

Determination of propagation constants in a buried ion-exchanged glass optical waveguide by using finite element method

V. A. POPESCU*, N. N. PUSCAS

Physics Department, University "Politehnica" of Bucharest, Splaiul Independentei 313, 060042 Bucharest, Romania

The finite element method is used to determine the propagation constants in a buried ion-exchanged glass waveguide when an electric field is applied during the diffusion anneal. For a high applied electric field and short field-assisted annealing time, we have obtained a well confined optical mode and a shift of the electric field intensity peak towards higher values of the depth, which is based on a similar behaviour of the refractive index profile maximum. In the case of the planar waveguide, the depth fundamental mode profile is narrower than those of the channel waveguide. Our equivalent ionic mobility is better in comparison with the ionic mobility based on the Nernst-Einstein equation.

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1. Introduction

The fabrication of buried ion-exchanged waveguides using field-assisted annealing (FAA) in erbium-doped phosphate glasses has been shown to be advantageous compared with the thermal exchange because of an important reduction in the diffusion time, compatibility with single-mode fibers, a high gain of the waveguide amplifiers within a short waveguide length, lower loss and lower cost [1-2]. The K^+-Na^+ ion-exchange and field-assisted annealing enables a higher control of the geometric and optical waveguide parameters and to construct single-mode waveguides suitable for communications and optical sensors [3].

The determination of normalized propagation constant of a waveguide structure is very important in integrated optics [3-6]. For example, by using nanowires as single-mode subwavelength-diameter optical waveguides, it is possible to use a high fraction of the guided energy as evanescent wave for optical sensor with high sensitivity [3].

In this paper the finite element method is used to determine the propagation constants in a buried ion-exchanged glass channel waveguide and field-assisted annealing with the reconstruction of refractive index profile (in depth) from near field measurements [1-2].

2. Planar waveguide

The refractive index profile $n(y)$ of the ion-exchanged glass planar waveguides using field-assisted annealing has been shown to be [1,7,8]

$$n(y) = n_s + \frac{\Delta n}{\sqrt{\pi}} \sqrt{\frac{Dt}{D_f t_f}} \left\{ \int_0^{\infty} \operatorname{erfc}(\xi) \exp \left[- \left(\frac{y - \xi(2\sqrt{Dt}) - a}{2\sqrt{D_f t_f}} \right)^2 \right] d\xi + \int_0^{\infty} \operatorname{erfc}(\xi) \exp \left[- \left(\frac{y + \xi(2\sqrt{Dt}) + a}{2\sqrt{D_f t_f}} \right)^2 \right] d\xi \right\}, \quad (1)$$

where n_s is the substrate index, Δn is the maximum index change, D is the diffusion coefficient of K^+ ions during the ion-exchange, D_f is the interdiffusion constant of the potassium ions inside the glass substrate during the annealing process, t is the time in the first step process in which K^+-Na^+ ion-exchanged channel waveguides were fabricated in pure KNO_3 molten bath, t_f is the field-assisted annealing time, $\xi = y_0 / (2\sqrt{Dt})$ (the ions which are present at a depth y_0 inside the guide diffuse symmetrically in the inward and outward directions) and x and y represent the lateral and depth directions, respectively. The second integral takes into account the back-reflection experienced by the ions diffusing in the outward direction when they reach the air-glass interface [1, 8]. The diffusion coefficient D is not a physical constant but an observed phenomenological coefficient. If the glass substrate layer is sandwiched between electrodes and diffusion anneal is performed while the electric field E_0 , the maximum in the concentration profile of exchanged ions is displaced a distance $a = \mu_e E_0 t_f$ relative to the profile in the absence of the applied field where we have defined an equivalent ionic mobility μ_e . In [1], $a = \mu E_0 t_0 [1 - \exp(-t_f/t_0)]$ where μ is the ionic mobility, E_0 is applied electric field in the y direction and t_0 is the exponential fitting parameter. Thus, we can use a smaller value of the effective applied electric field $E_0 - E_0 \exp(-t_f/t_0)$ and a shorter value of the effective

field-assisted annealing time t_0 due to the alkali-depleted layer at the positive electrode, which block the exchanged ions to migrate further [1], or we can use the initial electric field E_0 and field-assisted annealing time t_f , but with the fitting parameter $\mu_e < \mu$. In the absence of applied electric field ($a = 0$) we obtain the same results as in [8].

