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RADIATION POWER SPECTRAL DISTRIBUTION OF CHARGED PARTICLES MOVING IN A SPIRAL IN MAGNETIC FIELDS

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The expressions for the momentary and average radiation powers of the charged particles moving on an arbitrary determined trajectory in transparent isotropic media and in vacuum are studied by using the Lorentz's self-interaction method. Special attention is given to the research of the fine structure of the synchrotron radiation spectral distribution of two electrons moving in a spiral in vacuum. The spectra of synchrotron, Cherenkov and synchrotron-Cherenkov radiations for a separate electron are analyzed.

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1. Introduction

The investigation of the radiation spectra of charged particles moving in magnetic fields in transparent isotropic medium and in vacuum is important from the point of view of the applications in electronics, astrophysics, plasma physics, physics of storage rings etc. [1-3].

Under moving charged particles in magnetic field three kinds of radiation are possible in a medium [4-6]: synchrotron, Cherenkov, and synchrotron-Cherenkov ones whereas in vacuum only synchrotron radiation takes place.

A question requiring further investigations is the coherence of synchrotron radiation [1-3]. A laser radiation is emitted when an electron beam moves through a spiral undulator [7]. The properties of free-electron lasers were considered in papers [8-10]. The investigations of the fine structure of synchrotron, Cherenkov, and synchrotron-Cherenkov radiation spectra in vacuum and in transparent media for the low-frequency spectral range is of great interest [11]. The fine structure of the Cherenkov radiation spectrum in non-transparent media was investigated in papers [12-13].

The aim of this paper is to investigate the spectral distribution of the radiation power for the charged particles moving along an arbitrary defined trajectory using the Lorentz`s self-interaction method. Using the exact integral relationships for the spectral distribution of radiation power of two electrons moving one after another along a spiral in vacuum, the fine structure of the synchrotron radiation spectrum was investigated by means of analytical and numerical methods. The influence of the Doppler effect on the peculiarities of the radiation spectrum of a separate electron during its motion in a spiral in transparent media and in vacuum is also investigated.

2. Instantaneous and time-averaged radiation power of charged particles

The instantaneous radiation power of charged particles $P^{rad}(t)$ in an isotropic transparent medium and in vacuum is expressed in [14,15] as

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$$P^{rad}(t)=\int_{\tau}\left(\vec{j}(\vec{r},t)\frac{1}{c}\frac{\partial\vec{A}^{Dir}(\vec{r},t)}{\partial t}-\rho(\vec{r},t)\frac{\partial\varphi^{Dir}(\vec{r},t)}{\partial t}\right)d\vec{r}\,. \quad (1)$$

Here $\vec{j}(\vec{r},t)$ is the current density and $\rho(\vec{r},t)$ is the charge density. The integration is over some volume τ. According to the hypothesis of Dirac [14–18], the scalar $\varphi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials:

$$\varphi^{Dir} = \frac{1}{2}\left(\varphi^{ret}-\varphi^{adv}\right), \qquad \vec{A}^{Dir} = \frac{1}{2}\left(\vec{A}^{ret}-\vec{A}^{adv}\right). \quad (2)$$

After substituting (2) into (1) we obtain the relationship for instantaneous radiation power of charged particles moving in isotropic transparent media as a function of spectral distribution

$$P^{rad}(t)=\int_0^{\infty}d\omega W(t,\omega), \quad (3)$$

$$W(t,\omega)=\frac{1}{\pi c^2}\int_{-\infty}^{\infty}d\vec{r}\int_{-\infty}^{\infty}d\vec{r}\,'\int_{-\infty}^{\infty}dt'\omega\mu(\omega)\frac{\sin\left[\frac{n(\omega)\omega}{c}\left|\vec{r}-\vec{r}\,'\right|\right]}{\left|\vec{r}-\vec{r}\,'\right|}\cos\omega(t-t')\times$$
$$\times\left\{\vec{j}(\vec{r},t)\vec{j}(\vec{r}\,',t')-\frac{c^2}{n^2(\omega)}\rho(\vec{r},t)\rho(\vec{r}\,',t')\right\}, \quad (4)$$

where $\mu(\omega)$ is the magnetic permeability, $n(\omega)$ is the refraction index, ω is the angular frequency, and c is the speed of light in vacuum.

