

## MULTIPLASMON LASER GAIN SPECTRA OF QUANTUM WELLS

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Results of computer simulations concerning gain spectra of  $\text{In}_{0.05}\text{Ga}_{0.95}\text{As}$  quantum wells are presented. A novel multi-plasmon concept of a light absorption and laser gain of low-dimensional structures is comprehensively discussed. A generalized theory of multi-plasmon optical transitions in direct gap quantum wells is developed using the cumulant expansion method and fluctuation-dissipation theorem. Multi-quantum LO-phonon-plasmon optical transitions are investigated with account on coherent memory effects in quantum wells. It is shown that a red shift of the absorption edge can be caused not only by the well-known mechanism of band gap shrinkage but also by multi-plasmon transitions. The comparison with other theories and experimental data measured in  $\text{In}_{0.05}\text{Ga}_{0.95}\text{As}$  quantum wells is given.

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### 1. Microscopic theory

Generalizing the results of papers [6-10] for low-dimensional structures, we find for coefficient of interband absorption of light in the symmetric quantum wells with a width of  $d_0$  next expression :

$$\alpha(\omega) = \alpha_0 \frac{\omega_g}{\omega} \sum_{N=0}^{\infty} \int \{1 - f_N^e(\omega_{k_{\perp}}) - f_N^h(\omega_{k_{\perp}})\} C(k_{\perp}) \text{Re} \int_0^{\infty} e^{i(\omega - \omega_g - \omega_N - \omega_{k_{\perp}})t - g(t)} dt d\omega_{k_{\perp}} \quad (1)$$

The gain spectrum is determined by a known relation  $g(\omega) = -\alpha(\omega)$ . Here  $\hbar\omega_g = E_g$  - the band gap of semiconductor  $\omega_{k_{\perp}} = \hbar k_{\perp}^2 / 2m_r$ ,  $m_r = m_e m_h / (m_e + m_h)$  - the reduced mass of an electron and a hole.  $f_N^e$  and  $f_N^h$  - Fermi-Dirac distributions of electrons and holes, accordingly. A successions of energy subbands are labeled by the quantum numbers  $N=1,2,3\dots$ . The constant  $\alpha_0$  looks like:

$$\alpha_0 = \frac{2^{5/2} e^2 P_{cv}^2 m_r^{3/2}}{c n m_0^2 \hbar^{5/2} \omega_g} \left( \frac{\hbar}{2m_r d_0^2} \right)^{1/2} \quad (2)$$

Here  $c$  is the speed of light, and  $n$  – a refraction index,  $m_0$  - mass,  $e$  - a charge of an electron,  $P_{cv}$  is a matrix element of a projection of an momentum on a direction of light polarization.

Functions  $C(k_{\perp})$  and  $g(t)$  characterize interaction of an electron and a hole with plasma and lattice vibrations.

Without taking into account a Coulomb interaction in free carrier approximation  $C(k_{\perp}) = 1$ , and  $g(t) = 0$  and we come to known result for coefficient of interband absorption in quantum wells as a superposition of step functions [1-5]. At low concentrations of plasma Coulomb or Sommerfeld

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factor  $C(k_{\perp})$  looks like [1] :

$$C(k_{\perp}) = \frac{\exp\{q(k_{\perp})\}}{\cosh\{q(k_{\perp})\}}, \quad q(k_{\perp}) = \frac{\pi}{a_B k_{\perp}}, \quad a_B = \frac{\hbar^2 \epsilon_0}{m_r e^2}, \quad (3)$$

Let's consider, for a screened two-dimensional exciton  $C(k_{\perp})$  looks like (3), but  $q$  we shall calculate with account of interaction with plasma. Using screened Coulomb interaction as perturbation, we find

$$q(k_{\perp}) = \frac{2\lambda_0}{\pi a_B} \int \frac{1}{1+\lambda_0|k'_{\perp}-k_{\perp}|} \frac{dk'_{\perp}}{k_{\perp}^2 - k_{\perp}'^2} = 4 \frac{\lambda_0}{a_B} \operatorname{Re} \frac{1}{\sqrt{1-4\lambda_0 k_{\perp}^2}} \ln \frac{1+\sqrt{1-4\lambda_0 k_{\perp}^2}}{2\lambda_0 k_{\perp}} \quad (4)$$

