

Eigentensors and eigenelastic constants for woven composites

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The concept of an eigentensor was introduced by Kelvin and he determined the eigentensors for many elastic symmetries. The eigentensors and eigenelastic constants of particular elastic symmetries are significant because they identify preferred modes of deformation associated with particular anisotropic elastic symmetries and are useful in the design of composite materials. The proportional invariants for each linear anisotropic elastic symmetry have the same properties as the hydrostatic and deviatoric stresses or strains in the case of isotropic elastic symmetry, but the proportional invariants for the linear anisotropic elastic symmetry may depend on the components of elasticity or compliance tensor ($\hat{C}$ or $\hat{S}$). In the present work, the analysis is focused on an orthogonal unbalanced plain weave fabric lamina. To illustrate the invariant of the eigentensors concept, a woven composite lamina, containing a circular hole of radius 1 mm has been analysed. The analysis carried out the finite element method for orthotropic materials using the software package ALGOR provided information about stress and strain in different points of the finite element grid. The resulting grid enabled the usage of the MATLAB facilities for graphical representation of the surface as well as the mesh or contour plots for the three stress and strain invariants, respectively.

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1. Introduction

Fibre reinforced polymer (FRP) composites are advanced materials in which fibres with high strength and high stiffness are embedded in and bounded together by a low modulus continuous matrix. The need for fibre placement in different directions according to the particular applications leads to various types of fibrous composites.

The properties of a fibre reinforced composite material depend on the properties of its constituents and their distribution as well as their physical and chemical interactions.

Properties of FRP composites can be determined through experimental measurements; however, one set of experimental measurements evaluates the properties of a certain fibre-matrix system produced by a single fabrication process; when any change in the system variables occurs additional measurements are required. Thus experiments may become time consuming and cost prohibitive.

The simplest fibre reinforced composite (reinforced with unidirectional fibres) can be analysed using the lamina illustrated in Fig. 1.

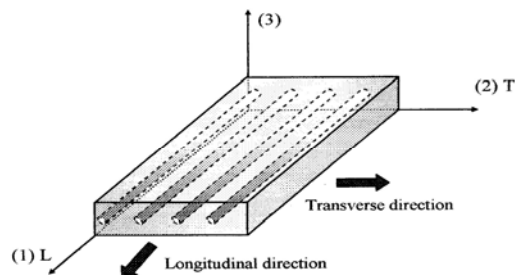


Fig. 1. Schematic representation of a unidirectional composite.

The direction parallel to the fibres is called the longitudinal direction (axis 1 or L) and the direction perpendicular to it is the transverse direction (axis 2 or T). Any direction in the 2-3 plane is also a transverse direction. These axes are referred to as the material axes of the lamina.

A unidirectional lamina has maximum value of mechanical properties (strength and modulus) along the fibres direction and minimum values of the same properties in the direction normal to the fibres direction. When some kind of balanced properties are required in two perpendicular directions, orthogonal bidirectional reinforcement is used.

Since experimental route of determining the mechanical properties is not always convenient it is advisable to use other methods to predict the required design characteristics. In terms of elastic constants these characteristics are: longitudinal modulus E_L , transverse modulus E_T , shear modulus G_{LT} , and Poisson ratios, ν_{LT} and ν_{TL} [12], [13]. Based on a micromechanical approach the following formulas can be utilised:

$$E_L = E_f V_f + E_m (1 - V_f) \quad (1.1)$$

Where E_f and E_m is the elastic modulus of the fibres and matrix, respectively;

V_f is the fibre volum fraction

$$E_T = \frac{E_f E_m}{E_m V_f + E_f (1 - V_f)} \quad (1.2)$$

$$G_{LT} = \frac{G_f G_m}{G_m V_f + G_f (1 - V_f)} \quad (1.3)$$

$$\nu_{LT} = \nu_f V_f + \nu_m (1 - V_f) \quad (1.4)$$

$$\nu_{TL} = [\nu_f V_f + \nu_m (1 - V_f)] \frac{E_f E_m / [E_m V_f + E_f (1 - V_f)]}{E_m V_f + E_f (1 - V_f)} \quad (1.5)$$

where G_f , G_m are the shear modulus of fibre and matrix respectively. When the components are isotropic they are calculated with:

$$G_f = \frac{E_f}{2(1 + \nu_f)} \quad (1.6a)$$

$$G_m = \frac{E_m}{2(1 + \nu_m)} \quad (1.6b)$$

in which ν_f and ν_m are the fibre and matrix Poisson's ratio, respectively.

