Radiation power spectral distribution of the system of electrons moving in a spiral in vacuum

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Using the expressions for the average radiation power of three and four electrons moving one by one in a spiral in vacuum the synchrotron radiation spectrum for the first time is obtained and studied. The synchrotron radiation spectrum of a single electron is compared to these of two, three and four electrons moving in a spiral in vacuum. The influence of the coherence factor on the spectrum of synchrotron radiation is studied.

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1. Introduction

Investigations of the radiation spectrum of a system of electrons moving in magnetic fields are important from the point of view of their applications in astrophysics, electronics, plasma physics, etc. [1-3].

The fine structure of synchrotron radiation spectrum for one and two electrons moving in magnetic field was studied in [4-12].

In this paper using the exact integral relationship for the spectral distribution of radiation power of three and four electrons moving one by one along a spiral in vacuum, the fine structure of synchrotron radiation spectrum is investigated for the first time by means of analytical and numerical methods in the case when the longitudinal component of velocity (the component parallel to magnetic induction vector) is much less than the velocity of light in vacuum. The synchrotron radiation spectrum of a single electron is compared with the radiation spectra of two, three, and four electrons moving in a spiral in vacuum. The influence of the Doppler effect and coherence factor on the spectrum of synchrotron radiation for one, two, three, and four electrons is studied.

2. Radiation power of the system of electrons moving along a spiral in vacuum

The time-averaged radiation power $\overline{P}_{rad}$ of charged particles moving in magnetic field is expressed in [4-6] as

$$\overline{P}_{rad} = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} \left( \frac{\partial q_{Dir}(\mathbf{r}, t)}{\partial t} - \frac{\partial A_{Dir}(\mathbf{r}, t)}{\partial t} \right) dt.$$  (1)

Here $\mathbf{J}(\mathbf{r}, t)$ is the current density and $\mathbf{P}(\mathbf{r}, t)$ is the charge density. The integration is over some volume $\tau$.

According to the hypothesis of Dirac [13], the scalar $q_{Dir}(\mathbf{r}, t)$ and vector $A_{Dir}(\mathbf{r}, t)$ potentials are defined as a half-difference of the retarded and advanced potentials:

$$q_{Dir} = \frac{1}{2} \left( q_{ret} - q_{adv} \right),$$

$$A_{Dir} = \frac{1}{2} \left( A_{ret} - A_{adv} \right).$$  (2)

Then according to [4], the source functions of $N$ charged point particles are defined as

$$\mathbf{j}(\mathbf{r}, t) = \sum_{l=1}^{N} \mathbf{j}_{l}(\mathbf{r}, t),$$

$$\rho(\mathbf{r}, t) = \sum_{l=1}^{N} \rho_{l}(\mathbf{r}, t),$$

$$\rho_{l}(\mathbf{r}, t) = e \delta(\mathbf{r} - \mathbf{r}_{l}(t)), (3)$$

where $\mathbf{r}_{l}(t)$ and $\mathbf{V}_{l}(t)$ are the motion law and the velocity of the $l^{th}$ particle, respectively.

The law of motion and the velocity of the $l^{th}$ electron moving in a spiral in vacuum are given by the expressions

$$\dot{\mathbf{r}}_{l}(t) = \left( \mathbf{e} \omega_{l} \mathbf{r}_{l}(t) + \frac{e \mathbf{B}_{ext} \times \mathbf{r}_{l}(t)}{m_{0} c} \right),$$

$$\dot{\mathbf{V}}_{l}(t) = \frac{d\mathbf{r}_{l}(t)}{dt}.$$  (4)

Here

$$\omega_{0} = V_{\perp} \omega_{0}^{(-1)},$$

$$E = c\sqrt{\mathbf{p}^{2} + m_{0}^{2} c^{2}}$$

and $\mathbf{B}_{ext} || OZ, V_{\perp}$ and $V_{\parallel}$ are the components of the velocity, $\mathbf{p}$ and $E$ are the momentum and energy of the electron, $c$ and $m_{0}$ are its charge and rest mass.

The time-averaged radiation power of system of electrons we obtain after substituting expressions (2) – (4) into (1). Then

$$\overline{P}_{rad} = \int_{0}^{\infty} W(\omega) d\omega,$$  (5)

$$W(\omega) = \frac{2c^{2} \pi}{\omega^{2}} \frac{\sin \left( \frac{\omega m_{0} c}{\eta_{e}} \right)}{\eta_{e}} \cos \left[ \sqrt{\omega^{2} c^{2} \cos^{2}(\omega_{0} x) + \omega^{2} - c^{2}} \right].$$  (6)
Here $\Delta t_{23}$ is the time shift between the second and third electrons.

The coherence factor $S_4(\omega)$ of four electrons is defined as

$$S_4(\omega) = 4 + 2\cos(\omega\Delta t_{12}) + 2\cos(\omega\Delta t_{23}) + 2\cos(\omega\Delta t_{34}) +$$
$$+ 2\cos[\omega(\Delta t_{12} + \Delta t_{23})] + 2\cos[\omega(\Delta t_{23} + \Delta t_{34})] +$$
$$+ 2\cos[\omega(\Delta t_{12} + \Delta t_{23} + \Delta t_{34})],$$

where $\Delta t_{34}$ is the time shift between the third and fourth electrons.

