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# CALCULATION OF SOME SPECTROSCOPIC PARAMETERS OF $LiNbO_3$:$Er^{3+}$OPTICAL WAVEGUIDES

N. N. Puscas[*], A. Ducariu, G. C. Constantin, D. Dinu

Physics Department, University "Politehnica" of Bucharest, Splaiul Independentei, 313, 0600042, Bucharest, Romania

Based on the transmission spectra of $Er^{3+}$-doped $LiNbO_3$ optical waveguides in this paper we report some experimental and theoretical results concerning the evaluation of some spectroscopic parameters which characterize the above mentioned waveguides. The absorption experimental spectra were used to determine the homogeneous absorption and emission cross sections (utilising the density matrix formalism and the McCumber's theory and taking into account the Stark splitting of the levels), the oscillator strength of the absorption transition, the spontaneous emission probabilities, radiative lifetime and the excitation energy in three regions of the optical spectrum: around 1550 nm, 980 nm and 550 nm, respectively. The obtained results are in good agreement with other ones published in the literature in the last few years and can be used for the theoretical modelling of the guided lasers and amplifiers, design of guided optoelectronic devices etc.

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## 1. Introduction

The absorption and emission cross sections of the rare earths doped $LiNbO_3$ optical waveguides are used for the theoretical modelling of the guided lasers and amplifiers, design of guided optoelectronic devices etc., the exact knowledge of them playing a very important role [1-10].

Using the obtained experimental transmission spectra in the case of $Er^{3+}$-doped $LiNbO_3$ optical waveguides, in this paper we report an exact determination of the homogeneous absorption and emission cross-sections in three regions of the optical spectra: around 1550 nm, 980 nm and 550 nm, respectively. For the evaluation of the homogeneous absorption and emission cross-sections from the inhomogeneous ones, obtained experimentally we used the density matrix formalism and the McCumber's theory [11] taking into account the Stark splitting of the levels [2,6].

In Section 2, we present the basic equations used for the evaluation of the homogeneous absorption and emission cross-sections. Section 3 is dedicated to the description of the the experimental setup and the obtained experimental results while in Sections 4 we evaluate some spectroscopic parameters such as: the homogeneous absorption and emission cross sections, the oscillator strength of the absorption transition, the spontaneous emission probabilities, radiative lifetime and the excitation energy in three regions of the optical spectra: around 1550 nm, 980 nm and 550 nm, respectively. In Section 5 the conclusions of this work are outlined.

## 2. Theoretical considerations

For the evaluation of the homogeneous absorption and emission cross-sections the interaction between the atomic system and the electric field may be described considering a two-level system Stark splitted and the semiclassical formalism presented in papers [2,6].

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[*] Corresponding author: pnt@ physics.pub.ro

The homogeneous absorption, $\sigma_a^H(\omega)$ cross section may be determined from the experimental (inhomogeneous) cross sections $\sigma_a^I(\lambda)$ through the inverse Fourier transform relation (and its properties) [2]:

$$\sigma_a^H(\omega) = F^{-1}\left[\exp\left(\frac{\Delta\omega_{inh}^2 x^2}{16\log 2}F\left[\sigma_a^I(\omega);x\right];\omega\right)\right] \tag{1}$$

where $\Delta\omega = 2\pi c\Delta\lambda_{inh}/\lambda^2$ is the inhomogeneous bandwidth and $\Delta\lambda_{inh}$ the inhomogeneous linewidth.

In Eq. (1) the inhomogeneous absorption cross sections are given by the relations [2]:

$$\sigma_a^I(\omega) = \int_{-\infty}^{+\infty} p_{inh}(\omega-\omega')\sigma_a^H(\omega')\mathrm{d}\omega' \tag{2}$$

where

$$p_{inh} = \sqrt{\frac{4\log 2}{\pi\Delta\omega_{inh}^2}}\exp\left[-4\log 2\left(\frac{\omega'}{\Delta\omega_{inh}}\right)^2\right] \tag{3}$$

represents the density distribution for inhomogeneous broadening of line frequencies in solids which is generally Gaussian.