Textile structural composites represent a class of advanced composites utilising fiber preforms produced by textile forming techniques, for structural applications. Textile structural composites offer the potential of providing adequate structural integrity as well as shapeability for near-net-shape manufacturing, [1,4,9]. For most applications of textile composites in structures, the motivation has primarily been the reduced cost of manufacturing. In addition, textile composites offer better dimensional stability over a large range of temperatures, better impact resistance and tolerance, conformability and deep draw moldability. Textile structural composites

provide the unique capability that the microstructure of fibre preforms can be designed to meet the needs of the performance of composite structures. The forming of textile preforms requires knowledge of the structure of yarn and fibres. Yarns are linear assemblages of fibers formed into continuous strands having textile characteristics, namely significant strength and flexibility. One of the major textile forming techniques for composites reinforcement is weaving, leading to woven products.

Woven fabric (WF) is characterised by better impact resistance, damage tolerance, toughness and dimensional stability over a large temperature range, and subtle conformability and easy of manufacturing. Orthogonal woven fabrics, Fig. 2, exhibit good dimensional stability in the warp and weft directions. Woven fabrics offer the highest yarn packing density in relation to fabric thickness. The pure and hybrid woven fabrics used in composites are mostly in the form of plain, basket, twill and satin weaves.

Woven fabrics provide more balanced properties in the fabric plane than unidirectional laminae; the bidirectional reinforcement in a single layer of a fabric gives rise to enhanced impact resistance. The ease of handling and low fabrication cost have made fabrics attractive for structural applications. On the other hand, the limited conformability, poor in-plane shear resistance, and reduced yarn-to-fabric tensile translation efficiency due to yarn crimp are some of the disadvantages of woven fabrics.

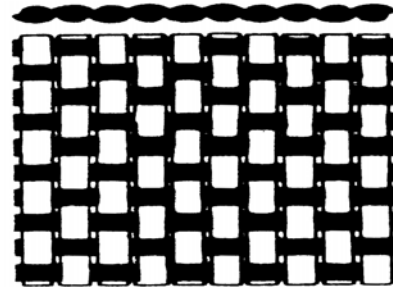


Fig. 2. Typical orthogonal woven.

An *orthogonal two-dimensional (2D)* woven fabric consists of two sets of interlaced yarns. The lengthwise set is called warp, and the crosswise set, fill (weft). Any weave repeats on certain number of warp and fill yarns. The repeat is a complete representative *unit cell* of weave. The regions, in Figure 3, enclosed by the dotted lines define the “*unit cells*” or the basic repeating regions for different weaving patterns [1, 3, 4, 9]. Different types of weaves can be identified by repeating patterns in both directions, defined by geometrical quantities n_w , n_f .

The number n_f denotes that the warp yarn is interlaced with every n_f filling yarn and n_w denotes that a fill yarn is interlaced with every n_w -th warp yarn. Both n_w , n_f represent counts on a length equal to 1 cm. The fabric geometry it is chosen so that it gives the best possible properties for the application under consideration. The important fabric parameters are *strands* cross-sectional

geometry, *finesse*, *number of counts* and the weaving condition. The fabric count is the number of yarns (strands) per unit length (usually cm) along the warp or fill direction. Linear density of the yarn is weight per unit length, which is a measure of *finesse* of the yarn and it is given by the tex number [g/km]. The yarn crimp is a measure of the degree of undulation. The undulated length within the interlacing region is termed “*u*”.

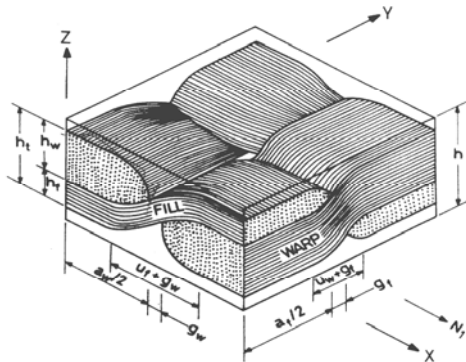


Fig. 3. The sections at the boundaries of the unit cell [9].

A plain weave fabric can be *balanced* or *unbalanced* depending upon the number of counts and yarn properties. If the yarn properties such as cross-sectional geometry, tex, crimp, the number of counts and the material properties are the same in both directions (warp and fill) the fabric is called balanced fabric.

