A detection method of laser-screen by considering the cross-section attenuation coefficient and its application in projectile's coordinate parameters measurement system

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To improve the detection ability of a laser-screen and optimize application in projectile's coordinate parameters measurement, this paper proposes a new design method and detection model of laser-screen, uses the high power line lasers as an active light source and one-dimensional multi-unit PIN detector as the core detection and receiving device in receiving optical path module to form laser detection screen with a fan-shaped detection plane, and gives the imaging principle of the reflected laser energy of projectile in laser detection screen and the calculation method of defocus parameters of projectile's image information. We establish the cross-section attenuation coefficient model and the calculation function of the surface laser reflection energy of the projectile, and deduce the characterization function of the output signal that contains projectile information by considering the influence of the radiation of the sun, the earth and other radiation sources on the detection system. In addition, combined with environmental noise, the correlation model of detection probability and false alarm probability of laser-screen was derived. By applying the design of laser-screen to the projectile's coordinate parameters measurement system, the feasibility and correctness of the proposed theoretical method are verified by the test of real projectile.

1. Introduction

The dynamic parameters, such as projectile flight velocity, spatial dispersion status of predetermined plane position, pitch and azimuth angles, are the core technical indexes used to evaluate the damage efficiency of weapons. Additionally, they are the necessary test parameters in conventional weapon testing, and the parameters required for weapon type finalization standards. The photoelectric detection sensor based on the photoelectric detection conversion mechanism is a widely used device in weapon range testing, with the advantages of fast response, high sensitivity and flexible structure. The existing detection screen sensors include sky-screen and light-screen[1-3]. The sky-screen uses natural light as the background light source and forms a detectable fan-shaped plane detection screen with certain thickness resulting from the lens, slit diaphragm, photoelectric detector and signal processing circuit[4]. The sky-screen can be used to design a large wide-angle detection field of view according to the requirements. Additionally, the detection distance of the sky-screen is relatively far, which is suitable for outdoor testing. When the projectile passes through the detectable fan-shaped plane detection screen, the projectile imaging changes the luminous flux to form the instantaneous change

signal into the photoelectric detector. This signal is used as the basis to discriminate the projectile.

The light-screen uses a line array emitting LED or semiconductor laser as the artificial light source, and utilizes the line array photoelectric receiving semiconductor tube to receive the light energy of the line array emitting LED, to form a small area detection screen[5]. When the projectile passes through the detection area, the projectile blocks the emitted LED light, disrupts the light reception of the photoelectric receiving semiconductor tube, then outputs variable signals in the detection circuit as the basis for identifying projectiles. Due to the limitation of the structure and mechanism of the light-screen, the detection area of the light-screen is relatively small, which is suitable for indoor testing. In practical application, multiple sky-screens or light-screens are generally arranged along the ballistic line according to the intersection space relationship, which can be used for simultaneous measurement of projectile velocity, dispersion position, attitude angle and other parameters. The measurement method by using six sky-screens or light-screens to measure projectile flight velocity and position were introduced in [6-7]. The method of measuring the velocity of a warhead fragment through six light-screens was introduced in Reference [8]. In the early stage of gun weapon development, because its performance

cannot be estimated and the projectile launched by the new gun produces random vibration, etc., it is impossible to use the theoretical shooting index to test the projectile flight parameters of the gun in a certain area. Because the detection area of the light-screen is relatively small, it is difficult to meet the current testing requirements. On the contrary, the sky-screen that has a large detection area can effectively address this problem. Therefore, the sky-screen is an important photoelectric detection sensor for the external ballistics of guns. However, the detection background of the sky-screen is passive natural light, which restricts the use of the sky-screen the entire day.

Weapon testing is performed both under high and low environmental illumination[9-11]. In an environment with low illumination, the detection performance of sky-screen is restricted. The laser-screen was designed aiming to replace the traditional sky-screen, to improve the detection ability of the sky-screen in low illumination environments. Based on the design principle of the laser-screen, this paper proposes a new design method and detection model, establishes the calculation function of the surface laser reflection energy of the projectile by introducing a cross-sectional attenuation coefficient and deduces the characterization function of the output signal of the photoelectric detection receiving module. The main contributions of this work are as follows:

(1)   This paper proposes a new design method and detection model of laser-screen by uses the high power line lasers as an active light source and one-dimensional multi-unit PIN detector as the core detection and receiving device in receiving optical path module to form laser detection screen with a fan-shaped detection plane, and gives the imaging principle of the reflected laser energy of projectile in laser detection screen and the calculation method of defocus parameters of projectile's image information.

