Journal of Optoelectronics and Advanced Materials Vol. 6, No. 1, March 2004, p. 63 - 69

# STATISTICAL PROPERTIES ANALYSIS OF $Er^{3+}$-DOPED Ti:$LiNbO_3$ $\mathcal{M}$-MODE STRAIGHT WAVEGUIDE AMPLIFIERS

N. N. Puscas*

Physics Department, University "Politehnica" of Bucharest,
Splaiul Independentei 313, 77206 - Bucharest, Romania

We report a theoretical analysis of some statistical parameters which characterize the single and double pass $Er^{3+}$-doped Ti:$LiNbO_3$ $\mathcal{M}$-mode straight waveguides. For the derivation and the evaluation of the Fano factor, the statistical fluctuation and the spontaneous emission factor we used a quasi-two-level model in the small gain approximation and the unsaturated regime. The simulations show the evolution of these parameters under various pump regimes and waveguide lengths. The obtained results can be used for the design of complex rare earth doped integrated circuits.

(Received May 8, 2003; accepted February 17, 2004)

*Keywords*: $Er^{3+}$:Ti:$LiNbO_3$ $\mathcal{M}$-mode straight waveguides, Fano factor, Statistical fluctuation, Spontaneous emission factor

## 1. Introduction

The theoretical and experimental studies of the gain spectral distribution, noise spectral distribution and the output statistics in a system where coherent and chaotic (thermal) fields are superimposed play an important role in the fabrication of integrated amplifiers with low noise and high optical gain [1-5]. In the last decade several papers presenting the behaviour of $Er^{3+}$-doped Ti:$LiNbO_3$ optical waveguides have been published in the last years [4-10].

Based on the models presented in [2,5,9] and [10] in this paper, we evaluate the output statistics of $\mathcal{M}$ mode straight $Er^{3+}$-doped Ti:$LiNbO_3$ amplifiers, like: the Fano factor, the statistical fluctuation and the spontaneous emission factor. This analysis is determined by the fact that often in the waveguides is excited not only the fundamental mode but also other high order ($\mathcal{M}$) modes, which influence not only the output gain, noise figure, signal-to-noise ratio but also the statistical properties of the waveguide.

The simulations of the output statistics of the amplifier are performed under the small gain approximation for *z* propagating crystals, pumped near 1484 nm, for various pumping regimes, and waveguide lengths. The results thereby computed are compared to those calculated for single mode straight waveguides.

The paper is organized as follows. Section 2 is devoted to the basic equations used to evaluate the Fano factor, the statistical fluctuation and the spontaneous emission factor which characterize the output statistics of the $\mathcal{M}$-mode waveguides. In Section 3 the discussion of the computed results is presented, while the conclusion of this work is outlined in Section 4.

## 2. Basic equations

In order to evaluate the output statistics in straight $\mathcal{M}$-mode $Er^{3+}$-doped Ti:$LiNbO_3$ waveguide amplifiers we use a quasi-two-level model under the small gain approximation in the wavelength range 1.44 µm < $\lambda$ <1.64 µm [2,5,9,10].

* Corresponding author: pnt@physics1.physics.pub.ro

In our model we considered that the waveguide is pumped by a radiation having a Gaussian distribution of the field using an optical fiber. For the determination of the optical field distribution in the waveguide we calculated first the refractive index profiles using the Fick's diffusion law [11,13]. After that, the optical mode fields were calculated numerically for both TE and TM polarisation using the effective index method presented in papers [13,14].