The time-averaged radiation power of charged particles is defined by the expression

$$\overline{P}^{rad} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P^{rad}(t)dt\,. \quad (5)$$

The values of $\overline{P}^{rad}$ can be obtained after substitution of the instantaneous radiation power expressed by relationships (3) and (4) into (5).

3. Systems of non-interacting point charged particles

Let us consider a system of point-like non-interacting particles with charges q_1, q_2,..., q_N and rest masses m_{01}, m_{02}, ..., m_{0N} moving along arbitrary trajectories. Then the source functions of N charged point-like particles are defined as [5,15]

$$\vec{j}(\vec{r},t)=\sum_{l=1}^{N}\vec{V}_l(t)\rho_l(\vec{r},t), \qquad \rho(\vec{r},t)=\sum_{l=1}^{N}\rho_l(\vec{r},t), \qquad \rho_l(\vec{r},t)=q_l\,\delta(\vec{r}-\vec{r}_l(t)), \quad (6)$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} particle, respectively.

By substituting relationships (6) into (3) and (4) we obtain the expression for the instantaneous radiation power of charged particles system in transparent media (magnetic permeabilities, $\mu(\omega)$, and dielectric permittivities $\varepsilon(\omega)$, are real):

$$P^{rad}(t)=\frac{1}{\pi c^2}\int_0^{\infty}d\omega\omega\mu(\omega)\int_{-\infty}^{\infty}dt'\sum_{l,j=1}^{N}q_l q_j\frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_l(t)-\vec{r}_j(t')\right|\right\}}{\left|\vec{r}_l(t)-\vec{r}_j(t')\right|}\times$$

$$\times \cos \omega(t-t')\left\{\vec{V}_l(t)\vec{V}_j(t')-\frac{c^2}{n^2(\omega)}\right\}. \tag{7}$$

The time-averaged radiation power can be obtained from the expression

$$\bar{P}^{rad} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} P^{rad}(t)dt = \frac{1}{\pi c^2}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\int_0^{\infty}d\omega\omega\mu(\omega)\int_{-\infty}^{\infty}dt'\times$$
$$\times\sum_{l,j=1}^{N} q_l q_j \frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_l(t)-\vec{r}_j(t')\right|\right\}}{\left|\vec{r}_l(t)-\vec{r}_j(t')\right|}\cos\omega(t-t')\left\{\vec{V}_l(t)\vec{V}_j(t')-\frac{c^2}{n^2(\omega)}\right\}. \tag{8}$$

Let us consider a system of point-like non-interacting charged particles ($q_l = n_l e$, $m_{0l} = n_l m_0$) moving one by one along an arbitrary defined trajectory. Then the motion law and the velocity of the l^{th} particle of this system are determined by the relationships

$$\vec{r}_j(t) = \vec{r}_p(t+\Delta t_j), \qquad \vec{V}_j(t) = \vec{V}(t+\Delta t_j). \tag{9}$$

Relationships (9) were obtained taking into account that the magnitudes $m_{0l} = n_l m_0$ and $q_l = n_l e$ are linearly included into the movement equation of a charged particle in electromagnetic field. In this case we obtain the averaged radiation power after substitution the expressions (9) into (8):

$$\bar{P}^{rad} = \frac{e^2}{\pi c^2}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\int_{-\infty}^{\infty}dt'\int_0^{\infty}d\omega\mu(\omega)\omega S_N(\omega)\times$$
$$\times\frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_p(t)-\vec{r}_p(t')\right|\right\}}{\left|\vec{r}_p(t)-\vec{r}_p(t')\right|}\cos\omega(t-t')\left[\vec{V}(t)\vec{V}(t')-\frac{c^2}{n^2(\omega)}\right], \tag{10}$$

where the coherence factor $S_N(\omega)$ is defined as

$$S_N(\omega) = \sum_{l,j=1}^{N} n_l n_j \cos\{\omega(\Delta t_l - \Delta t_j)\}. \tag{11}$$

Relationships (10) and (11) obtained here may be also applied to low-length bunchs of charged particles.