(2) The ratio function between the cross section of the projectile and the cross section of the detection screen at the same height was used to characterize the intensity of the reflected laser energy on the surface of the projectile and establish the cross-section attenuation coefficient model. A bidirectional reflection distribution function with four parameters was used to establish the calculation model of the surface laser reflection energy of the projectile by considering the laser emission power, cross-section attenuation coefficient and the projectile position.

(3) From the laser energy reflected on the surface of the projectile and the photoelectric conversion responsivity of the photoelectric detector, the characterization function of the output signal that contains projectile information was deduced by considering the influence of the radiation of the sun, the earth and other radiation sources on the detection system. Additionally, the correlation model of detection probability and false alarm probability of laser-screen was derived, which was combined with environmental noise.

(4) By applying the design of laser-screen to the traditional projectile's coordinate parameters measurement system, the projectile coordinate solution model of six laser-screens was established, and through comparative experiments, the feasibility and correctness of the proposed theoretical method are verified by the test of real projectile..

The remainder of this paper is organized as follows. The detection and design principle of laser-screen is detailed in section 2. The cross-section attenuation coefficient model and calculation method on the surface reflected laser power of the projectile are described in section 3. The characterization function of the output signal of the laser-screen in section 4, the correlation model of detection probability and false alarm probability are described in section 5. In section 6, the application of laser-screen in projectile coordinate parameter measurement is given. Section 7 gives the experimental verification and analysis. Finally, Section 8 concludes the paper.

2. Design and detection principle of laser-screen

2.1. Design method and principle of laser-screen

The laser-screen is a detection sensor that uses the laser source as the detection background and applies the reflected laser energy on the surface of the projectile as the sensing information of the photoelectric detector to form the target detection principle. Fig. 1 shows a schematic diagram of the design and detection principle.
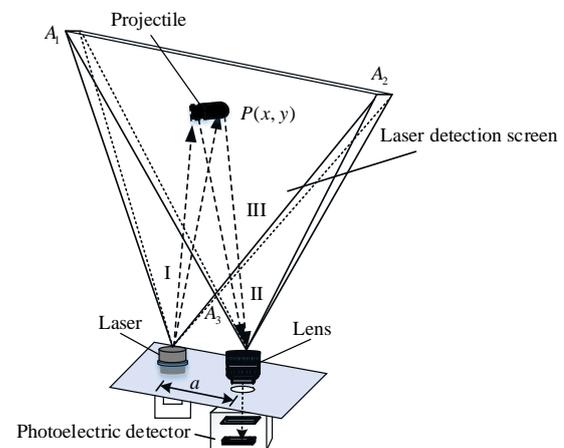
Fig. 1. The schematic diagram of the detection principle on the laser-screen (color online)

In Fig. 1, Plane I is the laser emission plane area, which is a fan-shaped plane laser light source with a certain thickness that is emitted by a high-power laser and expanded through the components of a refractive optical prism. Plane II is a detection screen of laser-screen, which is composed of a lens, a slit diaphragm, a photoelectric detector and a detection processing circuit. We call plane II as the receiving optical path module. The detection field of view and detection screen thickness of plane II are determined by the lens and slit diaphragm. During design, plane I and

plane II were made to be coincident to realize the laser detection screen, and their field of view and plane thickness were consistent, with both planes widening with the increase of the detection distance. Because the two planes coincide to form the effective detection area, namely plane III (i.e., area $A_1A_2A_3$), plane III is referred to as the laser detection screen, the thickness of the laser detection screen presents a gradient shape from the side view. From Fig. 1, the distance between the laser and the lens is a, compared with the projectile position in laser detection screen, we find that a is far lower than the detection distance in the laser detection screen, where the projectile is located. Therefore, a is negligible. From the formation principle of the laser detection screen of laser-screen, when the projectile passes through the laser detection screen, the laser energy of the surface of the projectile will reflect the photosensitive surface of the photoelectric detector. The photoelectric detector receives the changing reflected energy from the surface of the projectile, and converts the changed energy into electrical energy. After processing by the detection and amplification circuit, a dynamic projectile voltage signal can be sensed.

