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Determination of normalized propagation constants for the double-clad planar Nd:YAG and Yb:YAG waveguide lasers

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The exact normalized propagation constants for the five-layer slab waveguide laser and for its three-layer slab sub-structures (infinite clad waveguide and coreless structure) are determined at pumping and lasing wavelengths. Since the core dimensions of a Nd:YAG waveguide laser are higher compared with a Yb:YAG waveguide laser, the proportion of the fundamental mode that exists as an evanescent field and the losses are smaller to the Nd^{3+}-doped structure.

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1. Introduction

The determination of normalized propagation constant of a waveguide structure is very important in integrated optics [1-3]. For example, the lasing modes propagating in a planar waveguide laser define the output beam quality. Also, the laser threshold and efficiency characteristics are dependent on the pump and laser mode sizes [1].

In this paper the exact normalized propagation constants for the five-layer slab symmetric waveguide laser and for its three-layer slab sub – structures (infinite clad waveguide and coreless structure) are determined at pumping and lasing wavelengths. The structure of this waveguide laser consists of a core (Nd:YAG or Yb:YAG), a inner cladding (undoped YAG) and an outer cladding (sapphire). If the core-to-cladding-size ratios are 0.67 for Nd:YAG and 0.44 for Yb:YAG waveguides, then the gain mode selection is controlled by the confinement of the doping rather than the optical confinement of the core (the pump light is gradually absorbed each time it passes through the doped region) and a single spatial mode laser output is obtained [1].

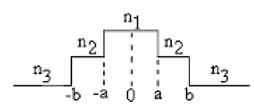
2. Theory

The scalar - wave equation of the waveguide (TE polarization) is given by

$$\frac{d^2\Psi(x)}{dx^2} + k^2 n^2(x)\Psi(x) = \beta^2\Psi(x), \qquad (1)$$

where n(x) is the refractive index profile, β is the propagation constant and k is the free space wave number. The refractive index n(x) for a finitely clad slab waveguide is given by the relation

$$n(x) = n_1, \; -a \le x \le a$$
$$n(x) = n_2, \; -b \le x < -a \text{ and } a < x \le b$$
$$n(x) = n_3, \; -\infty < x < -b \text{ and } b < x < \infty$$



(2)

where n_1, n_2 and n_3 are the refractive indices of the core, the inner-cladding and the surrounding (outer-cladding) material, respectively ($n_1 > n_2 > n_3$), 2a and 2b are the thickness of the core and the inner-cladding, respectively.

$$[2a_1b_1\cos a_3 + (a_1^2 - b_1^2)\sin a_3](b_1 - b_2)^2 - [2a_1b_1\cos a_4 + (a_1^2 - b_1^2)\sin a_4]$$
$$(b_1 + b_2)^2 - 2(a_1^2 + b_1^2)(b_1^2 - b_2^2)\sin(2ab_2) = 0, \qquad (3)$$

$$a_1 = \sqrt{\beta^2 - (n_3k)^2},\; b_1 = \sqrt{(n_2k)^2 - \beta^2},\; b_2 = \sqrt{(n_1k)^2 - \beta^2}, \qquad (4)$$

$$a_3 = 2[b_1b - a(b_1 + b_2)],\; a_4 = 2[b_1b + a(b_2 - b_1)], \qquad (5)$$

The exact eigenvalue equation is obtained from the boundary conditions of the wave function.

A different exact eigenvalue equation

$$b_1 \tan[b_1(a-b) + \arctan(A_3/b_1)] = -a_1, \qquad (6)$$

$$A_3 = -b_2\tan[2ab_2 - \arctan(A_2/b_2)],$$
$$A_2 = -b_1\tan[b_1(b-a) - \arctan(a_1/b_1)] \qquad (7)$$

is obtained by solving a corresponding first-order differential equation with the initial conditions for each region (similar with the numerical Runge-Kutta method).