For a wave propagating (TE polarization) along the z axis, assuming that the refractive index change is small, the electric field of a mode in an ion-exchanged planar waveguide is determined by the scalar one-dimensional wave equation:

$$\frac{d^2\Psi(y)}{dy^2} + [k_0^2 n^2(y) - \beta^2] \Psi(y) = 0, \quad (2)$$

where $n(y)$ is the refractive index profile, β is the propagation constant and k_0 is the free space wave number.

The dimensionless form of the refractive index profile in the y (depth) direction is

$$n(Y) = n_s + \frac{\Delta n}{\sqrt{\pi}} \sqrt{\frac{Dt}{D_f t_f}} \left\{ \int_0^\infty \text{erfc}(\xi) \exp \left[-\frac{Dt}{D_f t_f} \left(Y - \xi - \frac{a}{d_y} \right)^2 \right] d\xi + \int_0^\infty \text{erfc}(\xi) \exp \left[-\frac{Dt}{D_f t_f} \left(Y + \xi + \frac{a}{d_y} \right)^2 \right] d\xi \right\}, \quad Y = \frac{y}{d_y}, \quad d_y = 2\sqrt{Dt}, \quad Y > 0$$

$$n(Y) = n_c, \quad Y < 0$$

where $d_y = 2(Dt)^{1/2}$ is the diffusion depth, n_c is the index of the cover region.

The dimensionless form of the wave equation for a direction perpendicular to the substrate surface ($y = 0$) can be written as an eigenvalue equation (with the eigenvalue $\beta^2 d_y^2$)

$$\frac{d^2\Psi}{dY^2} + k_0^2 d_y^2 n^2(Y) \Psi = \beta^2 d_y^2 \Psi, \quad Y > 0, \quad (4a)$$

$$\frac{d^2\Psi}{dY^2} + k_0^2 d_y^2 \Psi = \beta^2 d_y^2 \Psi, \quad Y < 0, \quad (4b)$$

where $b = (N_m^2 - n_s^2) / (n_1^2 - n_s^2)$ is normalized propagation constant, $N_m = \beta/k_0$ is the effective refractive indices and n_1 is the maximum value of the refractive-index distribution. We can use the eigenvalue equations (4a) and (4b) to find the maximum value of β^2 (for a fixed d_y)

We have solved the equations (4a), (4b) for the given boundary conditions (the Dirichlet boundary condition at the ends of the interval where the wave function can be approximated with 0 and the Neuman boundary condition for $Y = 0$) by using the Galerkin's variant of the finite element method, with triangular grid and variable step [9].

The results concerning corresponding planar waveguide (one-dimensional variation of the dielectric constant) can be exploited to find the optimized parameters for the channel waveguide (bidimensional variation of the dielectric constant).

3. Channel waveguide

For a wave propagating (TE polarization) along the z axis, assuming that the refractive index change is small, the electric field of a mode in an ion-exchanged waveguide is determined by the scalar - wave equation:

$$\frac{\partial^2\Psi(x, y)}{\partial x^2} + \frac{\partial^2\Psi(x, y)}{\partial y^2} + [k_0^2 n^2(x, y) - \beta^2] \Psi(x, y) = 0 \quad (5)$$

where $n(x, y)$ is the refractive index profile, β is the propagation constant and k_0 is the free space wave number.

The refractive index profile $n(x, y)$ of the buried ion-exchanged glass channel waveguides using field-assisted annealing has been shown to be [1, 7-8]

$$n(x, y) = n_s + \frac{\Delta n}{\sqrt{\pi}} \sqrt{\frac{Dt}{D_f t_f}} \exp[-(x/d_x)^2] \left\{ \int_0^\infty \text{erfc}(\xi) \exp \left[-\left(\frac{y - \xi(2\sqrt{Dt}) - a}{2\sqrt{D_f t_f}} \right)^2 \right] d\xi + \int_0^\infty \text{erfc}(\xi) \exp \left[-\left(\frac{y + \xi(2\sqrt{Dt}) + a}{2\sqrt{D_f t_f}} \right)^2 \right] d\xi \right\}, \quad (6)$$

where d_x is the half-width of diffusion in the x direction.