The unbalanced fabric can be used if different properties are required along the warp and fill directions. The elastic characteristics of WF composites depend upon the type of weave fabric geometry, fibre volume fraction, laminate configuration and the material system used.

2. Eigentensors in anisotropic elasticity

The concept of an eigentensor was introduced by Kelvin [Enciclopedia Britannica article on Elasticity, 2 section 41] who called eigentensors „the principal types of stress or of strain “and he determined the eigentensors for many elastic symmetries.

Mehrabadi and Cowin [8] present the properties of the isotropic elastic eigentensors that are generalised to the eigentensors for the anisotropic elastic symmetries. The extension of the property mentioned above to include the eigentensors of elastic symmetries other than isotropy is stated as follows:

Property A: For any elastic symmetry the total stress tensor and the total strain tensor can be additively decomposed into of a sum of six or fewer eigentensors of identical form.

Property B: For any elastic symmetry each stress eigentensor is directly proportional to its strain eigentensor of identical form.

Property C: For any elastic symmetry there is an additive decomposition of the total strain energy into a sum of six or fewer terms, each term being a scalar – valued product of the stress and strain eigentensors of identical form.

Property D: For any elastic symmetry the trace of the stress and strain eigentensors of identical form, or the trace of the squares of the stress and strain eigentensors of identical form, are directly proportional.

The formation of anisotropic form of the linear Hooke's law, provides the correct generalisation to the various anisotropic symmetries of hydrostatic and deviatoric concepts employed so successfully in isotropic elasticity and plasticity theories based on isotropic elasticity [5-8].

The eigentensors and eigenelastic constants of particular elastic symmetries are important because they identify preferred modes of deformation associated with particular anisotropic elastic symmetries, [6, 8]. The maximum number of eigentensors for any elastic symmetry is therefore six.

The anisotropic form of Hook's law often written as linear transformation in six dimensions, has the following representation [8]:

$$\hat{\sigma} = \hat{c} \hat{\varepsilon} \quad (2.1)$$

$$\hat{\varepsilon} = \hat{s} \hat{\sigma} \quad (2.2)$$

where $\hat{\sigma}$ and $\hat{\varepsilon}$ are the six -dimensional stress and strain vectors and the symmetric matrix $\hat{c}$ represents the components of a elasticity second rank tensor. The inverse of elasticity tensor $\hat{s}$, $\hat{s} = \hat{c}^{-1}$, represent the compliance tensor.

The eigenvalues of the elasticity tensor $\hat{c}$ or $\hat{s}$ are the six number λ_k or $\frac{1}{\lambda_k}$ satisfying the equation

$$(\hat{c} - \lambda \mathbf{1}) \hat{N} = 0 \quad (2.3)$$

$$\left(\hat{s} - \left(\frac{1}{\lambda} \right) \mathbf{1} \right) \hat{N} = 0 \quad (2.4)$$

where the vectors $\hat{N}$ represent the *normalized eigenvectors* of $\hat{c}$ or $\hat{s}$.

These six eigenvalues λ_k are called the *six principal elasticities of the material* or *eigenelastic constants* and are ordered by inequalities $\lambda_1 > \lambda_2 > \dots > \lambda_6$ ($k=1 \dots 6$), [6],[8]. If the eigensystems for various anisotropic elastic symmetries have a number of distinct elastic constants which exceeds the number of eigenvalues, these other elastic constants are called *elasticity distributors*. The distributors are associated with eigenmodes [6]. In the case of **orthotropic symmetry** the number of the distinct elastic constants is equal 9, 6 eigenvalues and 3 distributors.

The results of the paragraph above show that there exist six eigentensors of stress, denoted by $\hat{\sigma}^{(k)}$ $k=1\dots 6$, in the 6-dimensional space, and six eigentensors of strain, denoted by $\hat{\epsilon}^{(k)}$ $k=1\dots 6$, which are expressed by the six equations

$$\hat{\sigma}^{(k)} = \lambda_k \hat{\epsilon}^{(k)} \quad (2.5)$$

For orthotropic symmetry there are nine distinct elastic constants (c_{11} , c_{12} , c_{13} , c_{22} , c_{23} , c_{33} , c_{44} , c_{55} , c_{66}), six constants are the distinct eigenvalues and the other three are elastic distributors. The eigenvalues of the matrix $\hat{\mathbf{c}}$ satisfying the equation