Also, due to the symmetry of the waveguide, the even and odd modes can be found by applying the boundary conditions of continuity to the electric field [1]

$$\tan(b_2a) - \frac{a_1a_2 + a_2^2\tanh[a_2(b-a)]}{b_2a_2 + b_2a_1\tanh[a_2(b-a)]} = 0 \text{ , even modes,} \qquad (8)$$

$$\cot(b_2a) + \frac{a_1a_2 + a_2^2\tanh[a_2(b-a)]}{b_2a_2 + b_2a_1\tanh[a_2(b-a)]} = 0 \text{ , odd modes,} \qquad (9)$$

where

$$a_2 = \sqrt{\beta^2 - (n_2k)^2} \qquad (10)$$

The propagation constant β_∞ in the infinitely clad waveguide (the inner-cladding of the waveguide can be assumed to be infinite) is given by the eigenvalue equation [2]

$$U_1 \tan U_1 = W_1 \text{, even modes, } U_1 = a\sqrt{(n_1 k)^2 - \beta_\infty^2} \text{, } W_1 = a\sqrt{\beta_\infty^2 - (n_2 k)^2}$$
$$U_1 \tan(U_1 - \pi/2) = W_1 \text{, odd modes,} \tag{11}$$

and is a good approximation if the wavelength is much shorter than the core of the waveguide.

The propagation constant β_c for the fundamental mode in the coreless structure (an infinite outer cladding layer and no index change in the core due to the rare-earth doping) is given by the eigenvalue equation [2] U

$$U_2 \tan U_2 = W_2 \text{, even modes, } U_2 = b\sqrt{(n_2 k)^2 - \beta_c^2}\text{, } W_2 = b\sqrt{\beta_c^2 - (n_3 k)^2}$$
$$U_2 \tan(U_2 - \pi/2) = W_2 \text{, odd modes,} \tag{12}$$

and is a good approximation if the wavelength is much longer than the core of the waveguide.

The propagation constant β_{ts} for the fundamental mode (m = 0) in the two-sided perturbation method [2]

$$\beta_{ts}^2 = \beta_c^2 + \left(\frac{V_1}{a}\right)^2 \frac{F_1}{F_1 + F_2 + F_3}, \quad V_1 = ak\sqrt{n_1^2 - n_2^2} \quad , \tag{13}$$

where

$$F_1 = \frac{T_1}{U_1^2 - \left(\frac{a}{b}\right)^2 U_2^2}, F_2 = \frac{T_1 + T_2}{W_1^2 + \left(\frac{a}{b}\right)^2 U_2^2}, F_3 = \frac{T_2}{\left(\frac{a}{b}\right)^2 W_2^2 - W_1^2},$$

$$T_1 = W_1 \frac{\cos\left(\frac{aU_2}{b}\right)}{\cos(U_2)} - \frac{aU_2}{b}\frac{\sin\left(\frac{aU_2}{b}\right)}{\cos(U_2)}, T_2 = \left(\frac{a}{b}W_2 - W_1\right)\exp\left[-\left(\frac{b}{a} - 1\right)W_1\right]$$

is a good approximation for large, small and medium a/λ. The solution for the five-layer slab waveguide can thus be obtained from the solutions for three-layer slab structures [2].

For the comparison of effective refractive indices given by different methods, the normalized propagation constant P_m of the TE_m-mode was found to be much more suitable than just the direct comparison of propagation constants

$$P_m = \frac{\left(\frac{\beta_m}{k}\right)^2 - n_3^2}{n_1^2 - n_3^2} \quad , 0 < P_m < 1 \tag{14}$$

This normalized propagation constant is different from [2] where for $\beta/k = n_2$, $P = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} = 0$. The criterion to separate the ranges of applicability for various approximations is determined by the value of the parameter p (in the place of P = 0 in [2])

$$p = \frac{n_2^2 - n_3^2}{n_1^2 - n_3^2} \Rightarrow \begin{cases} P_m > p\text{, infinite-cladding approximation} \\ P_m < p\text{, no-core approximation} \end{cases} \tag{15}$$

To evaluate the exact normalized propagation constant for the five-layer slab symmetric waveguide laser we need to solve the eigenvalue equation (3) or (6) or (8-9).

Also, we have solved the equations (1) for the given boundary conditions (the Dirichlet boundary condition at the ends of the interval where the wave function can be approximated with 0 and the Neuman boundary condition for x = -b, -a, a, b) by using the Galerkin's variant of the finite element method, with triangular grid and variable step [4].

3. Numerical results and conclusions

The calculated values of the normalized propagation constants P_m by using the finite element method are very close to the exact values from the eigenvalue equation (3) or (6) or (8-9) (their equivalence has been verified by the numerical results).