In a limit when the screening length  $\lambda_0$  satisfies to an inequality  $\lambda_0 k_{\perp} \gg 1$ , function  $q(k_{\perp})$  (4) transfers in the next one:  $q(k_{\perp}) = \pi/a_B k_{\perp}$  (3). According to expression (4), plasma screening decreases the influence of Coulomb attraction between an electron and a hole on a transition probability. As a result of screening, near to a threshold of light absorption ( $k_{\perp} = 0$ ), the power dependence  $q \sim 1/k_{\perp}$  (3) is replaced logarithmic (4). The screening length is determined by expression:

$$\frac{1}{\lambda_0} = \frac{2}{a_B} \left\{ \frac{m_e}{m_r N} \sum f_N^e(0) + \frac{m_h}{m_r N} \sum f_N^h(0) \right\} \quad (5)$$

At performance of an inequality  $\hbar^2 k_{\perp}^2 / 2m k_0 T \ll 1$  the screening length depends on electrons and holes number, which momentum along the interface  $\hbar k_{\perp}$  is equal to zero.

In case of strong interaction of the electron - hole pair with plasma and lattice vibrations the main contribution to integral on time (1) is given by values  $t \rightarrow \infty$  and absorption coefficient  $\alpha(\omega)$  can be represented as a superposition of Lorentzian line shape functions. At a strong e-h pair coupling with plasma actual there is a limit of small  $t$ , when function  $g(t)$  can be represented by expression:

$$g(t) = \frac{1}{2} \sigma t^2 + N_{LO} e^{i\omega_{LO} t}, \quad N_{LO} = \alpha_e \left( 1 + \sqrt{\frac{m_h}{m_e}} \right) n(\omega_{LO}) \quad (6)$$

Here the weak coupling with longitudinal optical phonons is taken into account.  $\alpha_e$  is Frohlich coupling constant for electrons,  $n(\omega_{LO})$  - number of phonons, and  $\sigma$  - the second moment of absorption band which looks like:

$$\sigma = \frac{2}{\pi \hbar} \sum_{\kappa_{\perp}} V_{\kappa_{\perp}} \int_0^{\infty} \operatorname{Im} \left\{ \frac{\epsilon_0}{\epsilon^*(\kappa_{\perp}, \omega)} \right\} cth \frac{\hbar \omega}{2k_0 T} d\omega, \quad V_{\kappa_{\perp}} = \frac{2\pi e^2}{\epsilon_0 A \kappa_{\perp}} \quad (7)$$

Two-dimensional Coulomb interaction  $V_{\kappa_{\perp}}$  (7) is dynamically screened by plasma which vibration frequencies are determined by zeros of the dielectric function  $\epsilon(\kappa_{\perp}, \omega)$ . At high temperatures

$$k_0 T > \hbar \omega_p, \quad \omega_p = \left( \frac{4\pi n e^2}{\epsilon_0 m_r} \right)^{1/2},$$

using the approximation

$$cth \frac{\hbar \omega}{2k_0 T} \approx \frac{2k_0 T}{\hbar \omega} \text{ and a sum rule one obtains}$$

$$\sigma = \frac{e^2 k_0 T}{\hbar^2 \epsilon_0} \int_0^\infty d\kappa_\perp \left( 1 - \frac{\epsilon_0}{\epsilon(\kappa_\perp, 0)} \right) = \frac{4^{5/3} \pi^{4/3}}{3^{3/2} \hbar^2} Ry k_0 T \left\{ \frac{d_0 a_B^2}{m_r} (m_e + m_h) n \right\}^{1/3} \quad (8)$$

Here  $n$  is plasma concentration. Substituting  $g(t)$  (6) in expression (1) for absorption coefficient and integrating on time, we find with account of LO-phonon satellites:

$$g(\omega) = \sqrt{\frac{\pi}{2\sigma}} \alpha_0 \frac{\omega_g}{\omega_{N0}} \sum_j C(k_\perp) \{f_N^e(k_\perp) + f_N^h(k_\perp) - 1\} \sum_{j=0}^\infty \frac{(N_{LO})^j}{j!} \exp\left[-\frac{(\omega - \omega_g - \omega_N - \omega_{k_\perp} + j\omega_{LO})^2}{2\sigma}\right] d\omega_{k_\perp} \quad (9)$$

The strong coupling of electrons and holes with plasmons are valid for the inequality

$$k_0 T Ry < (\hbar\omega_p)^2 \left( Ry = \frac{e^2}{2\epsilon_0 a_B} \right) \quad (10)$$

## 2. Results and discussion

The light gain frequency dependence in case of strong charge carriers interaction is determined by the processes of absorption and radiation of several oscillation quanta of two-dimensional plasma and of several optical phonons.