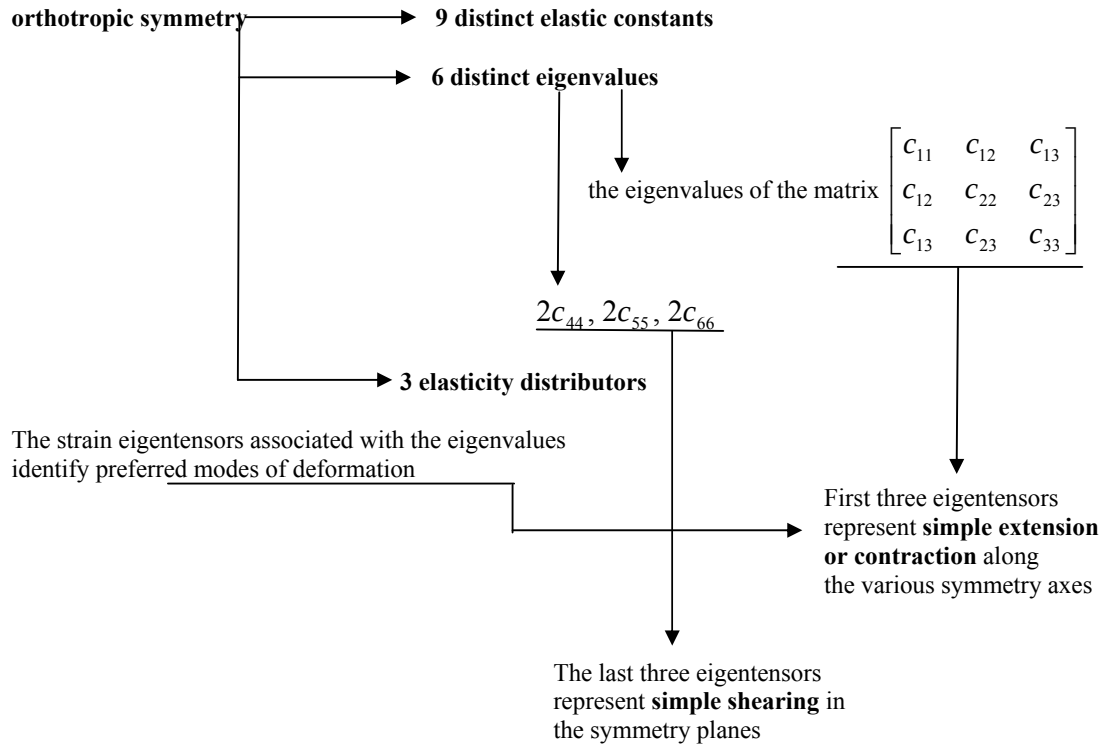
$$\begin{bmatrix} c_{11}-\lambda & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22}-\lambda & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33}-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2c_{44}-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 2c_{55}-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 2c_{66}-\lambda \end{bmatrix} \begin{bmatrix} \hat{\epsilon}_1 \\ \hat{\epsilon}_2 \\ \hat{\epsilon}_3 \\ \hat{\epsilon}_4 \\ \hat{\epsilon}_5 \\ \hat{\epsilon}_6 \end{bmatrix} = 0 \quad (2.6)$$

The first three, $\lambda_1, \lambda_2, \lambda_3$, satisfy the relation

$$\begin{bmatrix} c_{11}-\lambda & c_{12} & c_{13} \\ c_{12} & c_{22}-\lambda & c_{23} \\ c_{13} & c_{23} & c_{33}-\lambda \end{bmatrix} \begin{bmatrix} \hat{\epsilon}_1 \\ \hat{\epsilon}_2 \\ \hat{\epsilon}_3 \end{bmatrix} = 0 \quad (2.7)$$

and the last three are

$$\lambda_4 = 2c_{44}, \quad \lambda_5 = 2c_{55}, \quad \lambda_6 = 2c_{66} \quad (2.8)$$



3. The proportional invariants

In order to give a definition, of a **proportional invariant** it is introduced the notation $f(\mathbf{A}, \mathbf{a})$ to denote a scalar-valued invariant function of a symmetric second rank tensor in 3-dimension, denoted by $\mathbf{A}$, and a symmetric second rank tensor in six dimensions, denoted by $\mathbf{a}$. Cowin, Mehrabadi and Sadeh [5] give the definition

of a proportional invariant: **A function $f(\mathbf{A}, \mathbf{a})$ is said to be a proportional invariant for a particular anisotropic linear elastic material symmetry, if $f(\sigma, \hat{s})$ is directly proportional to $f(\epsilon, \hat{c})$.** Those functions of the invariants of second-rank tensor for which the corresponding stress and strain invariants of the identical form are proportional. In general there are six proportional invariants for each symmetry, because the invariants are associated with the

