

Collective modes and disorder in two-dimensional systems

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Using a simple repulsive contact interaction model we calculated the dispersion relation for the zero sound mode in two dimensional interacting Fermi systems. The almost linear dispersion relation reduces to the linear form in the long-wavelength limit. The damping of the mode appears at a critical wavevector which was calculated. The electron plasma frequency was calculated for a two dimensional system in the presence of disorder described by a phenomenological diffusive response function. The collective modes were analyzed in the case of a non-Coulombian interaction in both cases, without and with disorder. The case with a logarithmic interaction in disordered systems is also analyzed.

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1. Introduction

The problem of the existence of the collective modes was one of the most important problem since the beginning of the condensed matter theory. Among these modes there are of the particular importance the collective modes occurring in the interacting Fermi systems [1-3], such as zero sound and electron plasma modes. They have been intensively analyzed in many systems, such as low dimensional interacting systems [4-6], or bulk systems (see e.g. Ref.[2]). Since the discovery of the high-temperature superconductors the interest on the problem of collective modes was renewed. In the scenario of a non-Fermi liquid (where the standard Fermi liquid theory breakdown), the low dimensional interacting systems cannot be described by quasiparticles due to the presence of a branch cut structure in the electron propagator, and the collective modes are important [7-9]. Recently, Bill et.al.[10] pointed out the importance of the electron plasma modes in the superconductivity of the metal-intercalated halide nitrides, which are layered systems (quasi-two-dimensional systems). In this paper we analyzed the collective modes (zero sound, and plasma modes) in several different models that can have a particular importance for clean and disordered low dimensional systems, such as high-temperature materials and other systems.

2. Collective modes in two dimensions

2.1 Zero sound mode

Zero sound mode is a collective mode that occurs in a neutral Fermi system at zero temperature. A repulsive contact interaction (short-range interaction, $V(r) \sim \delta(r)$) is sufficient to produce such a mode [1,2]. The zero sound

mode represent an oscillation of the Fermi surface with a velocity larger than Fermi velocity v_F , and with a linear dispersion relation. Landau analyzed this collective mode, and in three dimensions the dispersion relation is given by Landau equation [3]:

$$(\omega/qv_F) \ln|(\omega+qv_F)/(\omega-qv_F)| = 2 + (2/N(0)V) \quad (1)$$

as long as the collective mode is well defined ($\omega > \omega_{pq}^0$, with $\omega_{pq}^0 = p_F q/m + q^2/2m$). Here $N(0)$ is the electron density of states, V the strength of the fermion-fermion interaction, p_F the Fermi momentum, and q the wavevector. In the weak-coupling limit ($N(0)V \ll 1$), Eq.(1) possesses the solution:

$$\omega = qv_F [1 + (2/e^2) \exp(-2/N(0)V)] \quad (2)$$

The damping of the zero sound mode occurs at a critical wavevector q_c that is given by the condition when the collective mode spectrum overlaps the electron-hole excitation spectrum:

$$q_c = (4p_F/e^2) \exp(-2/N(0)V) \quad (3)$$

Here we will reconsider the problem of the zero sound mode in two dimensional interacting Fermi systems, using the equation of motion method for an electron-hole pair, and the Random Phase Approximation (RPA). In this approximation, and in the absence of damping, the dispersion relation is given by:

$$1 = 2V(q) \int \frac{d^2p}{(2\pi)^2} \left(\frac{1}{\omega - (pq/m) \cos \theta - q^2/2m} - \frac{1}{\omega + (pq/m) \cos \theta + q^2/2m} \right) \quad (4)$$

$V(q)$ is the Fourier transform of the contact interaction. The integrals from Eq.(4) will be calculated in a similar way of Stern [4], and Ando et.al.[5], concerning the

properties of two dimensional systems. For the angular integral we use:

$$\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = \begin{cases} \frac{2\pi}{\sqrt{a^2 - b^2}}; a > |b| \\ 0; |a| < |b| \end{cases} \quad (5)$$