For the single and the double pass configuration of the optical waveguide amplifier having the length $L$ the corresponding equations for the Fano factor and the statistical fluctuation are [1,10]:

$$f(L,\nu) = 1 + 2G(L,\nu)\int_0^L \frac{\gamma_e(z',\nu)}{G(z',\nu)}dz', \tag{1}$$

$$e(L,\nu) = \frac{1}{\sqrt{G(L,\nu)\langle n(0)\rangle}}\left[1 + 2G(L,\nu)\int_0^L \frac{\gamma_e(z',\nu)}{G(z',\nu)}dz'\right]^{1/2}, \tag{2}$$

and

$$f(0,\nu) = G(L,\nu)R(L)\left[1 + 2G(L,\nu)\int_0^L \frac{\gamma_e(z',\nu)}{G(z',\nu)}dz'\right] + 2\int_0^L \gamma_e(z',\nu)G(z',\nu)dz' - G(L,\nu) + 1, \tag{3}$$

$$e(0,\nu) = \frac{1}{\sqrt{R(L)G^2(L,\nu)\langle n(0)\rangle}} \times \left[1 + 2\left(\int_0^L \gamma_e(z',\nu)G(z',\nu)dz + R(L)G^2(L,\nu)\int_0^L \frac{\gamma_e(z',\nu)}{G(z',\nu)}dz\right)\right]^{1/2} \tag{4}$$

while the expression of the spontaneous emission factor for single pass configuration is given by:

$$n_{sp}(L,\nu) = \frac{G(z,\nu)}{G(z,\nu)-1}\int_0^L \frac{\gamma_e(z',\nu)}{G(z',\nu)}dz'. \tag{5}$$

In Eqs. (1)-(5)

$$G(z,\nu) = \exp\left\{\int_0^z [\gamma_e(z',\nu) - \gamma_a(z',\nu)]dz'\right\} \tag{6}$$

represents the gain function at the $z$ section,

$$\begin{aligned} \gamma_e &= \gamma_e(z,\nu) = \sigma_e(\nu)\int_A N_2(x,y,z)i(x,y)dxdy \\ \gamma_a &= \gamma_a(z,\nu) = \sigma_a(\nu)\int_A N_1(x,y,z)i(x,y)dxdy \end{aligned} \tag{7}$$

$N_1, N_2$ are the populations of the ground and the excited levels, $\sigma_a(\nu), \sigma_e(\nu)$ are the absorption and emission cross sections, respectively and $\langle n(z)\rangle$ is the photon mean value. In the above mentioned equations the polarization subscript *m* has been omitted for simplicity [5,10].

Taking into account the relations between the forward $P^+(z) = P^+(0)G(z)$ and backward $P^-(0) = P^-(z)G(z)$ powers we calculated the gain $G(z,\nu)$ for the single pass (mirror reflectivities in

$z = L$, $R_P(L) = R_S(L) = 0$) and the double pass ($R_P(L) = R_S(L) = R$) configuration of the linear optical amplifier.

The expressions for the output signal gain are:

$$G(L,\nu) = P_S^+(L)/P_S(0), \tag{8}$$

for the single pass and

$$P_S^-(0)/P_S(0) = G^2(L,\nu)R(L), \tag{9}$$

for the double pass configurations.

In the derivation of Eqs. (1)-(5) we considered that the normalized mode intensity profile $i(x,y)$ in the $\mathcal{M}$ mode straight $Er^{3+}$-doped Ti:LiNbO$_3$ waveguide amplifiers is not uniform over its transversal section [10] and also that the input signals are characterized by Poissons statistics (coherent light) $P_n(0) = \frac{\langle n(0)\rangle^n}{n!} e^{-\langle n(0)\rangle}$ for which is well known that $\sigma^2(0) = \langle n(0)\rangle$. In linear regime, stimulated emission by itself determines changes in the light statistics, in addition to those introduced by spontaneous emission which are characterized by Bose-Einstein statistics. In Eqs. (2)-(3) the factor $\left[1 + 2G(L,\nu)\int_0^L \frac{\gamma_e(z',\nu)}{G(z',\nu)} dz'\right]^{1/2}$ characterizes the deviation of the output statistics from Poisson. It is well known that for Poisson statistics the Fano factor is $f = 1$ and $e = (\langle n\rangle)^{-1/2}$ $e = (\langle n\rangle)^{-1/2}$, while for Bose Einstein statistics $f = \langle n\rangle + 1$, and $e = (1 + 1/\langle n\rangle)^{-1/2}$ [1], [5].