magnitudes of the eigenvectors of $\hat{\mathbf{C}}$ in the case of strain and with the inverse of this matrix $\hat{\mathbf{S}}$ in the case of stress. In the case of orthotropic elastic materials, the linear proportional invariants are

$$\mathbf{I}_k(\boldsymbol{\varepsilon}, \hat{\mathbf{C}}) = f(\lambda_K, \lambda_I, \lambda_J) [c_{12}c_{23} + c_{12}c_{13} - c_{12}^2 - c_{23}(c_{11} - \lambda_K) - c_{13}(c_{22} - \lambda_K) + (c_{11} - \lambda_K)(c_{22} - \lambda_K)] \quad (3.1)$$

where $I, J, K=1, 2, 3$ ($I \neq J \neq K$):

$$f(\lambda_K, \lambda_I, \lambda_J) = \frac{1}{(\lambda_K - \lambda_I)(\lambda_K - \lambda_J) [c_{12}(c_{13}^2 - c_{23}^2) - c_{13}c_{23}(c_{11} - c_{22})]} \times \{ [c_{12}c_{13} - c_{23}(c_{11} - \lambda_I)] [(c_{11} - \lambda_J)\varepsilon_{11} + c_{12}\varepsilon_{22} + c_{13}\varepsilon_{33}] - [c_{11}c_{23} - c_{13}(c_{22} - \lambda_I)] [c_{12}\varepsilon_{11} + (c_{22} - \lambda_J)\varepsilon_{22} + c_{23}\varepsilon_{33}] \} \quad (3.2)$$

The proportional invariants associated with $\lambda_4, \lambda_5, \lambda_6$, are

$$\mathbf{I}_4(\boldsymbol{\varepsilon}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{23} \\ 0 & \varepsilon_{23} & 0 \end{bmatrix} \quad \mathbf{I}_5(\boldsymbol{\varepsilon}) = \begin{bmatrix} 0 & 0 & \varepsilon_{13} \\ 0 & 0 & 0 \\ \varepsilon_{13} & 0 & 0 \end{bmatrix} \quad \mathbf{I}_6(\boldsymbol{\varepsilon}) = \begin{bmatrix} 0 & \varepsilon_{12} & 0 \\ \varepsilon_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.3)$$

It easily verified that the sum of the six strain eigentensors is the strain tensor. The stress eigentensors are given by similar formulas, in which the components of strain $\boldsymbol{\varepsilon}$ are replaced by the components of stress $\boldsymbol{\sigma}$, and the components $\hat{\mathbf{C}}$ are replaced by the components of $\hat{\mathbf{S}}$ (compliance matrix).

$$\mathbf{I}_k(\boldsymbol{\sigma}, \hat{\mathbf{S}}) = f(\lambda_K, \lambda_I, \lambda_J) [s_{12}s_{23} + s_{12}s_{13} - s_{12}^2 - s_{23}(s_{11} - \lambda_K) - s_{13}(s_{22} - \lambda_K) + (s_{11} - \lambda_K)(s_{22} - \lambda_K)] \quad (3.4)$$

where $I, J, K=1, 2, 3$ ($I \neq J \neq K$):

$$f(\lambda_K, \lambda_I, \lambda_J) = \frac{1}{(\lambda_K - \lambda_I)(\lambda_K - \lambda_J) [s_{12}(s_{13}^2 - s_{23}^2) - s_{13}s_{23}(s_{11} - s_{22})]} \times \{ [s_{12}s_{13} - s_{23}(s_{11} - \lambda_I)] [(s_{11} - \lambda_J)\sigma_{11} + s_{12}\sigma_{22} + s_{13}\sigma_{33}] - [s_{11}s_{23} - s_{13}(s_{22} - \lambda_I)] [s_{12}\sigma_{11} + (s_{22} - \lambda_J)\sigma_{22} + s_{23}\sigma_{33}] \} \quad (3.5)$$

In order to demonstrate the proportionality between strain and stress eigentensors, the paper presents the graphical representation of these for a square plate of woven composite materials.