Using the density of states in two dimensions:

$$N(0) = m/2\pi \quad (6)$$

and taking the Fourier transform of the contact interaction:

$$V(q) = V/2 \quad (7)$$

the dispersion relation, obtained from Eq.(4), is:

$$\omega = qv_F \sqrt{1 + \frac{N(0)V}{2} \cdot \frac{1}{1 + (1/2N(0)V)} + \left(\frac{q}{2p_F}\right)^2 \left(1 + \frac{1}{N(0)V}\right)^2} \quad (8)$$

with the condition:

$$\omega > qv_F (1 + q/2p_F) \quad (9)$$

In the long-wavelength limit ($q \rightarrow 0$, $q/2p_F \ll 1$), Eq.(8) becomes:

$$\omega \cong qv_F \sqrt{1 + \frac{N(0)V}{2} \cdot \frac{1}{1 + (1/2N(0)V)}} \quad (10)$$

In the weak-coupling limit (consistent with RPA), $N(0)V \ll 1$, and Eq.(10) reduces to:

$$\omega \cong qv_F \left[1 + \frac{1}{2} (N(0)V)^2 \right] \quad (11)$$

The almost linear dispersion relation (8) becomes linear in the long-wavelength limit, and the velocity is larger than Fermi velocity v_F . Using Eq.(11), the critical wavevector in two dimensions becomes:

$$q_c = p_F [N(0)V]^2 \quad (12)$$

In the strong-coupling limit, the dispersion relation is given by:

$$\omega \cong qv_F \sqrt{\frac{N(0)V}{2}} \quad (13)$$

a result with a similar form to the relation in three dimensions [1].

2.2 Plasmon modes in disordered two-dimensional systems

The problem of plasma modes in the presence of disorder and in two dimensions has been analyzed by Giuliani and Quinn [11]. The main idea was that these collective modes can exist for wavevectors larger than a critical value. The density-density response function used in Ref.[11] was an improved form for a diffusive response function determined by many-body methods. Here we will give an alternative way to obtain plasma modes in two dimensions, using a much simple phenomenological model, given by Kadanoff and Martin [12]:

$$\chi(q, \omega) = -N(0) \frac{Dq^2}{Dq^2 - i\omega - \omega^2 \tau_0} \quad (14)$$

Here, $D = v_F^2 \tau_0 / 2$ is the diffusion coefficient in two dimensions, and τ_0 characterizes the lag in the response of the system. The dielectric response function $\varepsilon(q, \omega)$ is given by:

$$\varepsilon(q, \omega) = 1 - V(q)\chi(q, \omega) \quad (15)$$

where $V(q)$ is the Fourier transform of the Coulomb potential:

$$V(q) = 2\pi e^2 / q \quad (16)$$

The frequency of the collective modes is determined by the condition $\varepsilon(q, \omega) = 0$, which gives the following result:

$$\omega = -\frac{i}{2\tau_0} + \sqrt{-\frac{1}{4\tau_0^2} + \frac{v_F^2 q_s^2 \tau_0}{2\tau_0} x(x+1)} \quad (17)$$

where $q_s = 2\pi e^2 N(0)$ is the screening wavevector in two dimensions, and $x = q/q_s$. Eq.(17) represent a damped mode, similar to that from Ref.[11]. In order to avoid the overdamping, one should consider the condition:

$$v_F^2 q_s^2 \tau_0 x(x+1)/2\tau_0 - 1/4\tau_0^2 \geq 0 \quad (18)$$

which is equivalent to:

$$q \geq q_c \equiv \frac{q_s}{2} \left[\sqrt{1 + \frac{1}{Dq_s^2 \tau_0}} - 1 \right] \quad (19)$$

This condition ensure for a collective mode as long as $\text{Re } \omega > |\text{Im } \omega|$. However, in the case $\tau_0 = 0$ (classical diffusive response function [13]), there are no collective modes in two dimensions and in the presence of disorder ($|\text{Im } \omega| \rightarrow \infty$, and $q_c \rightarrow \infty$).