For N electrons ($q_l = e$, $m_{0l} = m_0$) the coherence factor takes the form [6,11]:

$$S_N(\omega) = \sum_{l,j=1}^{N} \cos\{\omega(\Delta t_l - \Delta t_j)\}. \tag{12}$$

The coherence factor $S_N(\omega)$ determines a redistribution of the charged particles radiation power between harmonics.

4. Fine structure of the radiation spectra of two electrons moving along a spiral in vacuum

Peculiarities of the radiation spectra of two electrons moving one by one in a spiral in vacuum can be investigated combining analytical and numerical methods. The law of motion and the velocity of the electron are given by the expressions:

$$\vec{r}_j(t)=r_0\cos\{\omega_0(t+\Delta t_j)\}\vec{i}+r_0\sin\{\omega_0(t+\Delta t_j)\}\vec{j}+V_\parallel(t+\Delta t_j)\vec{k}, \qquad \vec{V}_j(t)=\frac{d\vec{r}_j(t)}{dt}. \tag{13}$$

Here $r_0=V_\perp\omega_0^{-1}$, $\omega_0=ceH^{ext}\tilde{E}^{-1}$, $\tilde{E}=c\sqrt{p^2+m_0^2c^2}$, the magnetic intensity vector $\vec{H}^{ext}\|0Z$, $V_\perp$ and $V_\parallel$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass.

After substitution of the expressions (13) into (10) one obtaines the time-averaged radiation power of two electrons

$$\overline{P}^{rad}=\int_0^\infty W(\omega)d\omega, \tag{14}$$

$$W(\omega)=\frac{2e^2}{\pi c^2}\int_0^\infty dx\omega S_2(\omega)\frac{\sin\left\{\frac{1}{c}\omega\eta(x)\right\}}{\eta(x)}\cos\omega x\left[V_\perp^2\cos(\omega_0x)+V_\parallel^2-c^2\right], \tag{15}$$

where $\eta(x)=\sqrt{V_\parallel^2x^2+4\frac{V_\perp^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

The coherence factor $S_2(\omega)$ of two electrons is defined as

$$S_2(\omega)=2+2\cos(\omega\Delta t). \tag{16}$$

Here $\Delta t=\Delta t_2-\Delta t_1$ is the time shift of the electrons moving along a spiral.

At the frequencies $\omega=2i\pi/(\Delta t)$ (i=0, 1, 2,…) the coherence factor of two electrons (16) is equal to 4 and at the frequencies $\omega=(2i+1)\pi/(\Delta t)$ (i=0, 1, 2,…) the coherence factor is equal to zero. The analogous expression for the coherence factor was investigated by Bolotovskii [19].

From the relationships (14) and (15), after some transformations, the contributions of separate harmonics to the averaged radiation power can be written as

$$\overline{P}^{rad}=\frac{e^2}{c^3}\sum_{m=1}^\infty\int_0^\infty d\omega\omega^2\int_0^\pi\sin\theta d\theta 2\left[1+\cos\{\omega(\Delta t)\}\right]\times$$
$$\times\delta\left\{\omega\left(1-\frac{1}{c}V_\parallel\cos\theta\right)-m\omega_0\right\}\left\{V_\perp^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m'^2(q)\right]+\left(V_\parallel^2-c^2\right)J_m^2(q)\right\}, \tag{17}$$

where $q=\frac{V_\perp}{c}\frac{\omega}{\omega_0}\sin\theta$, $J_m(q)$ and $J'_m(q)$ are the Bessel function with integer index and its derivative, respectively.