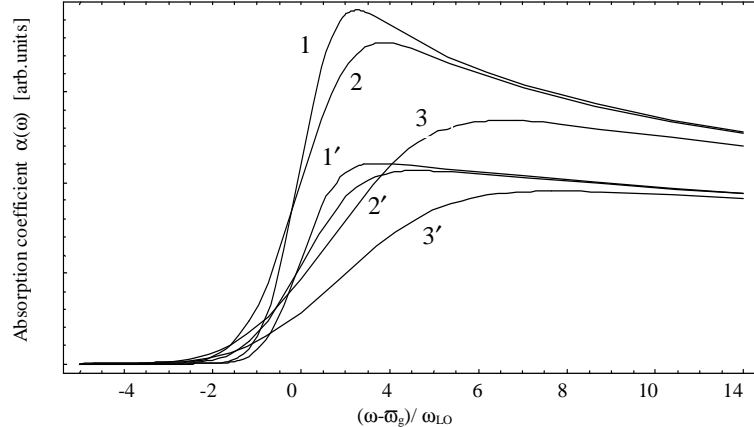


Fig. 1. Absorption coefficient  $\alpha(\omega)$  frequency dependence of 10 nm GaAs quantum well at  $T = 300$  K and at different plasma concentrations 1,1' -  $n = 10^{16} \text{ cm}^{-3}$ , 2,2' -  $n = 10^{17} \text{ cm}^{-3}$  and 3,3' -  $n = 10^{18} \text{ cm}^{-3}$ ,  $\hbar\omega_g = E_g + \hbar^2 \pi^2 / 2m_r d_0^2$ .

Fig. 1 is a plot of the absorption spectra for different plasma concentration. In the absence of doping, the total carrier density is the same for electrons and holes. For each of two groups the top curves 1,1' are for  $n = 10^{16} \text{ cm}^{-3}$ , the middle 2,2' for  $n = 10^{17} \text{ cm}^{-3}$  and the bottom 3,3' for  $n = 10^{18} \text{ cm}^{-3}$ . Curves 1,2,3 are the absorption spectra calculated with account of Sommerfeld enhancement factor, 1', 2', 3' at  $C(k_\perp) = 1$ . The attractive Coulomb interaction between electrons and holes is responsible for the increase in the absorption when compared to the free-carrier approximation  $C(k_\perp) = 1$ , but plasma screening leads to the decrease in it. At low concentrations of plasma  $n = 10^{16} - 10^{17} \text{ cm}^{-3}$  an absorption coefficient  $\alpha(\omega) = -g(\omega)$  (9) looks like a step. With growth of plasma concentration  $n$  the frequency dependence is flattened, and the threshold of absorption is shifted in the long-wave region of a spectrum. This features are represented at the Fig.1. And at last at  $n \cong 2 \times 10^{18} \text{ cm}^{-3}$  there appears an amplification.

The numerical calculation results of laser amplification spectrum of 8 nm  $\text{In}_{0.05}\text{Ga}_{0.95}\text{As}$

quantum well under the formula (9), using (3), (4) and (8) formulas, at a surface plasma density  $nd_0 = 1.56, 1.60, 1.64 \times 10^{12} \text{ cm}^{-2}$  are shown in Fig. 2. Apparently, the theoretical results and experimental data [11], shown in Fig. 2, are in agreement. The gain value  $g = 50 \text{ cm}^{-1}$  is obtained at  $nd_0 = 1.64 \times 10^{12} \text{ cm}^{-2}$ , that is approximately half the value given by the theory [11]. The experimental value of  $n$  in paper [11] is not given. Injection currents are 5, 7.5, 10, 15, 20 mA [11].