2.3 Collective modes in non-Columbian systems

In this section we will discuss the problem of the collective modes in two dimensional interacting systems, with the following model for the interaction potential:

$$V(r) \sim \frac{1}{r^\alpha} \quad (20)$$

The non-Coulombian model (that reduces to Coulomb interaction when $\alpha = 1$), has been discussed in Refs.[8,14] in connection to the Luttinger liquid behavior, and high-temperature superconductors with non-magnetic impurities, and in Ref.[15] in connection to the Thomas-Fermi model. In order to obtain the frequency of the collective modes we use Eq.(15). The Fourier transform of the non-Coulomb potential, in two dimensions, is:

$$V(q) = \int d^2r \cdot V(r) \cdot \exp(-iqr \cdot \cos\theta) \quad (21)$$

(with appropriate units, in Eq.(20)). Eq.(21) is equivalent to:

$$V(q) = 2 \int_0^\infty dr \cdot r^{1-\alpha} \int_0^\pi d\theta \cdot e^{-iqr \cos\theta} \quad (22)$$

The angular integral is $\pi J_0(qr)$, where $J_0(x)$ is the Bessel function, and the remaining integral is calculated using the formula:

$$\int_0^\infty dx \cdot x^\mu \cdot J_\nu(ax) = 2^\mu a^{-\mu-1} \frac{\Gamma\left(\frac{1}{2} + \frac{\nu}{2} + \frac{\mu}{2}\right)}{\Gamma\left(\frac{1}{2} + \frac{\nu}{2} - \frac{\mu}{2}\right)} \quad (23)$$

$$- \operatorname{Re} \nu - 1 < \operatorname{Re} \mu < 1/2, \quad a > 0$$

where $\Gamma(x)$ is the Gamma function. The result is:

$$V(q) = \frac{C(\alpha)}{q^{2-\alpha}} \quad (24)$$

where the factor $C(\alpha)$ is given by:

$$C(\alpha) = 2^{2-\alpha} \left[\Gamma\left(1 - \frac{\alpha}{2}\right) \right]^2 \sin\left(\frac{\pi\alpha}{2}\right) \quad (25)$$

Due to the restrictions in Eq.(23), Eq.(24) is valid for $1/2 < \alpha < 2$. For the response function $\chi(q, \omega)$ we will adopt the classical RPA result [2]:

$$\chi(q, \omega) = \frac{2q^2}{m} \int_{p < p_F} \frac{d^2p}{(2\pi)^2} \cdot \frac{1}{\left(\omega - \frac{pq}{m} \cos\theta\right)^2 - \frac{q^4}{4m^2}} \quad (26)$$

With Eqs.(24-26), and in the long-wavelength limit, the frequency of the collective modes becomes:

$$\omega = \sqrt{\frac{NC(\alpha)}{2m}} \cdot q^{\alpha/2} \cdot \sqrt{1 + \frac{6\pi}{mC(\alpha)} q^{2-\alpha}} \quad (27)$$

where $N = 2 \int_{p < p_F} d^2p / (2\pi)^2$ is the particle density. In the

$\alpha = 1$ limit, Eq.(27) reduces to the classical form of the electron plasma frequency, $\omega \sim \sqrt{q}$. Very close to $\alpha = 2$, ω increases rapidly, that can show an instability in the system.