Each harmonic is a set of the frequencies, which are the solution of the equation

$$\omega\left(1-\frac{1}{c}V_\parallel\cos\theta\right)-m\omega_0=0. \tag{18}$$

The limits of the m^{th} harmonic are determined by the frequencies

$$\omega_m^{\min}=\frac{m\omega_0}{1+\frac{V_\parallel}{c}}, \qquad \omega_m^{\max}=\frac{m\omega_0}{1-\frac{V_\parallel}{c}}, \tag{19}$$

and the total radiation power emitted by a separate electron is determined according to [20] as

$$P_m^{tot} = \frac{2}{3}\frac{e^2}{c^3}\frac{\omega_0^2 V_\perp^2}{\left(1-\frac{V^2}{c^2}\right)^2}, \tag{20}$$

where $\omega_0 = \frac{eH^{ext}}{m_0 c}\sqrt{1-\frac{V^2}{c^2}}$.

Our numerical calculations of the radiation power spectral distribution were performed at $H^{ext} = 1\,\text{Oe}$.

For the velocities components $V_{\perp vac} = 0.2\times10^{10}$ cm/s and $V_{\parallel vac} = 0.2\times10^{11}$ cm/s the radiation power spectral distributions of two electrons in vacuum depending on their location along a spiral are shown in Figs. 1–3 (curves 1–6).

It is interesting to compare the radiation power spectral distribution for two electrons with the radiation power spectral distribution of a separate electron (curve 0 in Fig. 1). The radiation power of the separate electron in vacuum $P_{vac0}^{tot} = 0.1283\times10^{-16}$ erg/s calculated according to relationship (20) is in good agreement to the power $P_{vac0}^{int} = 0.1294\times10^{-16}$ erg/s determined after integration of relationships (14) and (15). For a separate electron the coherence factor $S_1 = 1$. For the time difference $\Delta t_1 = 0.01\pi/\omega_{01}$ (curve 1 in Fig. 1) the wavelength corresponding to the basic frequency $\lambda_{01} = 2\pi c/\omega_{01} = 14435\,\text{cm}$ is higher by a factor of around 300 than the distance between the electrons. In this case the coherence factor $S_2(\omega) = 4$ and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four more than a separate electron.