### 3. Simulation results and discussion

Solving the system of coupled first order differential equations for the populations and the optical powers, (pump, signal and amplified spontaneous emission, ASE) by numerical methods (Runge-Kutta formula-4 th order, 4 stages) with an iterative procedure we obtained the gain for single and double pass configuration and the Fano factor, the statistical fluctuation and the spontaneous emission factor (Eqs. (1)-(5)) [5], [10]. In our simulations, the spontaneous emission spectrum is divided into 100 slots which corresponds to a wavelength resolution $\Delta\lambda$=2 nm in the region 1450-1650 nm.

In our model the lowest four order mode intensity profiles, $i(x,y)$ for both TE and TM polarisations are calculated numerically with the method described in paper [10], while the dopant distribution $d(x,y)$ (with a surface concentration of about $1\times10^{26} m^{-3}$ a diffusion depth of 20 $\mu$m and of 5.12 $\mu$m in width (defined at 1/e) is approximated by an erfc but also by Gaussian or constant functions in depth (labelled 1, 2 and 3, respectively), and a Gaussian function in width. These different Er concentration profiles correspond to various diffusion conditions (non complete or complete diffusion, or doping during the crystal growth).

The computer simulations for the evaluation of the statistical properties of $Er^{3+}$-doped LiNbO$_3$ waveguide amplifiers have been performed using the parameters found in the literature [9,10]. We assumed a 1484 nm pump and a signal at $\lambda$=1531 nm, having 1 $\mu$W input power. In computing the profile of the optical modes we used the following parameters characteristic of Ti waveguides: 7 μm for waveguide width, 0.1 μm for Ti thickness, 10 hours for diffusion time.

In our simulations we used for TE and TM pump (*p*) and signal (*s*) absorption (*a*) and emission (*e*) cross sections the values presented in papers [9] and [10]. The waveguide length is assumed to be $L$ = 5.4 cm. when not specified; the pump and signal input reflectivity always $R(0) = 0$ while the output is $R(L) = 0$ or $R(L) = 0.98$ for the single and double pass configuration, respectively.

Scattering losses $\alpha$=3.7 dBm$^{-1}$ for TE, $\alpha$=4.8 dBm$^{-1}$ for TM and spontaneous emission lifetime $\tau$=2.6 ms have been assumed. We limit our presentation to the TE pump and signal polarization; similar results can be obtained for the other cases.

The peak values of the small signal gain $G(z)=\ln\left[\frac{P_{signal}(z)}{P_{signal}(0)}\right]$ for the double pass configuration, $G(0,\nu)$ for the input pump power of 100 mW (high pump regime) at $\lambda$=1531 nm for the erfc (2.55 dB), Gaussian (5.57 dB) and constant (8.30 dB) dopant profiles in the case of $\mathcal{M}$-mode operation are smaller than in the single one, of about 14.5 dB for Gaussian profile in depth [5,10].

The peak values of the noise figure for the erfc (5.58 dB), Gaussian (6.13 dB) and constant (6.24) dopant profiles in the case of $\mathcal{M}$-mode operation are greater than those corresponding to the single one of about 5.3 dB for Gaussian profile in depth [5,10].

Fig. 1 shows the wavelength dependence of the Fano factor $f(0,\lambda)$ for the double pass configuration (Eq. (3)) for input pump power $P(0)$=100 mW. As can be seen the behaviour of the Fano factor is determined also in this case by the absorption and emission cross sections of $Er^{3+}$-doped $LiNbO_3$ [5,9,10]. The values of the Fano factor for Gaussian (97.45) and constant (167.55) dopant profiles in the case of $\mathcal{M}$-mode operation are greater than those corresponding to the single mode operation which is about 90 for Gaussian profile in depth, but that for erfc profile (32.51) is smaller, respectively [5,10].