2.4 Collective modes in non-Columbian systems with disorder

In this case, we model the two dimensional system in the presence of disorder, with the density-density response function given by Eq.(14), and the interaction given by Eq.(24). With Eq.(15), the frequency of the collective modes is given by the following equation:

$$\frac{\tau_0}{\tau} \omega^2 + \frac{i}{\tau} \omega - \frac{v_F^2}{2} F(\alpha) q^\alpha \left[1 + \frac{q^{2-\alpha}}{F(\alpha)} \right] = 0 \quad (28)$$

where $F(\alpha) = N(0)C(\alpha)$, and we obtain:

$$\omega = -\frac{i}{2\tau_0} + \sqrt{-\frac{1}{4\tau_0^2} + \frac{v_F^2 \tau}{2\tau_0} F(\alpha) q^\alpha \left[1 + \frac{q^{2-\alpha}}{F(\alpha)} \right]} \quad (29)$$

In order to have collective modes in the system, the real part of the Eq.(29) should exceed appreciably the imaginary part. However, the following condition (together with $\operatorname{Re} \omega > |\operatorname{Im} \omega|$) should be fulfilled:

$$q^2 + F(\alpha) q^\alpha - \frac{1}{2v_F^2 \tau \tau_0} \geq 0 \quad (30)$$

Close to $\alpha \approx 2$, $\alpha < 2$, the following condition is found:

$$q \geq \sqrt{\frac{1}{2v_F^2 \tau \tau_0 [1 + F(\alpha)]}} \quad (31)$$

Again, when $\tau_0 \rightarrow 0$, there are no collective modes in the system. For finite τ_0 , and α close to 2 ($F(\alpha)$ - very large), the collective modes can exist for very small values of the wavevectors.

2.5 Collective modes in a strictly two-dimensional world with disorder

The case discussed above, with $V(q) \sim 1/q$, corresponds to a Coulomb interaction in a plane that is embedded in a three-dimensional space. In a strictly two-dimensional world, the Coulomb potential has a logarithmic form:

$$V(r) \sim \ln |r| \quad (32)$$

The Fourier transform of Eq.(32) is easily evaluated, and gives the following result:

$$V(q) = \frac{A}{q^2} \quad (33)$$

where A is a constant. In the presence of disorder, described by Eq.(14), the dielectric function (15) is:

$$\varepsilon(q, \omega) = 1 + \frac{AN(0)D}{Dq^2 - i\omega - \omega^2\tau_0} \quad (34)$$

The frequency of the collective modes is now given by:

$$\omega = -\frac{i}{2\tau_0} + \sqrt{-\frac{1}{4\tau_0^2} + \frac{AN(0)D}{\tau_0} \left(1 + \frac{q^2}{AN(0)}\right)} \quad (35)$$

These collective modes are well defined as long as the wavevector q exceeds appreciably the critical value q_c :

$$q \geq q_c \equiv q^* \sqrt{\frac{1}{2v_F^2\tau_0(q^*)^2} - 1} \quad (36)$$

where $(q^*)^2 = AN(0)$. Additionally, one should have the condition for τ_0 :

$$\tau_0 \leq \frac{1}{4D(q^*)^2} \quad (37)$$

When $\tau_0 \rightarrow 0$, there are no collective modes in this two-dimensional system.

3. Conclusions

In conclusion, we calculated, using a simple model, the dispersion relation for the zero sound mode in two dimensions. Such a mode can occur in interacting Fermi systems (e.g. ^3He thin layers). However, the ^3He case seems to be much more complicated, and have to be carefully analyzed [16]. Additionally, strong interacting systems (e.g. high-temperature superconductors) have a non-Fermi liquid character (even in two dimensions) [7],

and require a separate treatment. We also have discussed a phenomenological theory of the plasma oscillations in a two-dimensional disordered system. As in the case of a microscopic theory [11], we predict the existence of a wave-vector cutoff for these modes. The plasma excitations in quasi-two-dimensional systems have been observed experimentally, via infrared transmission and emission techniques [17,18]. We also discussed the problem of the collective modes in the non-Coulombian systems, without and with disorder, and the collective modes in a strictly two-dimensional world in the presence of disorder. Here, further experimental investigations are necessary in order to give a relevance to the predictions we made.

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