We have calculated the normalized propagation constants P_m as functions of the m order (TE_m-modes) at pumping and lasing wavelengths of the Nd:YAG and Yb:YAG waveguide lasers (Fig. 1). The Nd^{3+}-doped structures support 35 modes for pumping, λ_p = 0.808 μm and 27 modes for lasing wavelengths, λ_s = 1.064 μm and the Yb^{3+}-doped structures support 18 modes for pumping, λ_p = 0.941 μm and 17 modes for lasing wavelengths, λ_s = 1.029 μm respectively. The number of the modes in the Yb:YAG waveguide laser is smaller in comparison with the Nd:YAG waveguide laser since the core dimensions of a Nd:YAG laser are higher with respect to a Yb:YAG laser. The coreless approximation gives the same number of modes as the exact method.

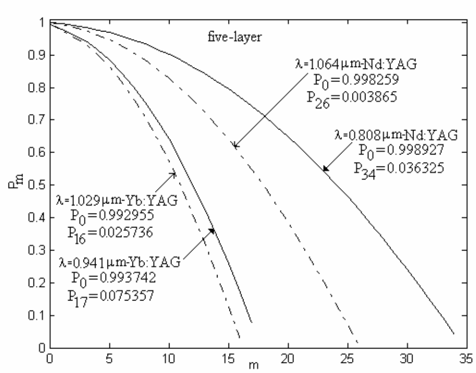


Fig. 1. Normalized propagation constants P_m as functions of the m order (TE_m-modes) at pumping and lasing wavelengths of the Nd:YAG and Yb:YAG waveguide lasers.

Fig. 2 shows the calculated electric field (normalized to a maximum value of 1) for the fundamental mode TE_0 at pumping and lasing wavelengths of the Nd:YAG and Yb:YAG waveguide lasers. In this figure ΔX is the complete width at the 1/e points of the electric field and the proportion of the fundamental mode that exists as an evanescent field is 0.36 at the pumping and 0.41 at the lasing wavelengths of the Nd:YAG waveguide lasers and 0.55 at the pumping and 0.58 at the lasing wavelengths of the Yb:YAG waveguide lasers, respectively. Since the core dimensions of a Nd:YAG waveguide laser are higher with respect to a Yb:YAG waveguide laser, the proportion of the fundamental mode that exists as an evanescent field and the losses are smaller to the Nd^{3+}-doped structure.

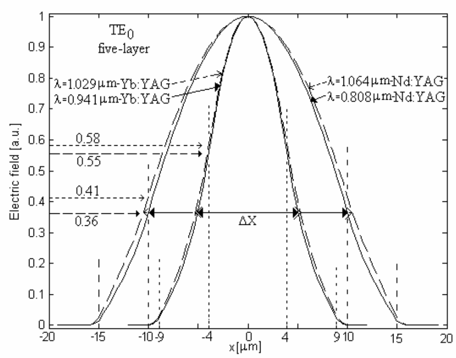


Fig. 2. The calculated electric field (normalized to a maximum value of 1) by using the finite element method for the fundamental mode TE_0 at pumping and lasing wavelengths of the Nd:YAG and Yb:YAG waveguide lasers. ΔX is the complete width at the 1/e points of the electric field.

Fig. 3 shows the calculated electric field for the fundamental mode TE_0, first-order mode TE_1 and the highest-order mode TE_{34} at pumping wavelength of the Nd:YAG waveguide laser. Fig. 4 shows the calculated electric field for the fundamental mode TE_0, first-order mode TE_1 and the highest-order mode TE_{17} at pumping wavelength of the Yb:YAG waveguide laser.

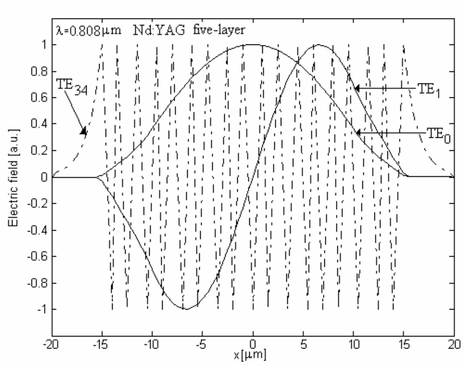


Fig. 3. The calculated electric field by using the finite element method for the fundamental mode TE_0, first-order mode TE_1 and the highest-order mode TE_{34} at pumping wavelength of the Nd:YAG waveguide laser.