2.2. Imaging principle of the reflected laser energy of projectile in laser detection screen

According to Fig. 1, the laser detection screen consists of laser emission plane and a detection screen of laser-screen, it is not difficult to find that when the laser transmitting power and transmitting field of view are determined, the detection ability of the laser-screen is determined by the receiving optical path module. Fig. 2 is the imaging principle of the reflected laser energy of projectile in the receiving optical path module of laser detection screen.
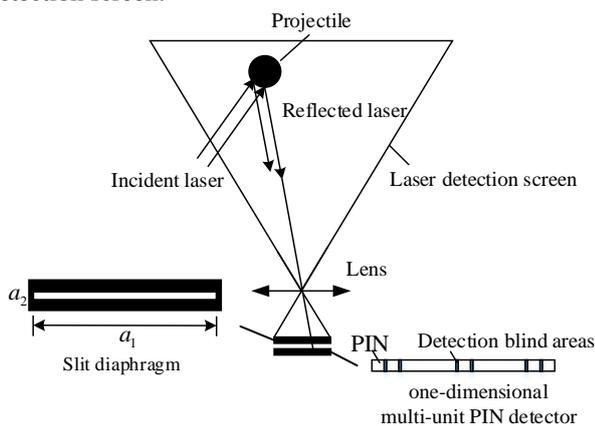
Fig. 2. The imaging principle of the reflected laser energy of projectile

In Fig. 2, we design a slit diaphragm, which is a rectangular window. The bottom of the slit diaphragm is a photoelectric detector. Because of the existence of the slit diaphragm, the imaging region of the projectile that the photoelectric detector can receive is only the region of the slit diaphragm, which is the detectable region of the

projectile, it's a fan shaped plane region. When the projectile passes through this fan shaped region, the surface of projectile reflects laser, and the reflected laser enters slit diaphragm, and then, photoelectric detector obtains the reflected laser energy of projectile, and detection processing circuit outputs a transient change signal, this changed signal provides the basis for calculation of the projectile's coordinate parameters measurement system.

According to the slit diaphragm structure, it is a rectangular window whose window size can be defined as $a_1 \times a_2$, a_1 is the length and a_2 is width. If the thickness of the laser detection screen is b, which is related to a_2, when the focal length of lens is f, $b = a_2 \times R / f$, R is the detection distance. If the coordinates of the projectile is determined, $R = \sqrt{x^2 + y^2 + z^2}$. z depends on the thickness of laser detection screen. In laser-screen, due to the relatively small width of the slit, for the flying projectile, its coordinates in laser detection screen only consider x and y, that is, $z=0$. In order to receive imaging information of the fan-shaped region formed by the slit diaphragm, we use one-dimensional multi-unit PIN detector to design its detection processing circuit. One-dimensional multi-unit PIN detector is made of sixteen unit PIN detectors. The size of the photosensitive surface on each unit PIN detector is $2.5mm \times 2.5mm$ and the photosensitive surface of the photoelectric detector is $41.5mm \times 2.5mm$. It is not difficult to see that the two adjacent PINs have detection blind areas with a size of $0.1mm \times 2.5mm$. If the image of the projectile just falls in the detection blind area, the detection sensitivity of the sensor will decrease. Table 1 shows the main designed parameters of the laser-screen.

Table 1. The main designed parameters of the laser-screen

The name of parameter	The value of parameter
Wavelength of PIN detector	480nm-670nm
Transmittance of optical system	0.8
Aperture of the lens	1:1.8
Focal length of lens	50mm
Responsiveness of detector	10^{-7}s
Size of slit diaphragm	40mm×0.25mm
Size of one-dimensional multi-unit PIN detector	41.5mm×2.5mm
Laser emission power	40W
Field of view of laser	50º

Table 1 is only a parameter of a conventional state detection capability of laser-screen. In order to further improve the detection distance or change detection state, we can choose to change some parameters to meet the requirements, such as, the power of laser, the short-focus lens, the amplification of detection circuit, etc.