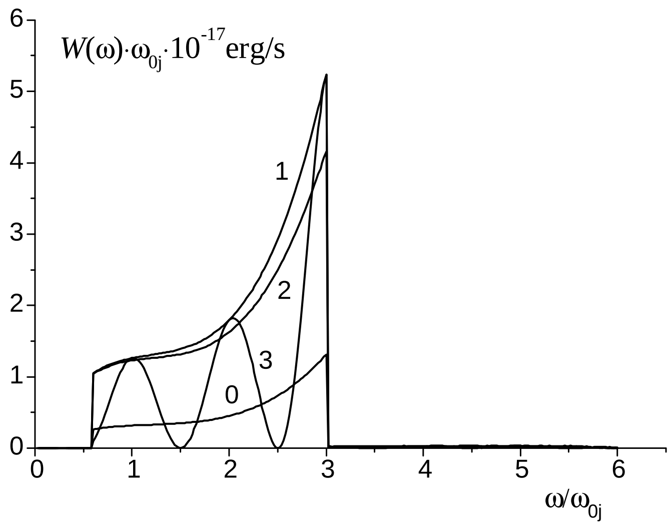

Fig. 1. Spectral distribution of radiation power for two electrons moving one by one in a spiral. ($V_{\perp vac} = 0.2\times10^{10}$ cm/s, $V_{\parallel vac} = 0.2\times10^{11}$ cm/s, curves 0–6). Curve 0 – the radiation spectrum of a separate electron, $P_{vac0}^{tot} = 0.1283\times10^{-16}$ erg/s $P_{vac0}^{int} = 0.1294\times10^{-16}$ erg/s. Curve 1: $\Delta t_1 = 0.01\pi/\omega_{01}$, $P_{vac1}^{int} = 0.517\times10^{-16}$ erg/s. Curve 2: $\Delta t_2 = 0.1\pi/\omega_{02}$, $P_{vac2}^{int} = 0.4544\times10^{-16}$ erg/s. Curve 3: $\Delta t_3 = 2\pi/\omega_{03}$, $P_{vac3}^{int} = 0.2741\times10^{-16}$ erg/s, $\omega_{00} = \omega_{01} = \omega_{02} = \omega_{03} = 0.1305\times10^8$ rad/s, $r_{00} = r_{01} = r_{02} = r_{03} = 153.3\,\text{cm}$.

In the case $\Delta t_3 = 2\pi / \omega_{03}$ the function of the radiation power spectral distribution has the maxima appoximately at the frequencies ω_{03}, $2\omega_{03}$ and $3\omega_{03}$ whereas the radiation at $1.5\omega_{03}$ and $2.5\omega_{03}$ is absent.

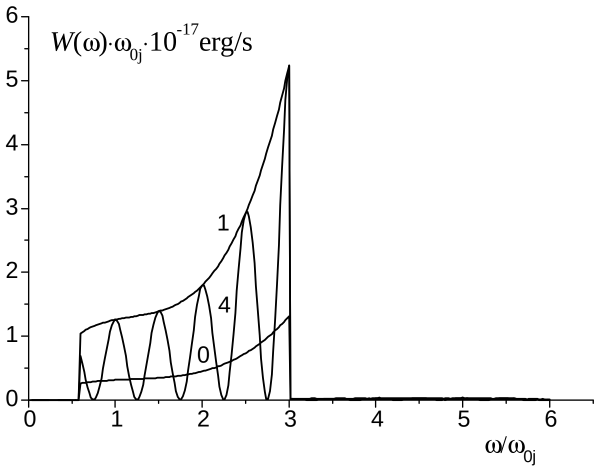

Fig. 2. Spectral distribution of radiation power for two electrons moving one by one in a spiral. Curve 4: $\Delta t_4 = 4\pi / \omega_{04}$, $P_{vac4}^{int} = 0.2595\times10^{-16}$ erg/s, $\omega_{04} = 0.1305\times10^{8}$ rad/s, $r_{04} = 153.3$ cm.

For the time difference $\Delta t_4 = 4\pi / \omega_{04}$ (curve 4 in Fig. 2) we have found the maxima of the spectral distribution function appoximately at the frequencies ω_{04}, $1.5\omega_{04}$, $2\omega_{04}$, $2.5\omega_{04}$ and $3\omega_{04}$ at the frequencies $0.75\omega_{04}$, $1.25\omega_{04}$, $1.75\omega_{04}$, $2.25\omega_{04}$, while at $2.75\omega_{04}$ the radiation is absent.

For the time differences $\Delta t_5 = \pi / \omega_{05}$ (curve 5 in Fig. 3) and $\Delta t_6 = 3\pi / \omega_{06}$ (curve 6 in Fig. 3) the radiation at the basic frequencies $\omega_{04} = \omega_{05}$ is absent.

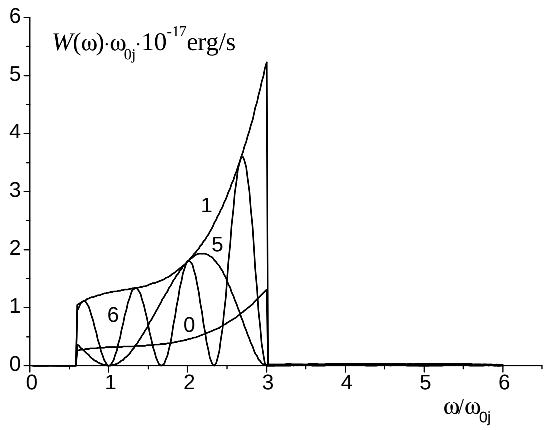

Fig. 3. Spectral distribution of radiation power for two electrons moving one by one in a spiral. Curve 5: $\Delta t_5 = \pi / \omega_{05}$, $P_{vac5}^{int} = 0.2081\times10^{-16}$ erg/s. Curve 6: $\Delta t_6 = 3\pi / \omega_{06}$, $P_{vac6}^{int} = 0.2561\times10^{-16}$ erg/s.