Also, we have calculated the normalized propagation constants $P_0^{n\,c}$ (no-core), $P_0^{\infty\,c}$ (infinite cladding), P_0^{tsp} (two-sided perturbation) and P_0^{exact} (exact) for the fundamental mode TE_0 at pumping and lasing wavelengths of the Nd:YAG waveguide laser (Table 1) and of the Yb:YAG waveguide laser (Table 2). The value of the parameter p is used as a criterion to separate the ranges of applicability for no-core and infinite cladding approximations.

Table 1. Comparison of normalized propagation constants $P_0^{n\,c}$ (no-core), $P_0^{\infty\,c}$ (infinite cladding), P_0^{tsp} (two-sided perturbation) and P_0^{exact} (exact) for the fundamental mode TE_0 at pumping and lasing wavelengths of the Nd:YAG waveguide laser. ΔX is the complete width at the 1/e points of the electric field. The value of the parameter p is used as a criterion to separate the ranges of applicability for no-core and infinite cladding approximations.

	Nd:YAG, a=10μm, b=15μm n_1=1.8216,n_2=1.8212, n_3=1.760 λ=0.808μm – pumping [1] p = 0.993396, ΔX = 20.32μm	Nd:YAG,a=10μm,b=15μm n_1=1.8151,n_2=1.8147, n_3=1.755 λ=1.064μm – lasing [1] p = 0.993233, ΔX = 21.25μm
$P_0^{n\,c}$	0.992603	0.991837
$P_0^{\infty\,c}$	0.998980	0.998460
P_0^{tsp}	0.998883	0.998160
P_0^{exact}	0.998927	0.998259

Table 2. Comparison of normalized propagation constants $P_0^{n\,c}$ (no-core), $P_0^{\infty\,c}$ (infinite cladding), P_0^{tsp} (two-sided perturbation) and P_0^{exact} (exact) for the fundamental mode TE_0 at pumping and lasing wavelengths of the Yb:YAG waveguide laser. ΔX is the complete width at the 1/e points of the electric field. The value of the parameter p is used as a criterion to separate the ranges of applicability for no-core and infinite cladding approximations.

	Yb:YAG, a=4μm, b=9μm n_1=1.81876,n_2=1.8175, n_3=1.757 λ=0.941μm – pumping [1] p = 0.980222, ΔX = 10.39μm	Yb:YAG, a=4μm, b=9μm n_1=1.8166,n_2=1.8154, n_3=1.755 λ=1.029μm – lasing [1] p = 0.980190, ΔX = 10.86μm
P_0^{nc}	0.977335	0.976751
$P_0^{\infty\,c}$	0.993884	0.993199
P_0^{tsp}	0.993531	0.992674
P_0^{exact}	0.993742	0.992955

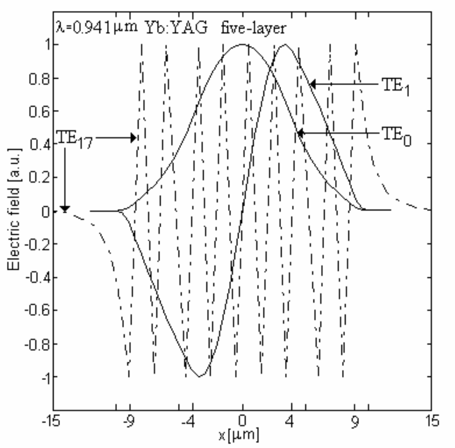


Fig. 4. The calculated electric field by using the finite element method for the fundamental mode TE_0, first-order mode TE_1 and the highest-order mode TE_{17} at pumping wavelength of the Yb:YAG waveguide laser.

Fig. 5 shows a comparison of the calculated electric field (normalized to a maximum value of 1) for the fundamental mode TE_0 at pumping wavelength of the Nd:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer). Fig. 6 shows a comparison of the calculated electric field for the fundamental mode TE_0 at pumping wavelength of the Yb:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer). The calculated electric field by using the infinite cladding approximation is very close to the exact electric field in the core region (Figs. 5 and 6) because the parameter $p < P_0$ (relation 15, see also the Tables 1-2). Also, the infinite cladding approximation gives a more accurate normalized propagation constant P_0 than the two-sided perturbation method at pumping wavelength of the Yb:YAG waveguide laser (Table 2).