The dimensionless form of the wave equation for a channel waveguide can be written as an eigenvalue equation (with the eigenvalue $-\beta^2 d_y^2$)

$$-\left(\frac{d_y}{d_x}\right)^2 \frac{\partial^2\Psi(X, Y)}{\partial X^2} - \frac{\partial^2\Psi(X, Y)}{\partial Y^2} - k_0^2 d_y^2 [n(X, Y)]^2 \Psi(X, Y) = -\beta^2 d_y^2 \Psi(X, Y), \quad Y > 0, \quad X = \frac{x}{d_x}, \quad Y = \frac{y}{d_y}, \quad (7a)$$

$$-\left(\frac{d_y}{d_x}\right)^2 \frac{\partial^2\Psi(X, Y)}{\partial X^2} - \frac{\partial^2\Psi(X, Y)}{\partial Y^2} - k_0^2 d_y^2 \Psi(X, Y) = -\beta^2 d_y^2 \Psi(X, Y), \quad Y < 0 \quad (7b)$$

$$n(X, Y) = n_s + \frac{\Delta n}{\sqrt{\pi}} \sqrt{\frac{Dt}{D_f t_f}} \left\{ \int_0^\infty \text{erfc}(\xi) \exp \left[-\frac{Dt}{D_f t_f} \left(Y - \xi - \frac{a}{d_y} \right)^2 \right] d\xi + \int_0^\infty \text{erfc}(\xi) \exp \left[-\frac{Dt}{D_f t_f} \left(Y + \xi + \frac{a}{d_y} \right)^2 \right] d\xi \right\} \exp[-X^2], \quad d_y = 2\sqrt{Dt}, \quad Y > 0$$

$$n(X, Y) = n_c, \quad Y < 0$$

which has the same form as the usual Hamiltonian eigenvalue equation.

We have solved the equations (7a) and (7b) for the given boundary conditions (the Dirichlet boundary condition at the ends of the interval where the wave function can be approximated with 0 and the Neuman boundary condition for $Y = 0$) by using the Galerkin's variant of the finite element method, with triangular grid and variable step [9]. For symmetry reasons in a channel waveguide, we can use only the positive part of $X(x)$ but with the Neuman boundary condition for $X = 0$ and $Y = 0$.

4. Numerical results and discussion

The accuracy of the results obtained from the finite element method depends on the choice of the parameters $a = \mu_e E_0 t_f$, d_x , d_y and Δn , where we have defined an equivalent ionic mobility $\mu_e = \mu(t_0/t_f) [1 - \exp(-t_f/t_0)]$ and value of the parameter a is dependent only of the μ_e for a

given E_0 and t_f . By varying the parameters μ_e , d_x , d_y , and Δn , for a given mask width (6 μm , [1]) and temperature (380°C, [1]), the electric field peak is matched against those of actual waveguides [1-2]. For an ion-exchanged channel waveguide, Δn is independent of the diffusion time and mask width [7].