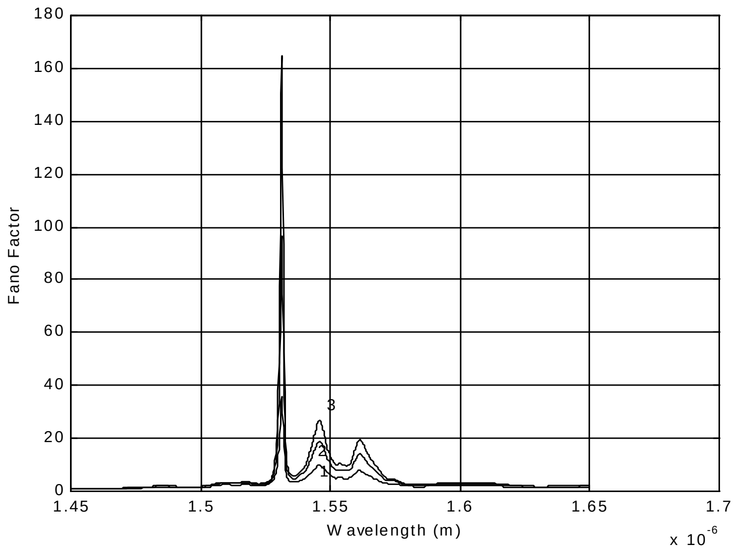



Fig. 1. Spectral behaviour of the Fano factor in the double pass configuration for input pump power: $P$(0)=100 mW in the case of erfc (curve 1), Gaussian (curve 1) and constant (curve 3) dopant profiles in depth, TE polarization and $L$=5.4 cm for the waveguide length.

The spectral behaviour of the statistical fluctuation in double pass configuration (Eq. (4)) is presented in Fig. 2 while Fig. 3 shows the spontaneous emission factor spectra in single pass configuration (Eq. (5)). The values of the last two parameters are greater in the $\mathcal{M}$-mode operation than in the single one [5,10]. For a high input pump power the spontaneous emission factor is less than 2 over most of the 60-nm spectral range considered in the figure, (i. e. for Gaussian profile of the dopant in the spectral ranges 1.45 $\mu$m $\div$ 1.48 $\mu$m and 1.62 $\mu$m $\div$ 1.65 $\mu$m, which correspond to the amplification of an equivalent input noise of one photon per unit frequency.

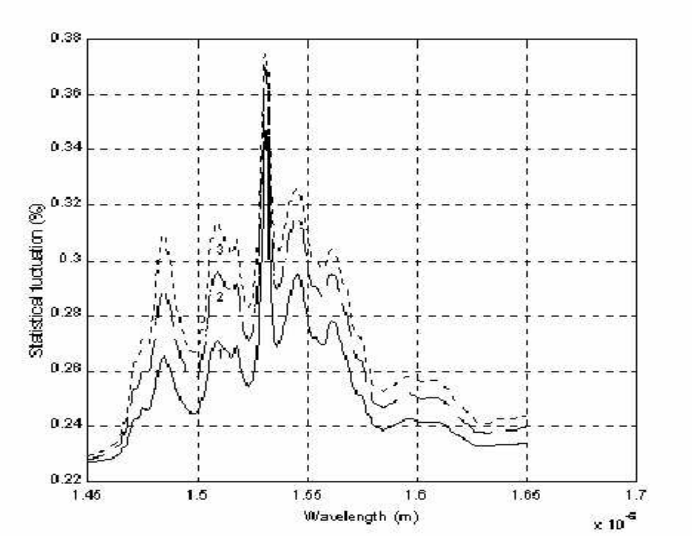



Fig. 2. The spectral dependence of the statistical fluctuation, as in Fig. 1.

The $\mathcal{M}$-mode operation in comparison with the single one determines the diminution of the gain and the enhancement of the noise figure because the overlap integral between the population of the excited level and the normalized intensity field profile is smaller in the case of $\mathcal{M}$-mode operation than in the case of the single one, (i. e. 1.21 times in the case of Gaussian profile of the dopant for a waveguide having 5.4 cm length and for an input pump power of 100 mW). Also, the $\mathcal{M}$-mode operation affects the output statistics diminishing the coherent properties of the light.