4. Methodology of analysis of woven lamina composite

In the present work, the analysis is focused on a two-dimensional (2D) orthogonal unbalanced plain weave fabric lamina. The unit cell, Fig. 4, consists of the interlacing region (where the warp and fill yarns pass one over the other) and the gap region. For this cell, the main geometrical parameters are: yarn width = a_w (a_f); yarn

thickness = h_w (h_f); interyarn gap = g_w (g_f); undulated length = u_w (u_f); fabric thickness = h_f ; lamina thickness = h , Fig. 4, Fig. 5. In order to determine the stiffness matrix or the compliance matrix, a unit cell presented below, Fig. 4, has been utilised. The cell is composed of two unidirectional (UD) laminae of equivalent thickness h_e . One of this UD lamina has the yarn along the x-axis (fill direction) and the other has the yarn along the y-axis (warp direction).

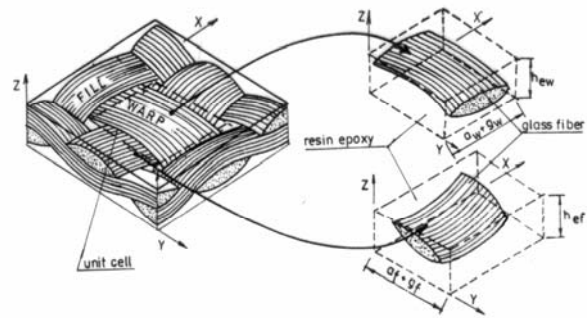


Fig. 6. The unit cell used for analysis.

Knowing the fabric counts (n_1 = warp yarn counts and n_2 = fill yarn counts) we have calculated the equivalent thickness for each other UD ply [1]:

$$h_{e/w} = h n_w / (n_w + n_f) = h \quad (3.6)$$

$$h_{e/f} = h n_f / (n_w + n_f) = h k_{2b}$$

where

h = thickness of lamina

h_f = thickness of UD ply which have the fibers on fill direction

h_w = thickness of UD ply which have the fibers on warp direction

k_i = the relative volume fraction of the i -th reinforcing system

Assuming that the fibers are aligned along the x-axis for one ply and for the other are aligned along y-axis, each UD ply present orthotropic symmetry in the x-y plane.

In this case, the stiffness constants $C^{w(f)}$ of each UD-ply are given by [1]:

$$C^{w(f)} = \begin{bmatrix} c_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}} & c_{12} = \frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}} & 0 \\ c_{21} = \frac{\nu_{TL}E_L}{1 - \nu_{LT}\nu_{TL}} & c_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}} & 0 \\ 0 & 0 & G_{TL} \end{bmatrix} \quad (3.7)$$

where

$C^{w(f)}$ is the local stiffness tensor.

For each of the two plies, the local stiffness matrix (C^w and C^f) can be calculated. Then, with the stiffness averaging,

the global composite stiffness matrix $\mathbf{C}_{\text{comp}}$ can be computed:

$$\mathbf{C}_{\text{comp}} = \mathbf{k}_1 \mathbf{C}^w + \mathbf{k}_2 \mathbf{C}^f \quad (3.8)$$

Alternatively, the global compliance matrix $\mathbf{S}_{\text{comp}}$, can be determined

$$\mathbf{S}_{\text{comp}} = \mathbf{k}_1 \mathbf{S}^w + \mathbf{k}_2 \mathbf{S}^f \quad (3.9)$$

Knowing the stiffness matrix or the compliance matrix, the elastic constants of composite (E_x , E_y , G_{xy} , ν_{xy}) in the global coordinates system can be calculated, [1]:

$$E_x = \frac{c_{11}}{H}, E_y = \frac{c_{22}}{H}, G_{xy} = c_{66}, \nu_{xy} = \frac{c_{21}}{c_{22}}, H = \left(1 - \frac{c_{12}}{c_{11}} \cdot \frac{c_{21}}{c_{22}} \right)^{-1} \quad (3.10)$$

5. Application

To illustrate the significance of the invariant of the eigentensors concept, we have considered a woven composite lamina, containing a circular hole of radius 1 mm at its centre. We consider a square plate of epoxy resin reinforced by E-glass unbalanced plain weave fabric structure. The fabric was assumed to be: *unbalanced plain weave* with the following geometry of yarn and fabric and the elastic properties of fiber and resin .

Table 5.1. Elastic properties of fiber and resin.