2.3. The calculation method of defocus parameters of projectile's image information

A one-dimensional multi-unit PIN detector has a blind area, if the imaging spot of the projectile is located in the detection blind area, the laser-screen may not be able to sense the information of laser reflected energy from the projectile surface. To improve the detection performance of the laser-screen and make the imaging of the projectile evenly distributed on the photosensitive surface of the one-dimensional multi-unit PIN detector, we propose a defocus imaging detection design method. The method is that one-dimensional multi-unit PIN detector and slit diaphragm is a short distance away from the focal plane. The position of the slit diaphragm affects how much of the reflected laser imaging energy shot of projectile is concentrated on the photosensitive surface of PIN, it can be located in the image plane of the moving projectile or the focal plane of the image of the lens. If the slit diaphragm is located in the image plane of the moving projectile, although the image of projectile is clear, due to the defocus amount, the infinite background light passing through the imaging lens will form diffuse spots in the image plane. Because the existence of diffuse spots, part of the light energy will be blocked by the slit diaphragm, reducing the light energy projected to the photoelectric detector. Therefore, the detection performance is best when the image spot of the projectile is close to the area size of the photoelectric detector. In Fig. 3, L is defocus parameter, which is expressed by formula (1).

$$L = f^2 / (R - f) \qquad (1)$$

The slit diaphragm is placed in defocus position and the photosensitive surface of the photoelectric detector is located below the slit diaphragm. Then the diffuse spot diameter is obtained on the photosensitive surface using formula (2).

$$g = L \cdot D / f = f \cdot D / (R - f) \qquad (2)$$

where g the diffuse spot diameter, and D is the aperture of the lens.

According to the selected one-dimensional multi-unit PIN detector, it is required that the diameter of the diffuse spot is not less than the outer tangent circle diameter of the photosensitive surface of the unit PIN, namely, $\phi \geq 3.54mm$. Obviously, the diffuse spot diameter is not infinitely larger than $3.54mm$, which needs to be determined by the size of the blind area of the one-dimensional multi-unit PIN detector. When the aperture of the lens and the diffuse spot diameter are determined, the defocus parameters can be determined.

3. The cross-section attenuation coefficient on the projectile surface reflected laser power

According to the design method and detection principle of the laser-screen, the laser detection screen is a fan-shaped plane with a certain thickness. When the detection distance increases, the thickness of detection screen also increases, as shown in Fig.3. In order to establish an intuitive projectile detection model, the parameters of plane II were calculated and analyzed based on the photoelectric receiving optical path.
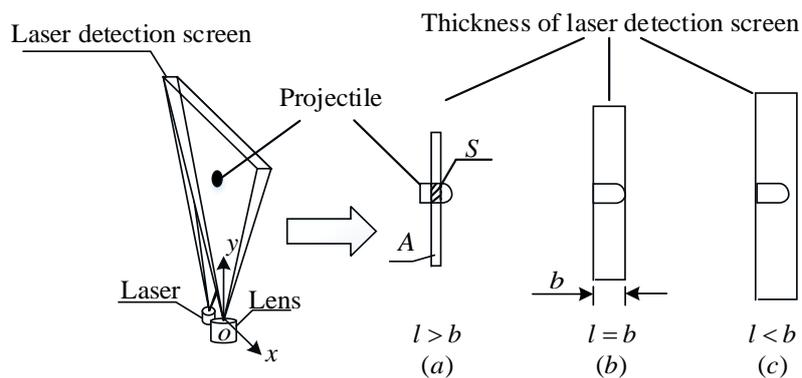
Fig. 3. The schematic diagram of thickness of laser detection screen and coordinate system

The position of the lens that receives the reflected laser energy from the surface of the projectile was defined as the coordinate origin of the laser detection screen plane. It was marked as point o, xoy is coordinate system of laser detection screen, oy is the detection height distance direction, ox is the center line between the laser and the lens. Let $P(x, y)$ be the position where the projectile passes through the laser detection screen, then x is the vertical distance between the projectile and the optical axis of the lens, and y is the vertical height. From plane III, the width of the laser detection screen is determined by the slit diaphragm and lens.