We have calculated by using finite element method, the normalized propagation constant b , the propagation constants β and the effective refractive indices N_m for the fundamental mode of an ion-exchanged glass planar waveguide using field-assisted annealing ($\lambda = 1.55\mu\text{m}$, $n_s = 1.50855$, $n_c = 1$, $\Delta n = 0.008$, $E_0 = 100\text{V/mm}$, $t = 1\text{h}$, $t_f = 45\text{ min}$, $t_0 = 20.5\text{ min}$, $d_y = 3.37\mu\text{m}$, $D = D_f = 7.88674 \times 10^{-16}\text{m}^2/\text{s}$, $\mu_e = 1.43419 \times 10^{-14}\text{m}^2/\text{Vs}$, $a = 3.87231\mu\text{m}$, Table 1) and for an ion-exchanged glass planar waveguide using field-assisted annealing ($\lambda = 1.55\mu\text{m}$, $n_s = 1.50855$, $n_c = 1$, $\Delta n = 0.008$, $E_0 = 120\text{V/mm}$, $t = 1\text{h}$, $t_f = 30\text{ min}$, $t_0 = 20.5\text{ min}$, $d_y = 4\mu\text{m}$, $D = D_f = 1.11111 \times 10^{-15}\text{m}^2/\text{s}$, $\mu_e = 2.47656 \times 10^{-14}\text{m}^2/\text{Vs}$, $a = 5.34937\mu\text{m}$, Table 1). Our equivalent ionic mobility μ_e is better in comparison with the ionic mobility μ based on the Nernst-Einstein equation [1]. The calculated electric field profile, normalized to a maximum value of 1 for our two samples are shown in Fig. 1. In Table 1, ΔY (in μm) is the complete width at the $1/e$ points of the field intensity in the y direction. A well confined optical mode can be supported in the waveguide for a high E_0 and short t_f (Fig. 1 and Table 1) in agreement with the results from [1]. The maximum of the field intensity (at $y = 5.89750\mu\text{m}$, [2]) is close to the maximum of the refractive index for the first sample ($E_0 = 100\text{V/mm}$, $t_f = 45\text{ min}$, $n_1 = 1.5111$, $y(n_1) = 5.16759\mu\text{m}$). Also, the maximum of the field intensity (at $y = 7.2\mu\text{m}$, [1]) is close to the maximum of the refractive index for the second sample ($E_0 = 120\text{V/mm}$, $t_f = 30\text{ min}$, $n_1 = 1.51152$, $y(n_1) = 6.81145\mu\text{m}$). It can be seen (Fig. 1) a shift of the electric field peak towards higher values of the depth, which is based on a similar behaviour of the refractive index profile maximum.

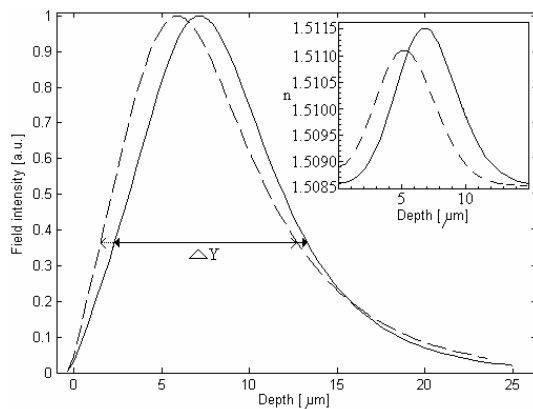


Fig. 1. Calculated electric field intensity by using the finite element method for the fundamental mode and the depth dependence of the refractive index profile of a FAA planar waveguide ($E_0 = 100\text{V/mm}$, $t_f = 45\text{ min}$, ---) and for a FAA planar waveguide ($E_0 = 120\text{V/mm}$, $t_f = 30\text{ min}$, —). ΔY (in μm) is the complete width at the $1/e$ points of the field intensity in the y direction.

Table 1. Comparison of normalized propagation constants b , the propagation constants β (in μm^{-1}) and the effective refractive indices N_m obtained by using finite element method for the fundamental mode of FAA planar and channel waveguides ($\lambda = 1.55\mu\text{m}$, $n_s = 1.50855$, $n_c = 1$, $\Delta n = 0.008$) for two applied electric field E_0 and different time t_f . ΔX (in μm) and ΔY (in μm) are the complete widths at the $1/e$ points of the field intensity in the x and y directions, respectively. In the parentheses we give the experimental results of the mode horizontal and vertical sizes from [2].

	b	β	N_m	ΔX	ΔY
$E_0 = 100\text{V/mm}$, $t_f = 45\text{ min}$, planar	0.30895	6.11836	1.50934		11.59
$E_0 = 100\text{V/mm}$, $t_f = 45\text{ min}$, channel	0.07159	6.11590	1.50873	16.39 (14.8)	12.53 (12.5)
$E_0 = 120\text{V/mm}$, $t_f = 30\text{ min}$, planar	0.41266	6.12013	1.50978		11.22
$E_0 = 120\text{V/mm}$, $t_f = 30\text{ min}$, channel	0.172207	6.11724	1.50906	15.28	11.56