The inhomogeneous or experimental cross sections are given by the best fit with Gaussian functions:

$$\sigma_a^I(\omega) = \sum_k a_k^a \exp\left[-4\log 2\frac{(\omega-\omega_k)^2}{\Delta\omega_k^2}\right]. \tag{4}$$

Taking into account that the inhomogeneous absorption cross section can be calculated from the measured absorption spectra of the Er ion in the medium as folowing:

$$\sigma_a^I(\nu) = \frac{1}{Nl}\ln\frac{I_0(\nu)}{I(\nu)} \tag{5}$$

where $N$ is the concentration of the ions, $l$ the length of the optical waveguide, $I(\nu)$ and $I_0(\nu)$ the light intensities transmitted and incident on the sample, the relation between the absorption and emission cross sections is [2] :

$$\sigma_a(\nu) = \sigma_e(\nu)\exp\left[\frac{h(\nu-\varepsilon)}{k_B T}\right] \tag{6}$$

where:

$$h\varepsilon \approx -k_B T\log(N_2/N_1) \tag{7}$$

$N_{1,2}$ being the populations of the levels (Stark splitted) implied in transitions, $T$ the temperature, $k_B$ the Boltzmann's constant and $\varepsilon$ representing the excitation energy.

The spectra can be numericaly generated through the fitting formula [2,6]:

$$I(\lambda) = \sum_i a_i \exp\left\{-4\log 2[(\lambda-\lambda_i)/\Delta\lambda_i]^2\right\}. \tag{8}$$

The experimental line shapes $I_{a,e}(\lambda) \approx \sigma_{a,e}^I(\lambda)$. The fitting curve is not unique but the deconvolution expressed in $\sigma_{a,e}^H(\omega)$ has a unique solution. Based on the theoretical models presented in papers [2, 5] the oscillator strength, $f(aJ,bJ')$ of the absorption transitions can be calculated in the form:

$$f(aJ,bJ') = \frac{mc^2}{\pi e^2 Nl} \ln\left[I_0(\nu)/I(\nu)\right] \tag{9}$$

where $m$ is the mass of electron, $c$ represents the speed of light and $e$ is its electric charge.

The oscillator strength is related to the electric dipole line strength, $s_{ed}(aJ,bJ')$ by:

$$f(aJ,bJ') = \frac{8m\pi^2\bar{\nu}\left(n^2+2\right)^2}{27nh(2J+1)e^2} s_{ed}(aJ,bJ') \tag{10}$$

where $\bar{\nu}$ represents the mean frequency of the transition, $h$ is the Planck's constant and $n$ is the refractive index of the sample. The electric dipole line strength can be expressed as folowing:

$$s_{ed}(aJ,bJ') = e^2 \sum_{t=2,4,6} \Omega_t \left(\left\langle f^N[\alpha SI]J \left\| U^{(t)} \right\| \left\langle f^N[\alpha Ss]IJ \right\rangle\right.\right)^2 \tag{11}$$

In Eq. (11) $\Omega_t\ (t = 2, 4, 6)$ represent the three Judd-Ofelt phenomenological parameters, $U^{(t)}$ are the reduced matrix elements of the unit tensors, whose intermediate-coupled eigenstate is represented by $\left| f^N[\alpha sL]J \right\rangle$ and are calculated in [13]. The Judd-Ofelt phenomenological parameters reflect the effect of crystal field on the transitions, electronic wavefunctions and energy level separation and the reduced matrix elements of the unit tensors are reflecting the so-called intermediate coupling approximation. The spontaneous emission probabilities of the transitions, $A(aJ,bJ')$ of the ions from the states $aJ$ to $bJ'$ can be calculate using the the electric dipole line strength in the form:

$$A(aJ,bJ') = \frac{64\pi^4\bar{\nu}^3 n\left(n^2+2\right)^2}{27hc^3(2J+1)e^2} s_{ed}(aJ,bJ'). \tag{12}$$

Knowing the emission cross section one can evaluate the radiative life time, $\tau$ using the following relation [9]:

$$\frac{1}{\tau} = A = \frac{8\pi n^2}{c^2} \int \nu^2 \sigma_e(\nu) d\nu\ . \tag{13}$$

The temperature dependent excitation energy can be evaluated using the equation:

$$\varepsilon = k_B T \ln\left\{\left(c^2 A\right) / \left[8\pi n^2 \int \nu^2 \sigma_a(\nu) \exp\left(-\frac{h\nu}{k_B T}\right) d\nu\right]\right\}. \tag{14}$$

### 3. Experimental setup and results

In the range 1480-1620 nm (using a laser amplifier L. A. (Fig. 1)), around 980 nm (using a spectral lamp) and around 550 nm (using a laser diode operating at $\lambda$ =980 nm (Fig. 2)) we measued the experimental inhomogeneous absorption and respectively emission spectra of $Er^{3+}$doped $LiNbO_3$ optical waveguides [6-8].