At the basic frequency ω_{0j} the function of the radiation power spectral distribution of two electrons is equal to zero if the time difference between them in a spiral is equal to $(2i+1)\pi / \omega_{0j}$ (i=0, 1, 2,…).

5. Spectral distribution of synchrotron-Cherenkov radiation power in low-frequency range

Let us consider the influence of the Doppler effect on synchrotron-Cherenkov radiation in transparent media. The expressions for the synchrotron-Cherenkov radiation power in such a medium can be obtained starting from (10). Then for the separate electron moving in a spiral we have found [6,11]

$$\overline{P}^{rad} = \int_0^\infty W(\omega) d\omega, \tag{21}$$

$$W(\omega) = \frac{2e^2}{\pi c^2} \int_0^\infty dx\, \mu(\omega)\omega \frac{\sin\left\{\frac{n(\omega)\omega}{c}\eta(x)\right\}}{\eta(x)} \cos(\omega x) \left[V_\perp^2 \cos(\omega_0 x) + V_\parallel^2 - \frac{c^2}{n^2(\omega)}\right], \tag{22}$$

where $\eta(x) = \sqrt{V_\parallel^2 x^2 + 4\frac{V_\perp^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

In the case of transparent media in low-frequency spectral range, i.e. at $\varepsilon = const$ and $\mu = 1$, the power of the Cherenkov radiation at rectilinear motion in a medium (n is the constant) is determined as [11]:

$$P_{ch}^{tot} = \frac{e^2}{2c^2} V \omega_{\max}^2 \left(1 - \frac{c^2}{V^2 n^2}\right). \tag{23}$$

For the refraction index $n = 2$ at the velocities $V_{\perp m} = 0.15\times10^{10}$ cm/s, $V_{\|m} = 0.1493\times10^{11}$ cm/s, and $V_{\perp m} = 0.12\times10^{10}$ cm/s, $V_{\|m} = 0.1496\times10^{11}$ cm/s (curves 7 and 8 in Fig. 4) the conditions for the existence of the synchrotron-Cherenkov radiation are fulfilled.

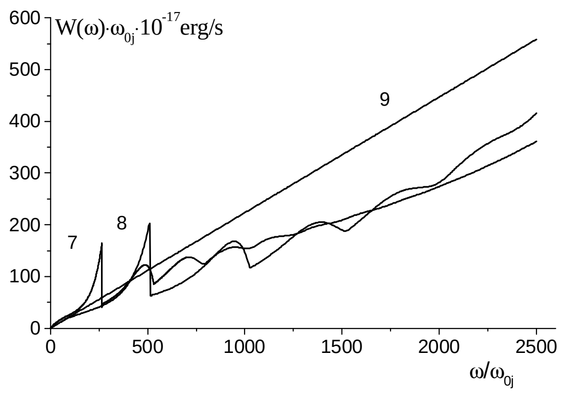

Fig. 4. Spectral distribution of synchrotron-Cherenkov radiation power with relative frequency. (For the curves 7-12: $B^{ext} = 1\,$Gs, $n = 2$, $\omega_{07} = \omega_{08} = \omega_{09} = \omega_{010} = \omega_{011} = \omega_{012} = 0.1523\times10^8$ rad/s, $c = 0.2997925\times10^{11}$ cm/s). Curve 7: $V_{\perp m} = 0{,}15\times10^{10}$ cm/s, $V_{\|m} = 0.149333\cdot10^{11}$ cm/s, $P_{m7}^{int} = 0.4688\cdot10^{-11}$ erg/s, $r_{07} = 98.5$ cm. Curve 8: $V_{\perp m} = 0.12\cdot10^{10}$ cm/s, $V_{\|m} = 0.1496035\cdot10^{11}$ cm/s, $r_{08} = 78.8$ cm, $P_{m8}^{int} = 0.469\times10^{-11}$ erg/s. Curve 9: $V_{\perp m} = 0.1\times10^8$ cm/s, $V_{\|m} = 0.1500839\times10^{11}$ cm/s, $P_{m9}^{int} = 0.699\times10^{-11}$ erg/s, $P_{ch9}^{tot} = 0.6979\times10^{-11}$ erg/s, $r_{09} = 0.66$ cm.