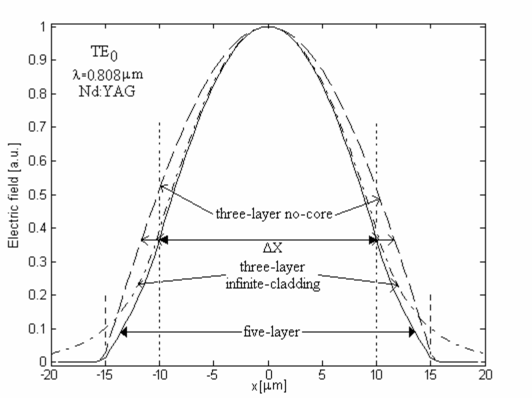


Fig. 5. Comparison of the calculated electric field (normalized to a maximum value of 1) by using the finite element method for the fundamental mode TE_0 at pumping wavelength of the Nd:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer). ΔX is the complete width at the 1/e points of the electric field.

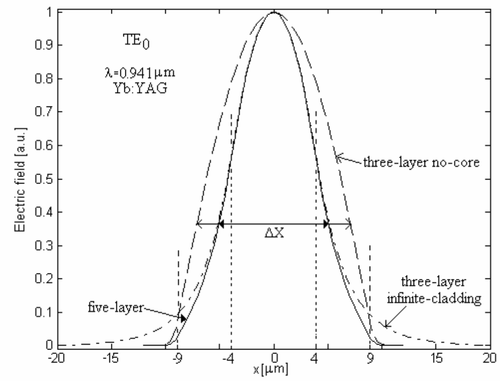


Fig. 6. Comparison of the calculated electric field (normalized to a maximum value of 1) by using the finite element method for the fundamental mode TE_0 at pumping wavelength of the Yb:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer). ΔX is the complete width at the 1/e points of the electric field.

Fig. 7 shows a comparison of the calculated electric field for first-order mode TE_1 at pumping wavelength of the Nd:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer). Fig. 8 shows a comparison of the calculated electric field for the first-order mode TE_1 at pumping wavelength of the Yb:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer). It can be seen that both waveguides allow only two propagation modes in the infinite cladding approximations [1] and the mode TE_1 of the Yb:YAG waveguide laser have appreciable evanescent intensity in comparison with the mode TE_1 of the Nd:YAG waveguide laser (Figs. 7-8).

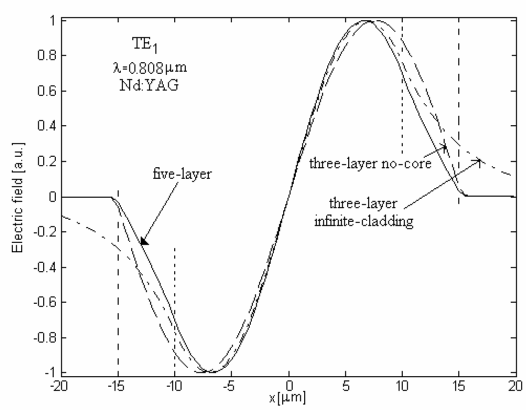


Fig. 7. Comparison of the calculated electric field by using the finite element method for the first-order mode TE_1 at pumping wavelength of the Nd:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer).

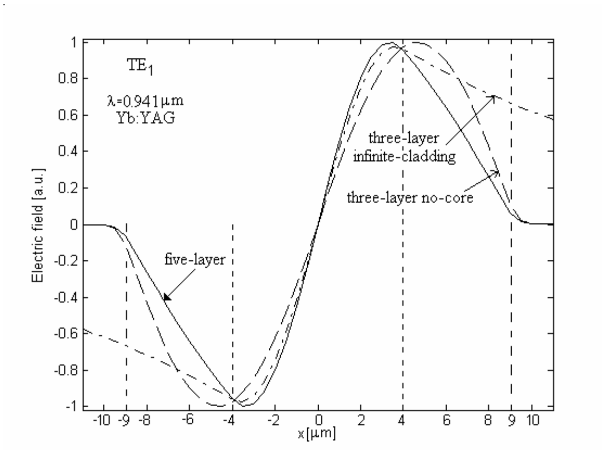


Fig. 8. Comparison of the calculated electric field by using the finite element method for the first-order mode TE_1 at pumping wavelength of the Yb:YAG waveguide laser for no-core and infinite cladding approximations with the exact method (five-layer).

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