In the experimental arrangement used to measure the transmission spectra shown schematically in Fig. 1 we used a He-Ne laser ($\lambda$=0.63 μm) for alignment and a laser amplifier (L. A.) at $\lambda$=1.55 μm for the optical signal, coupled together by a 3 dB coupler (C). The transmission spectra of some $LiNbO_3$:Er:Ti optical strip waveguides (W) X-cut 48 mm long and

Z-cut 52 mm long made by Pirelli-Cavi Laboratories (Milano-Italy) has been evaluated. The waveguides widths range from 5 $\mu$m to 9 $\mu$m.

The output signal from the waveguides has been detected by an optical spectrum analyzer (O. S. A.) used like a photodiode; then the measured data have been acquired by a computer (CO.).

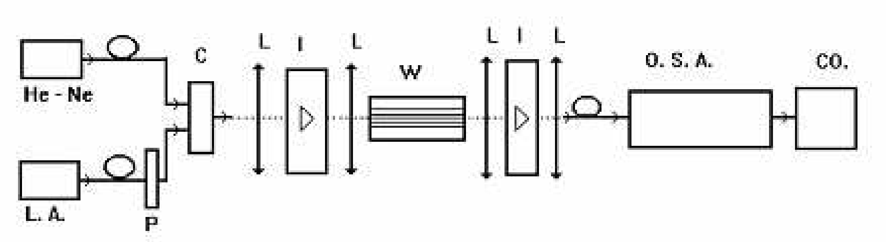

Fig. 1. The experimental arrangement used for the investigation of transmission spectra around 1530 nm.

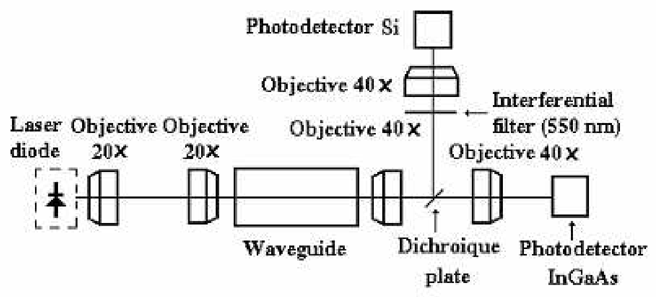

Fig. 2. The experimental setup used for the measurement of the flourescence spectra around 550 nm.

The experimental nonpolarised inhomogeneous absorption cross sections in the spectral regions above mentioned which correspond to the transitions between the levels $^4I_{15/2}\rightarrow{}^4I_{13/2}, (\lambda \approx 1480\,nm), {}^4I_{15/2}\rightarrow{}^4I_{11/2}, (\lambda \approx 980\,nm)$, and respectively $^4I_{15/2}\rightarrow{}^4S_{3/2}, (\lambda \approx 550\,nm)$ are presented in Figs. 3-5 [6-8].

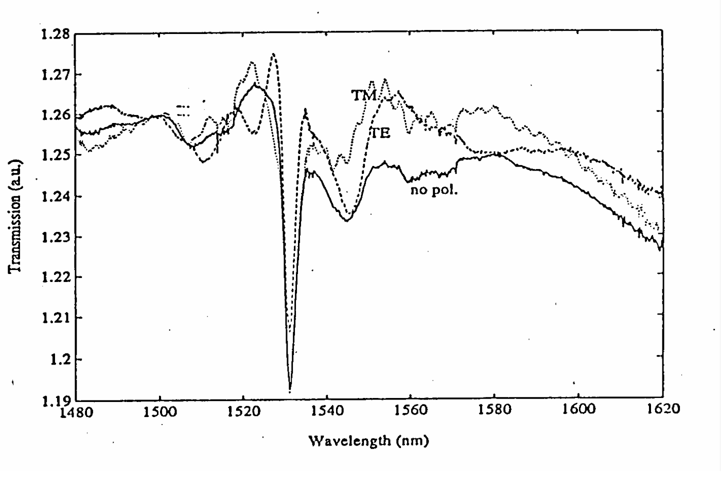

Fig. 3. The transmission spectra of a 7.5 μm $Er^{3+}$:Ti:$LiNbO_3$ optical waveguide in the range 1480-1620 nm, a) without polarization, b) TE and c) TM, respectively polarizations.

