\tau_2,\ldots\tau_n)\exp[-i(\omega_1\tau_1 + \omega_2\tau_2 + \ldots\omega_n\tau_n)]d\tau_1 d\tau_2\ldots d\tau_n \tag{6}$$

and by inverse transform results:

$$h_n(\tau_1, \tau_2, \ldots, \tau_n) =$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} H_n(\omega_1,\omega_2,\ldots\omega_n)\exp[i(\omega_1\tau_1 + \omega_2\tau_2 + \ldots\omega_n\tau_n)]d\omega_1 d\omega_2\ldots d\omega_n \tag{7}$$

We consider in the following that the excitation is of harmonic type ε = A exp($i\ \omega t$), where A is the amplitude of the strain and ω is the angular frequency.

By introducing eq. (5) in eq. (3) one may calculate different orders of the transfer function for harmonic excitation. In our case the response will have the form [5]:

$$x(t) = \sum_{n=1}^{\infty} (i\tau_0 \omega \delta E)^n H_n(\omega_1, \omega_2, \ldots, \omega_n) \exp(in\omega t) = \sum_{n=1}^{\infty} \xi_n \exp(in\omega t) \qquad (8)$$

where $\omega_1 = \ldots = \omega_n = \omega$. This solution represents a superposition of harmonics. The amplitudes ξ_n can be found from the constitutive equation (4).

Using the notation $F(\omega) = i\ \omega + \beta / \tau_0$ the following results are obtained:

$$\begin{aligned} \xi_1 &= \frac{1}{F(\omega)} \\ \xi_2 &= -\frac{1}{F(2\omega)} \frac{2\alpha_2}{\tau_0} \xi_1^2 \\ \xi_3 &= -\frac{1}{F(3\omega)} \left(6\frac{\alpha_2}{\tau_0} \xi_1 \xi_2 + 6\frac{\alpha_3}{\tau_0} \xi_1{}^2\right) \\ &\ldots \\ \xi_n &= -\frac{1}{F(n\omega)} \sum_{k=2}^{n} \alpha_k H(\omega, \omega, \ldots, \omega) \end{aligned} \qquad (9)$$

In the last equation $H_n^{(k)}$ (where $0 \le k \le n$) represents the modified transfer function which correspond to the y^k response [5]:

$$H^{(k)}{}_n(\omega_1, \omega_2, \ldots, \omega_n) =$$

$$= k! \sum_{(\nu,k,n)} \sum_N{}' H_{\nu_1}(\omega_1, \omega_2, \ldots, \omega_{\nu_1}) H_{\nu_2}(\omega_{\nu_1+1}, \omega_{\nu_1+2}, \ldots, \omega_{\nu_2}) \ldots H_{\nu_k}(\omega_\mu, \omega_2, \ldots, \omega_{\nu_n}) \qquad (10)$$

where $\omega_1 = \ldots = \omega_n = \omega$ and the notations:

$$\nu = \nu_1 + \nu_2 + \ldots + \nu_{k-1} + 1 = n - \nu_k + 1,$$

were used; (ν, k, n) indicates that the summation will be made for all integer values ν_j so that

$$\nu_1 + \nu_2 + \ldots + \nu_k = n, \qquad 1 \le \nu_1 \le \nu_2 \le \ldots \le \nu_k. \qquad (11)$$

The different amplitudes ξ_i and, finally the parameters of the rheologic model may be obtained from experimental data. Σ'_N indicates that the summation will be repeted N times, corresponding to all non-identical products which can be obtained by permutation of the index j of ω_j. The number of terms is:

$$N_0 = \frac{n!}{\nu_1! \nu_2! \ldots \nu_k! r_1! r_2! \ldots r_k!} \qquad (12)$$

where r_1 is the number of the first number of index equaly, and so on.

The Volterra - Fourier series above defined (8)-(9), represents a generalization of the standard Fourier transform rheology [4], applied to the linear case. If a strain ε = A exp($i\omega t$) is imposed by forced vibrations to a sample and the stress response $\sigma(t)$ is recorded, from its Fourier transform the different harmonics may be obtained and if compared with eqs. (8) and (9) the parameters $\alpha_2, ..., \alpha_m$ of the model result.

4. Conclusions

The phenomena of stress relaxation in the cast and solded metallic structures can be facilitated by external shocks and vibrations. These effects cannot be explained by linear anelastic models of solids. In this paper we present a general nonlinear model for description of these phenomena.

This simply model is in a good accordance with the known experimental data, i. e. nonlinearities have an influence only on the exponentially decaying terms.

As a generalization of the Fourier transform rheology we introduce the Volterra - Fourier series in order to express the response of this non-linear model. There was introduced the nonlinear $h_n(\tau_1, ..., \tau_n)$ transfer functions. The case of harmonic excitation was considered.

Finally, a method is proposed for the identification of the parameters of the constitutive equation from experimental data.

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