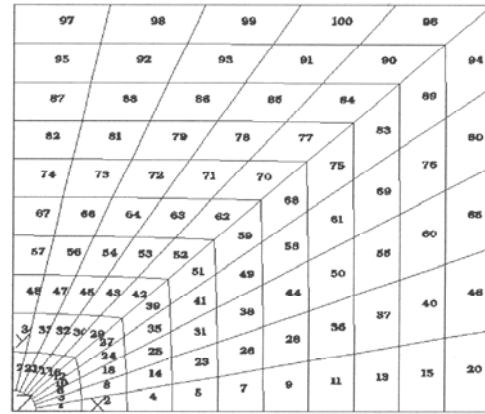
Material	E (GPa)	G_{TL} (GPa)	ν_{TL}	ρ (g/cm ³)
glass -E	72.0	27.7	0.30	2.58
epoxy resin	3.5	1.3	0.35	1.17

Table 5.2. Yarn and fabric geometry.

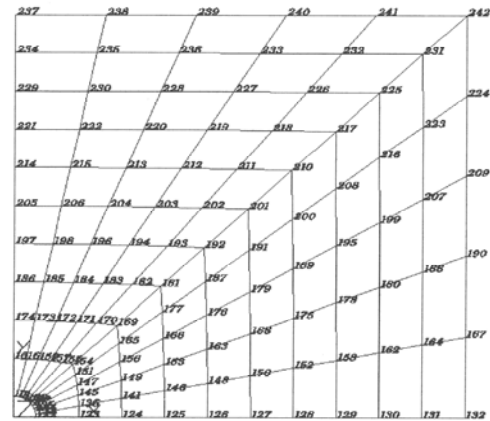
Fabric Thickness h_t (mm)	Overall fiber volume fraction	Counts/cm (w / f)	Lamina thickness h (mm)	Width of yarn (mm) (a_w / a_f)	Gap (mm) (g_w / g_f)	Fabric weight g/m ²
0.50	0.39	6.40 / 5.80	0.50	1.44 / 1.12	0.12 / 0.6	470

The analysis performed the finite element method for orthotropic materials in ALGOR provided information about stress and strain in different points of the finite element grid. The finite element model used, the *numbering of finite elements and the numbering of nodes* are represented in Fig. 5a and 5b.

The points of the element grid were non-uniformly spaced and therefore, they had to be transformed by 2-D spline interpolation into a regular grid, built by the MATLAB function *griddat*. The resulting uniform grid allowed the usage of the MATLAB facilities for graphical representation of the surface and the mesh or contour plots for the three stress and strain invariants, respectively. The attribute NaN (IEEE standard for Not-a-number) was assigned to those points of the regular grid which were not physically belonging to the lamina.