Also, we have calculated for the fundamental mode of a buried ion-exchanged glass channel waveguide using field-assisted annealing ($\lambda = 1.55\mu\text{m}$, $n_s = 1.50855$, $n_c = 1$, $\Delta n = 0.008$, $E_0 = 100\text{V/mm}$, $t = 1\text{h}$, $t_f = 45\text{ min}$, $t_0 = 20.5\text{ min}$, $d_x = 7\mu\text{m}$, $d_y = 3.37\mu\text{m}$, $D = D_f = 7.88674 \times 10^{-16}\text{m}^2/\text{s}$, $\mu_e = 1.43419 \times 10^{-14}\text{m}^2/\text{Vs}$, $a = 3.87231\mu\text{m}$, Table 1) and for a buried ion-exchanged glass channel waveguide using field-assisted annealing ($\lambda = 1.55\mu\text{m}$, $n_s = 1.50855$, $n_c = 1$, $\Delta n = 0.008$, $E_0 = 120\text{V/mm}$, $t = 1\text{h}$, $t_f = 30\text{ min}$, $t_0 = 20.5\text{ min}$, $d_x = 7\mu\text{m}$, $d_y = 4\mu\text{m}$, $D = D_f = 1.11111 \times 10^{-15}\text{m}^2/\text{s}$, $\mu_e = 2.47656 \times 10^{-14}\text{m}^2/\text{Vs}$, $a = 5.34937\mu\text{m}$, Table 1), the normalized propagation constant b , the cross section of the electric field (normalized to a maximum value of 1) at its peak ($X = 0$, $Y = 1.8$) and the complete widths ΔX and ΔY at $\psi = 1/e$ by using finite element method (Table 1, Fig. 2). The maximum of the field intensity is at $y = 7.2\mu\text{m}$ [1] for the first sample and at $y = 6.066\mu\text{m}$ [2] for the second sample. For a high applied electric field and short field-assisted annealing time, both the horizontal and the vertical fundamental mode profiles are narrower than those in a small applied electric field (Fig. 2, Table 1). Thus, a well confined optical mode can be supported in the waveguide for a high E_0 and short t_f in agreement with the results from [1]. In the case of the channel waveguide, both the horizontal and the vertical fundamental mode profiles are larger than those of the planar waveguide (Table 1). For y direction we have obtained $\Delta Y = 12.53\mu\text{m}$ which is very close to the experimental results [2] ($\Delta Y = 12.5\mu\text{m}$, Table 1).

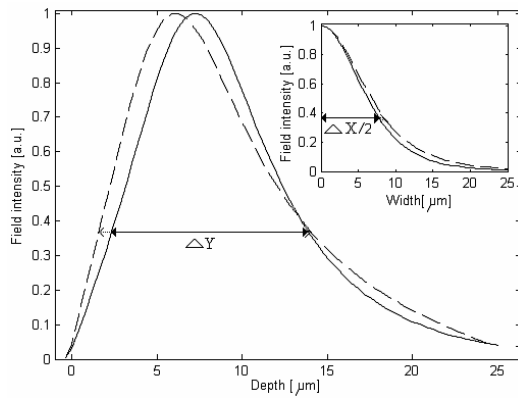


Fig. 2. Cross section of the field intensity at its peak in the y direction of the fundamental mode for a buried FAA channel waveguide ($E_0 = 100\text{V/mm}$, $t_f = 45\text{ min}$, ---) and for a buried FAA channel waveguide ($E_0 = 120\text{V/mm}$, $t_f = 30\text{ min}$, ____). The inset is a cross section of the field intensity at its peak in the x direction. ΔX (in μm) and ΔY (in μm) are the complete widths at the $1/e$ points of the field intensity in the x and y directions, respectively.

An analytical model shows how the finite element method is useful for the reconstruction of electric field profile in the waveguide. The computed results correspond to the expected intensity profile.

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*Corresponding author: cripoco@physics.pub.ro