The total radiation power emitted by a single electron is determined, according to [15], as

$$P_{\text{rad}}^{\text{tot}} = \frac{2}{3} \frac{e^2}{c^3} \frac{\omega_0^2 V_{\perp}^2}{c^2} \left(1 - \frac{V^2}{c^2}\right)^{-2} eB_{\text{ext}} \sin \theta \omega_0 \sqrt{1 - \frac{V^2}{c^2}}$$

(14)

It is interesting to compare the radiation power spectral distribution for two, three and four electrons (curve 2 in Fig. 2, curve 3 in Fig. 3, and curve 4 in Fig. 4, respectively) to that of a single electron (curve 1 in Fig. 1). Our numerical calculation of the radiation spectra were carried out on the basis of equations (5) and (6) taking into account the corresponding coherence factors. Spectral distribution of synchrotron radiation power is obtained for $B_{\text{ext}} = 1$ Gs, $V_{\perp\text{vac}} = 0.24 \cdot 10^{14}$ cm/s, $V_{||\text{vac}} = 0.15 \cdot 10^{10}$ cm/s, $c = 0.2997925 \cdot 10^{11}$ cm/s, $n_{0j} = 2285$ cm, $\omega_{0j} = 0.105 \cdot 10^8$ rad/s ($j=1,2,\ldots,7$).

The total radiation power of a single electron in vacuum $P_{\text{rad}}^{\text{tot}\text{vac1}} = 0.2852 \cdot 10^{-14}$ erg/s calculated according to relationship (14) is in good agreement to the power $P_{\text{rad}}^{\text{tot}\text{vac2}} = 0.2839 \cdot 10^{-14}$ erg/s determined after integration of relationships (5) and (6) at $S_{1}(\omega) = S_{1}(\omega) = 1$.

For the time shifts $\Delta t_{12}^{(2)} = 0.001 \cdot \pi \omega_0 / \omega_0$ the coherence factor $S_2(\omega) \equiv 4$ and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$ (curve 2 in Fig. 2), i.e. by a factor of four higher than a single electron ($P_{\text{rad}}^{\text{tot}\text{vac2}} \equiv 4 \cdot P_{\text{rad}}^{\text{tot}\text{vac1}} = 0.1136 \cdot 10^{-13}$). Here the upper index (2) and other similar ones further mark the corresponding curves in the plots.
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For the time shifts $\Delta t_{12}^{(2)} = 0.001 \cdot \pi / \omega_{02}$ with radiation power

$$P_{\text{vac2}}^{\text{int}} = 0.1135 \cdot 10^{-13} \text{ erg/s}.$$

For the time shifts $\Delta t_{12}^{(3)} = \Delta t_{23}^{(3)} = 0.001 \cdot \pi / \omega_{03}$ the coherence factor $S_3(\omega) \approx 9$ and three electrons radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$ (curve 3 in Fig. 3), i.e. by a factor of nine higher than a single electron ($P_{\text{vac3}}^{\text{int}} \approx 9 \cdot P_{\text{vac1}}^{\text{int}} = 2.555 \cdot 10^{-13} \text{ erg/s}$).

For the case of four electrons at the time shifts $\Delta t_{12}^{(4)} = \Delta t_{23}^{(4)} = \Delta t_{34}^{(4)} = 0.001 \cdot \pi / \omega_{04}$, the coherence factor $S_4(\omega) \approx 16$ and four electrons radiate as a charged particle with the charge $4e$ and the rest mass $4m_0$ (curve 4 in Fig. 4), i.e. by a factor of sixteen higher than a single electron ($P_{\text{vac4}}^{\text{int}} \approx 16 \cdot P_{\text{vac1}}^{\text{int}} = 0.4543 \cdot 10^{-13} \text{ erg/s}$).

In the case of uniform location of two electrons (one the opposite gides of a convolution) at the time shifts $\Delta t_{12}^5 = \pi / \omega_{05}$ any radiation at the frequencies $(2i − 1) \omega_{05} (i=1,2,\ldots,20)$ is absent (curve 5 in Fig. 5).
In the case of uniform location of three electrons along spiral at the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = 2\pi/(3 \cdot \omega_{06})$ we have found that any radiation also is absent at the frequencies $(3i-2)\omega_{06}$ and $(3i-1)\omega_{06}$ ($i=1,2,\ldots,13$) (curve 6 in Fig. 6).

Similar in the case of uniform location of four electrons along spiral at the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = \Delta t_{34}^{(7)} = \pi/(2 \cdot \omega_{07})$ any radiation is absent at the frequencies $(4i-3)\omega_{07}$, $(4i-2)\omega_{07}$ and $(4i-1)\omega_{07}$ ($i=1,2,\ldots,10$) (curve 7 in Fig. 7).

4. Conclusions

The influence of the Doppler effect determines the band’s boundaries of separate harmonics in the radiation spectra of a single, two, three, and four electrons moving one by one along a spiral in vacuum.

For small time shifts two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four higher than a single electron.

For uniform location of two electrons along a spiral at the time shifts $\Delta t_{12}^{(2)} = \pi/\omega_{05}$ between them the radiation at the frequencies $(2i-1)\omega_{05}$ ($i=1,2,\ldots,20$) is absent.

For small time shifts three electrons radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$, i.e. by a factor of nine higher than a single electron.

For uniform location of three electrons along a spiral at the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = \pi/(3 \cdot \omega_{06})$ between them the radiation at the frequencies $(3i-2)\omega_{06}$ and $(3i-1)\omega_{06}$ ($i=1,2,\ldots,13$) is absent.

At small time shifts four electrons radiate as a charged particle with the charge $4e$ and the rest mass $4m_0$, i.e. by a factor of sixteen higher than a single electron.

For uniform location of four electrons along a spiral with the time shifts $\Delta t_{12}^{(7)} = \Delta t_{23}^{(7)} = \Delta t_{34}^{(7)} = \pi/(2 \cdot \omega_{07})$ between them the radiation at the frequencies $(4i-3)\omega_{07}$, $(4i-2)\omega_{07}$ and $(4i-1)\omega_{07}$ ($i=1,2,\ldots,10$) is absent.

References

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