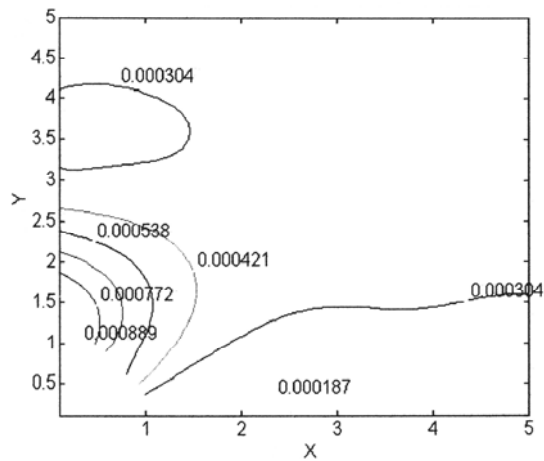
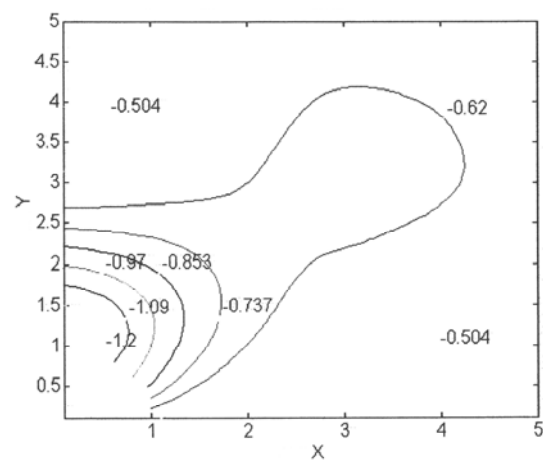
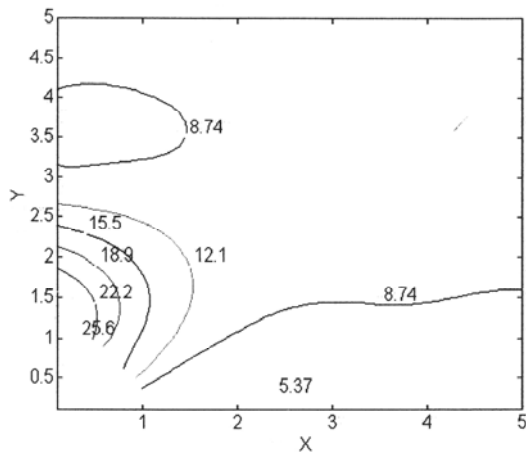
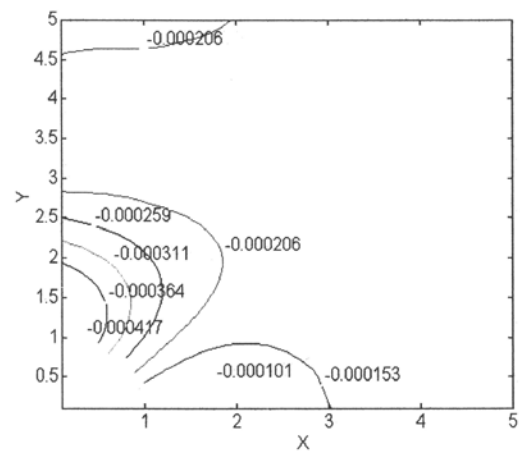
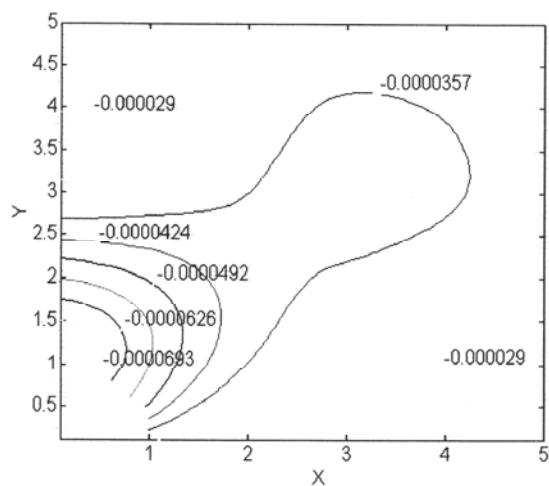
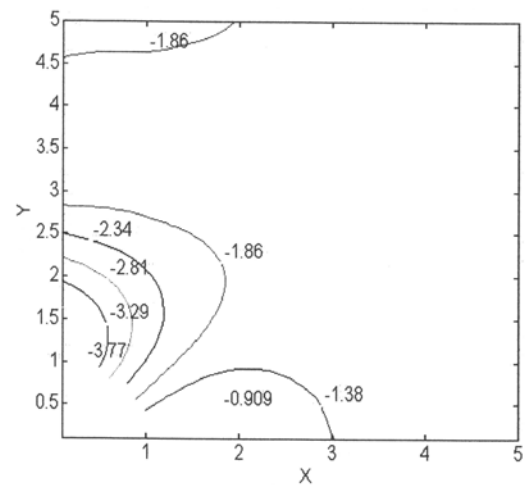
a)-numbering of finite elements



b)-numbering of nodes

Fig. 5. The finite element model.

In order to compare the proportionality of these invariants, in Figs. 6-8, the contour plots of the three linear invariants of the stress and strain eigentensors are shown. The proportionality constant, that is to say the ratio of the invariant of stress eigenvector to the invariant of the strain eigenvector, is equal to the corresponding eigenvalue $\lambda_1, \lambda_2, \lambda_3$ of $\hat{\mathbf{C}}$ for the linear invariants.

Fig. 6a. The linear proportional strain invariant I_k .Fig. 7b. The linear proportional stress invariant I_j .Fig. 6b. The linear proportional stress invariant I_k .Fig. 8a. The linear proportional strain invariant I_l .Fig. 7a. The linear proportional strain invariant I_j .Fig. 8b. The linear proportional stress invariant I_l .

6. Conclusions

The Kelvin formulation has at least two immediate practical applications for anisotropic materials. One application is for the study of the *sensitivity of the material constants* [2] and the other is *the graphical representation of stress and strain distributions* in linear anisotropic materials using the special set of stress and strain invariants called proportional invariants or invariants of eigentensors.

The eigentensors of particular elastic symmetries are significant because they identify preferred modes of deformation associated with particular anisotropic elastic symmetries.

The eigentensors will be useful in the design of composite materials and by understanding the functioning of natural composites.

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