## 4. Evaluation of the spectroscopic parameters

In Figs. 6-8 are presented the experimental nonpolarised inhomogeneous absorption cross sections in the spectral regions above mentioned which correspond to the transitions between the levels: ${}^4I_{15/2}\rightarrow{}^4I_{13/2}$, $(\lambda\approx 1530\text{nm})$, ${}^4I_{15/2}\rightarrow{}^4I_{11/2}$, $(\lambda\approx 980\text{ nm})$, and ${}^4I_{15/2}\rightarrow{}^4S_{3/2}$,$(\lambda\approx 550\text{ nm})$, respectively.

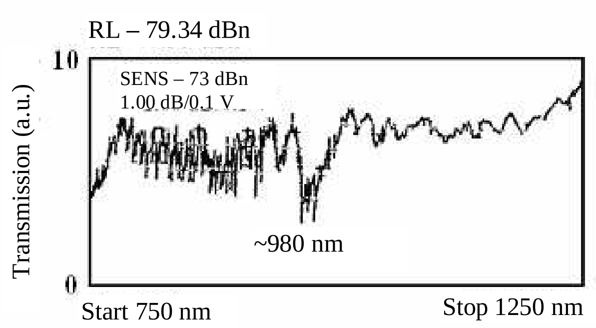

Fig. 4. The transmission spectra of a 7.5 μm $Er^{3+}$:Ti:$LiNbO_3$ optical waveguide in the range 750-1250 nm.

Based on the obtained experimental transmission spectra in the case of $Er^{3+}$-doped $LiNbO_3$ optical waveguides, presented in Section 3 of this paper we performed an exact determination of the homogeneous absorption and emission cross-sections in the regions above mentioned using the density matrix formalism, the McCumber's [11] theory taking into account the Stark splitting of the levels and the theoretical model outlined in Section 2 (Eqs. 1-8).

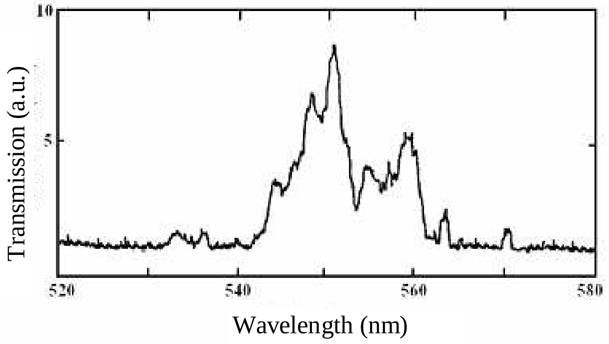

Fig. 5. The transmission spectra of an $Er^{3+}$:Ti:$LiNbO_3$ optical waveguide in the range 520-580 nm.

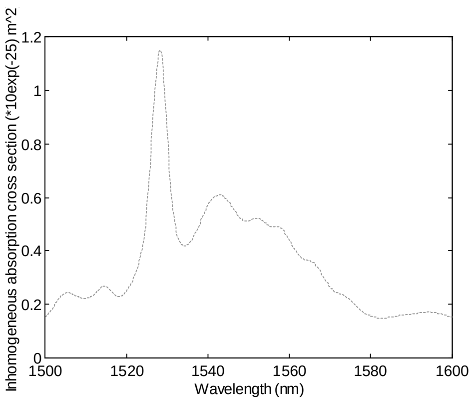

Fig. 6. The absorption cross section around 1530 nm.

The unpolarised homogeneous absorption and emission cross section spectra, respectively of an $Er^{3+}$:Ti:$LiNbO_3$ optical waveguide in the range 1500-1600 nm are presented in Figs. 9-10.

The peak values of the unpolarised homogeneous absorption, $\sigma_a^H$ and emission, $\sigma_e^H$ cross sections of an $LiNbO_3$:$Er^{3+}$:Ti optical waveguide in the range 1500-1600 nm, 960-1010 nm and 520-560 nm, respectively are presented in Table 1.