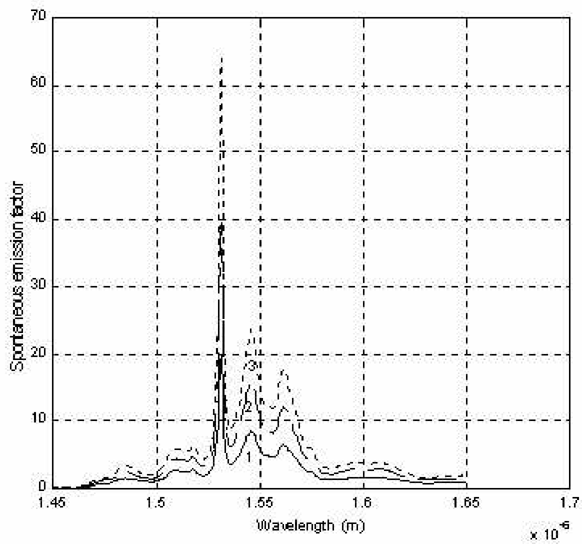
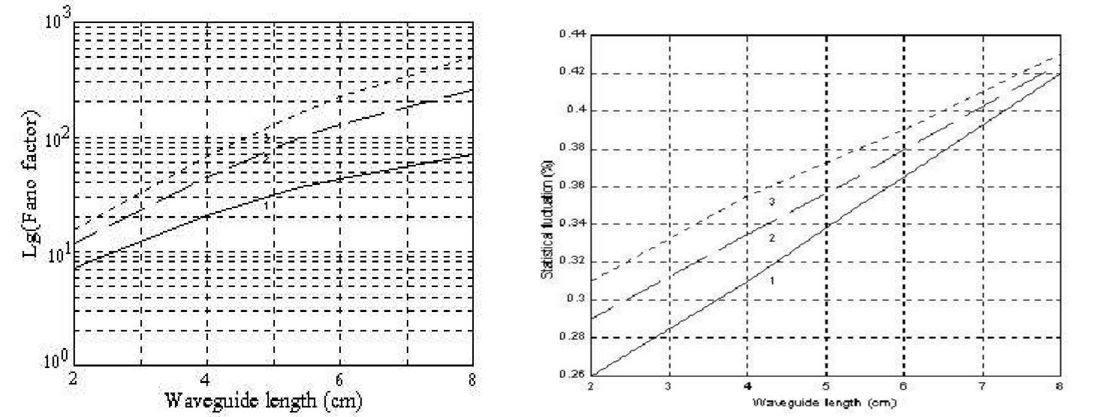



Fig. 3. The spontaneous emission factor spectra in single pass configuration for the input pump power of 100 mW in the case of erfc (curve 1), Gaussian (curve 1) and constant (curve 3) dopant profiles in depth, TE polarization and $L$ = 5.4 cm for the waveguide length.

In Figs. 4 a), b), 5 a), b) the dependences of the Fano factor (a) and the statistical fluctuation (b) in double pass configuration, TE polarization for a signal at $\lambda$=1531 nm versus the waveguide length, for input pump power $P(0)$=100 mW in the case of erfc (curve 1), Gaussian (curve 1) and constant (curve 3) dopant profiles in depth, (Fig. 4) and versus the pump power, respectively for an amplifier length $L$ =5.4 cm, (Fig. 5) are presented.