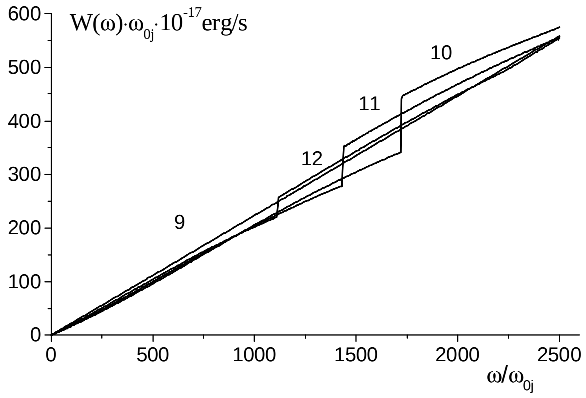

Fig. 5. Spectral distribution of synchrotron-Cherenkov radiation power with relative frequency. Curve 10: $V_{\perp m} = 0.55\times10^{9}$ cm/s, $V_{||m} = 0.1499832\times10^{11}$ cm/s, $r_{010} = 36.1$ cm, $P^{int}_{m10} = 0.6866\times10^{-11}$ erg/s. Curve 11: $V_{\perp m} = 0.5\times10^{9}$ cm/s, $V_{||m} = 0.1500007\times10^{11}$ cm/s, $r_{011} = 32.8$ cm, $P^{int}_{m11} = 0.6964\times10^{-11}$ erg/s. Curve 12: $V_{\perp m} = 0.4\times10^{9}$ cm/s, $V_{||m} = 0.150031\times10^{11}$ cm/s, $r_{012} = 26.3$ см, $P^{int}_{m12} = 0.6920\times10^{-11}$ erg/s.

The power of the Cherenkov radiation at rectilinear motion $P^{tot}_{ch9} = 0.6979\times10^{-11}$ erg/s (relation (23)) is in good agreement to the power of the synchrotron-Cherenkov radiation $P^{int}_{m} = 0.699\times10^{-11}$ erg/s calculated by applying the relationships (21) and (22) at the motion of the charged particle having a small ($V_{\perp m} = 0.1\times10^{8}$ cm/s) transverse velocity component (the absolute values of the velocities are the same).

The performed high-accuracy calculations of relationships (21) and (22) for the spectral distribution of the synchrotron-Cherenkov radiation power of electrons showed that the spectral distribution at $V_{||} < c/n$ (curves 7 and 8 in Fig. 4) essentially differs from that at $V_{||} > c/n$ (curves 10–12 in Fig. 5). The analytical and numerical calculations showed that the influence of the Doppler effect on the peculiarities of the radiation power spectral distribution of the electrons is essentially near the Cherenkov threshold.

Taking into account the frequency dispersion, this does not change essentially the radiation power spectral distribution in low-frequency range but leads to some interesting peculiarities in high-frequency spectral range [12,13].

6. Conclusions

The coherence factor leads to essential changes in the radiation power spectral distribution of a system of charged particles.

In the radiation spectrum of charged particles the Doppler effect establishes the limits between the bands of separate harmonics.

The obtained spectral distributions of the power of synchrotron, Cherenkov, and synchrotron-Cherenkov radiations of the electrons in low-frequency spectral range can be applied to develop new sources of electromagnetic energy and for interpretation of some phenomena in electronics, in astrophysics and plasma physics.

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