Also, using Eqs. (9)-(14) we evaluated the the oscillator strength of the absorption transition, the electric dipole line strength, the spontaneous emission probabilities, radiative lifetime and the excitation energy at room temperature $(T = 300\,\mathrm{K})$, which are presented in Table 1 while the Judd-Ofelt phenomenological parameters are presented in Table 2 of this section.

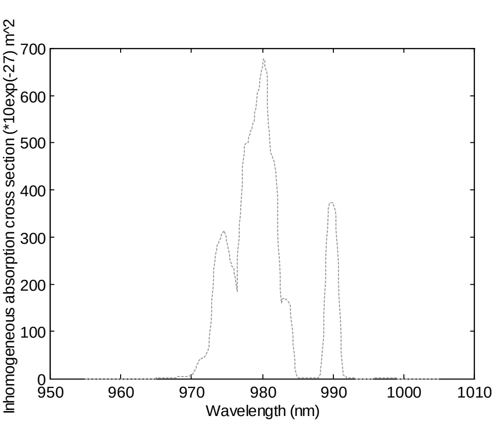

Fig. 7. The absorption cross section around 980 nm.

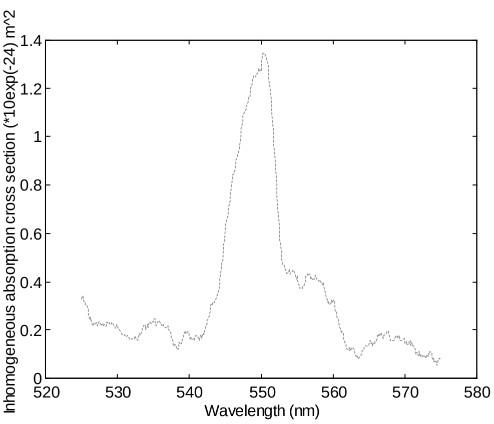

Fig. 8. The absorption cross section around 550 nm.

The Judd-Ofelt phenomenological parameters were evaluated using a least-square fit and the evaluated data presented in [13].

The dependences of the excitation energy vs the wavelength in the range 1500-1600 nm, 960-1010 nm and 520-560 nm, respectively for three values of the temperatures: 77 K, 300 K and 400K are presented in Figs. 12-14.

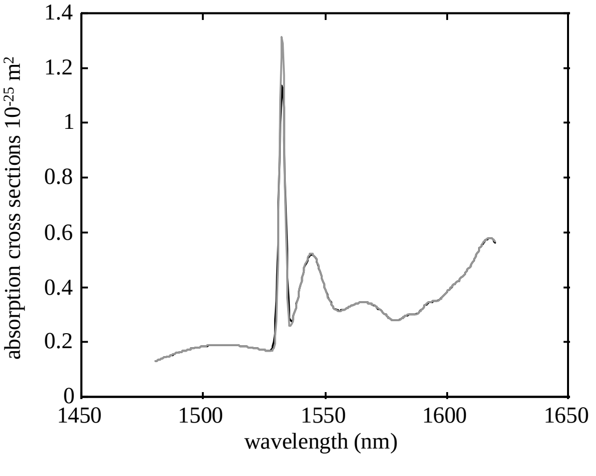

Fig. 9. The unpolarised homogeneous absorption cross section spectra of an $Er^{3+}$:Ti:$LiNbO_3$ waveguide.

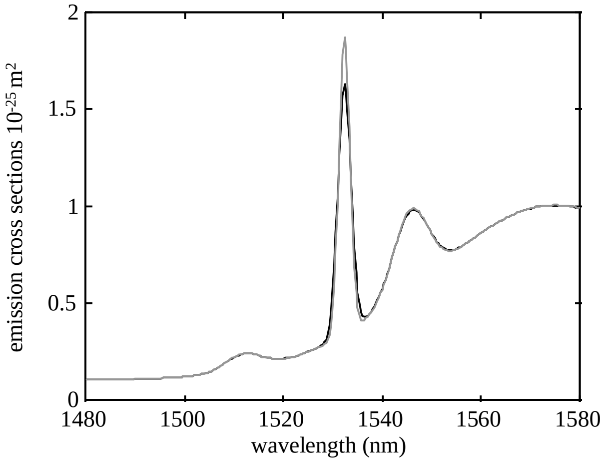

Fig. 10. The unpolarised homogeneous emission cross section spectra of an $Er^{3+}$:Ti:$LiNbO_3$ waveguide.