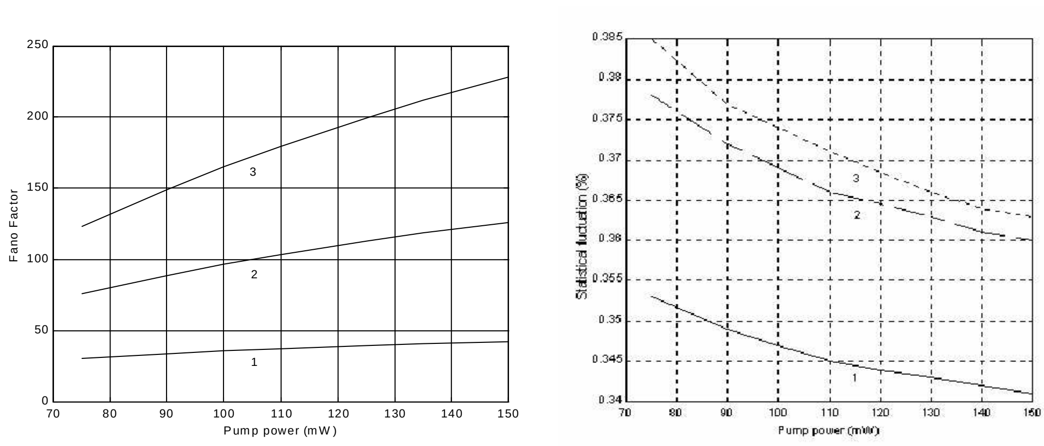



a) b)

Fig. 4 a), b). The Fano factor, a) and the statistical fluctuation, b) vs the waveguide length.

The Fano factor increases both the increasing of the waveguide length and the pump power (Figs. 4, 5 a)) while the statistical fluctuation increases with the waveguide length and decreases when the pump power is augmented (Figs. 4, 5 b)).

Therefore, it seems that the amplifier length is responsible for the reduction of the Poissonian aspect of the amplified light. This is not the case for shorter waveguide lengths but higher pumping levels, for which the statistical properties of the light are roughly maintained. A consequence concerns the design of complex structures, when the coherence of the amplified light must be conserved. If the miniaturization of the integrated devices have to be considered, it is preferable to choose short lengths and high level pumpings, rather than long arms and low pumping regimes.




a) b)

Fig. 5 a). The Fano factor, a) and the statistical fluctuation, b) vs the pump power.

## 4. Conclusions

In the small gain approximation an original analysis of the output noise statistical properties of a $\mathcal{M}$ -mode $Er^{3+}$-doped $LiNbO_3$ straight waveguide amplifier has been presented. The simulations concern the Fano factor, the statistical fluctuation and spontaneous emission factor. The study demonstrates that in calculating the Fano factor in the single and double pass configurations of the amplifier the Poissonian photon statistics are maintained for pump powers lower than 100 mW and waveguide lengths smaller than 5 cm. These parameters are of importance when the coherence of the amplified light is an issue for the doped structure under investigation. For example, we have shown that the coherence of the output amplified signal is more sensitive to the waveguide length than to the pumping power. The theoretical results of this simulation characterize the $Er^{3+}$-doped $LiNbO_3$waveguide amplifier from the point of view of noise statistical properties and can be used in the better understanding of the amplification process.

## References

[1] E. Desurvire, Erbium-Doped Fiber Amplifiers, New York: Wiley, (1994).
[2] P. Diament, M. C. Teich, IEEE J. Quant. Electron. **28**, 1325 (1992).
[3] A. Yariv, Optics Letters **15**, 1064 (1990).
[4] E. L. Goldstein, M. C. Teich, IEEE J. Quant. Electron. **25**, 2289 (1989).
[5] N. N. Puscas, D. Scarano, R. Girardi, I. Montrosset, Opt. and Quant. Electronics **29**, 799 (1997).
[6] I. I. Mansour, F. Caccavale, Applied Optics **35**, 1492 (1996).
[7] M. Majd, B. Schüppert, K. Petermann, IEEE Photonics Technology Letters **5**, 806 (1993).
[8] R. Tavlykaev, K. Kückelhaus, E. Voges, J. of Optical Communications **15**, 71 (1994).
[9] M. Dinand, W. Sohler, IEEE J. Quant. Electron **30**, 1267 (1994).
[10] N. N. Puscas, J. Optoelectron. Adv. Mater. **4**, 911 (2002).
[11] D. Marcuse, Light Transmission Optics, New York: Van Nostrand, (1972).
[12] I. Turek, I. Martinček, R. Stránsky, Optical Engineering **39**, 1304 (2000).
[13] N. N. Puscas, N. B. Wacogne, A. Ducariu, B. Grappe, Opt. and Quant. Electronics **32**, 1 (2000).
[14] G. B. Hocker, W. K. Burns, Applied Optics **16**, 113 (1977).