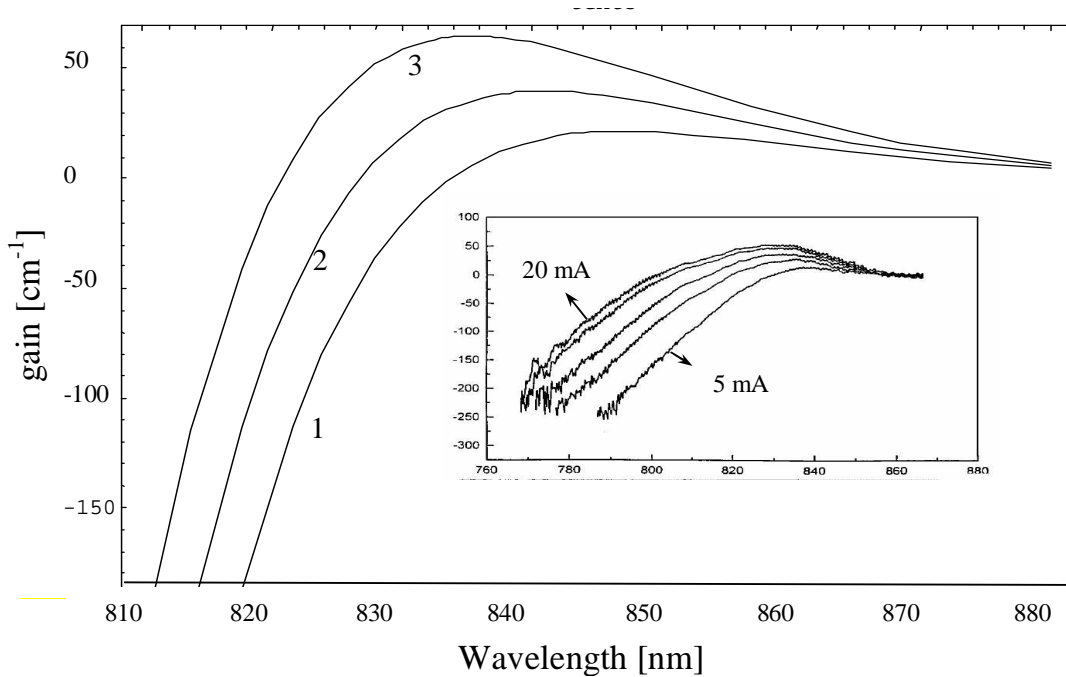


Fig. 2. Laser gain spectra designed under the formula (9) with account of transitions between two subbands ( $N = 1$ ) and functions (3), (4) and (8) at the following values of parameters:  $T = 300 \text{ K}$ ,  $m_e = 0,0648 m_0$ ,  $m_h = 0,476 m_0$ ,  $E_g = 1,42 \text{ eV}$ ,  $\hbar\alpha_{LO} = 35 \text{ meV}$ ,  $\alpha_e = 0,07$ ,  $\epsilon_0 = 13,13$ ,  $\epsilon_\infty = 11,1$ . Surface density of electrons are 1-1.56, 2-1.60, 3- $1.64 \times 10^{12} \text{ cm}^{-2}$ . Experimental gain spectra are measured at the next injection currents 5, 7.5, 10, 15, 20 mA [11].

The approach of two - dimensional plasma is valid for a narrow quantum well. In case of a well with arbitrary width, one obtains:

$$\omega_p^2(\kappa_\perp) = \frac{4\pi n e^2}{\epsilon_0 m_r} \left( 1 - e^{-\frac{\kappa_\perp d_0}{2}} \right) \quad (11)$$

In a limit of  $\kappa_\perp d_0 < 1$ , formula (10) gives the result known for two-dimensional plasma  $\omega_p^2 = 2\pi n e^2 d_0 \kappa_\perp / \epsilon_0 m_r$  [1-5], whereas at  $\kappa_\perp d_0 > 1$  plasma behaves as bulk one. In a three-dimensional case plasma oscillations are similar to longitudinal optical oscillations of a lattice and multi-plasmon processes give to equidistant thin structure of emission bands [6-10]. Thus, and in case of wide quantum wells multi-plasmon satellites of lines of radiation, under condition of existence of plasma oscillations  $\omega\tau > 1$  where a  $\tau$  - plasmon lifetime, may be the allowed. Optical properties of wide quantum wells were explored in papers [12-14]. Gossard with co-authors [14] have found out, that spectrums of photo excitation of wide (4000 Å) GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  quantum wells in which  $x$  it was incremented from zero close the interface, up to  $x = 0,3$  in middle of a well, consist of a series of equidistant peaks, parted energy distance about 3 meV. This equidistant structure was attributed by authors of paper [14] to the parabolic shape of a quantum well and equidistant an energy spectrum of electrons and holes. In our opinion the equidistant structure may be caused by resonant radiation of several plasmons alongside with a photon. The plasmon energy

$\hbar\omega_p \cong 3$  meV meets to a surface density of electrons  $nd_0 \cong 10^{12}$  cm<sup>-2</sup> given in paper [14]. The magnification of distance between peaks was observed for doping silicon layers donor. It is possible to explain this effect by the multi-plasmon mechanism of radiation, namely - magnification of energy plasmon with the increase of electron plasma concentration.

### 3. Conclusion

Multi-quantum LO-phonon-plasmon optical transitions were investigated with account on the coherent memory effect in quantum wells. It was shown that a red shift of the absorption edge can be caused not only by the well-known mechanism of band gap shrinkage but also by multi-plasmon transitions.

The results from our theory were compared with the experimental data on In<sub>0.05</sub>Ga<sub>0.95</sub>As and good agreement was obtained.

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