As can be seen from Figs. 11-13 the variation of the excitation energy vs the wavelength in the three ranges mentioned above for several values of the temperature (77 K, 300 K and 400K) is rather slow.

## 5. Conclusions

In this paper we presented a method to evaluate the homogeneous absorption and emission cross sections of $Er^{3+}$-doped $LiNbO_3$ optical waveguides from the transmission spectra. This method is based on the density matrix formalism and the McCumber's theory taking into account the Stark splitting of the levels.

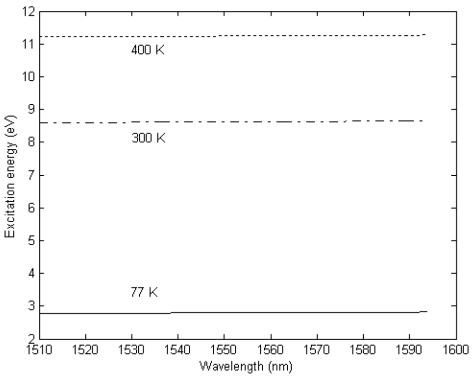

Fig 11. The excitation energy vs wavelength in the range 1510-1594 nm.

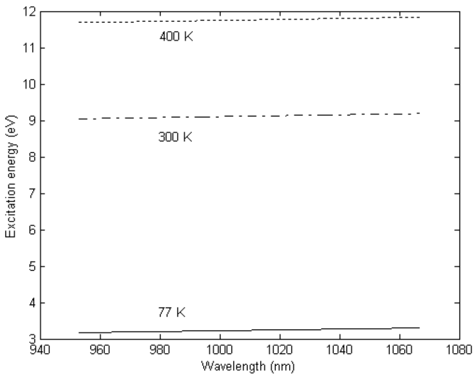

Fig 12. The excitation energy vs wavelength in the range 960-1010 nm.

Also, we evaluated the oscillator strength of the absorption transition, the electric dipole line strength, the Judd-Ofelt phenomenological parameters, the spontaneous emission probabilities, radiative lifetime and the excitation energy in three regions of the optical spectrum: around 1550 nm, 980 nm and 550 nm, respectively, which characterize the waveguide [12-14].

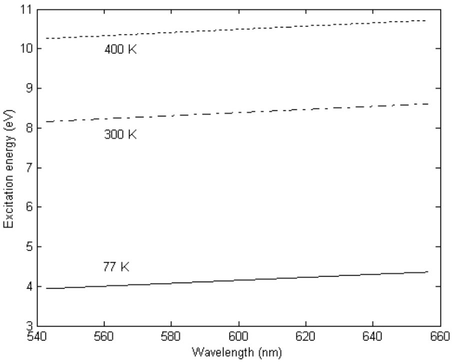

Fig 13. The excitation energy vs wavelength in the range 520-560 nm.

Table 1.

| Parameter | $\lambda$ =1530 (nm) | $\lambda$ =980 (nm) | $\lambda$ =550 (nm) |
|---|---|---|---|
| $\sigma_a^H\left(\mathrm{m}^2\right)\times 10^{-24}$ | 0.13 | 0.76 | 1.32 |
| $\sigma_e^H\left(\mathrm{m}^2\right)\times 10^{-24}$ | 0.18 | 0.65 | 0.58 |
| $f\times 10^{-6}$ | 0.19 | 0.27 | 1.19 |
| $s_{ed}\left(\mathrm{m}^2\right)\times 10^{-24}$ | 7.92 | 7.06 | 17.21 |
| $A\left(\mathrm{s}^{-1}\right)$ | 224.1 | 496.7 | 666.2 |
| $\tau(\mathrm{s})\times 10^{-3}$ | 4.46 | 2.01 | 1.51 |
| $\varepsilon(\mathrm{eV})$ | 8.61 | 8.68 | 8.19 |

Table 2.

| | | | |
|---|---|---|---|
| $\Omega\left(\mathrm{m}^2\right)\times 10^{-24}$ | $\Omega_2$=0.21 | $\Omega_4$ =2.12 | $\Omega_6$=4.72 |

The obtained results are in good agreement with other results obtained by several authors [3,5] and can be used in the design of optoelectronic integrated components.

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