Fig. 3 shows that the thickness of the laser detection screen is different at the various detection height distances. The same projectiles pass through the laser detection screen and show three different states. If l is the length of projectile, when $l > b$, it indicates that the surface area of the projectile reflecting laser in the laser detection screen is determined by the thickness of the laser detection screen (Fig. 3(a)). When $l = b$, the projectile surface is located in the laser detection screen, and the surface area of the projectile irradiated by the laser can contribute to the photoelectric detector entirely (Fig. 3(b)). When $l < b$, the projectile surface passes through the laser detection screen in a short period (Fig.3(c)). Under this state, the reflected area of the projectile surface is constant and it is determined by the cross-sectional area of the projectile itself. However, when the detection distance increases, the cross-sectional area of the laser detection screen increases at the height of the projectile.

These three states show that the ability of the laser-screen to detect projectile related to the laser emission power and detection distance in addition to the projectile reflection surface area at detection distance R. Let S be the reflected area of the projectile surface by the laser at the detection distance R, then the ratio of the reflected area of the projectile surface and the cross-sectional area of the laser detection screen at the detection distance R is defined as the cross-sectional attenuation coefficient[12], recorded as $\sigma(R)$. Formula (3) shows the calculation function of $\sigma(R)$.

$$\sigma(R) = \frac{S}{A} = \frac{S}{a_1 a_2 R^2 / f^2} \qquad (3)$$

where A is the cross-sectional area of the laser detection screen at detection distance R, which takes values in the interval of $[0,1]$. Then, the reflected laser illumination by the projectile surface can be obtained by formula (4).

$$E(R) = \frac{P_t \varepsilon \sigma(R) e^{-2\delta R}}{S} \qquad (4)$$

where ε is the efficiency of the optical receiving system, δ is the atmospheric attenuation coefficient, P_t is the laser emission power, and the reflected luminous flux that is emitted from the projectile surface in the laser detection screen is expressed by formula (5).

$$\Phi(R) = \frac{E(R)S}{\cos \alpha_0} \qquad (5)$$

where α_0 is the reflected angle of laser on the projectile surface.

To obtain the reflected laser power, the four parameter bidirectional reflection distribution function (BRDF) was used to represent the projectile reflection characteristics[13]. BRDF represents the reflection characteristics of the projectile surface at any observation angle, under different incident angles. It is an important function that describes the diffusion reflection characteristics of the materials of the projectile and is expressed by the ratio of reflected and incident irradiance of projectile light radiation. Its mathematical expression is as follows:

$$f_r(\theta, \varphi, \alpha, \beta) = \frac{dL(\theta, \varphi, \alpha, \beta)}{dE(\theta, \varphi)} \qquad (6)$$

In (6), θ and φ are the incident angle and azimuth angle of laser incident light, respectively, α and β are the reflection angle and azimuth angle of the reflected laser of the projectile's surface, respectively, $L(\theta, \varphi, \alpha, \beta)$ is the irradiance of the projectile in direction (α, β) of laser irradiationand its unit is $W / m^2 \cdot sr \cdot um$; $E(\theta, \varphi)$ is the irradiance of the projectile surface generated by the emitting laser incident light in the direction (θ, φ) and its unit is $W / m^2 \cdot um$. The unit of $f_r(\theta, \varphi, \alpha, \beta)$ is sr^{-1}, and its physical meaning is the ratio of the irradiance emitted along direction (α, β) to the irradiance incident on the projectile surface in direction (θ, φ). The laser reflection characteristics of the projectile are denoted by f_r, which can be obtained by formula (7).

$$f_r = \frac{C_1}{\cos^6 \theta} \exp(\tan^2 \theta / \varpi^2) + C_2 \cos^m \theta \qquad (7)$$

where the first component is the specular reflection component and the second is the diffuse reflection component, C_1, C_2, ϖ and m are undetermined coefficients determined by the material of the projectile surface, C_1 is the specular reflection amplitude, C_2 is the diffuse reflection amplitude, ϖ is the surface inclination coefficient and m is the diffuse reflection coefficient[14-16]. The radiation intensity of the projectile can be calculated by formula (8).

$$I(R) = f_r \cdot \Phi(R) / \pi \qquad (8)$$

The reflected laser power on the projectile surface is calculated as follows:

$$P_r = \frac{P_t \varepsilon e^{-2\delta R} f_r A_0}{\pi R^2} \frac{S}{a_1 a_2 R^2 / f^2} \qquad (9)$$

where A_0 is the optical aperture area of the lens.

From the design principle of laser-screen, the main influence of the thickness parameter of the fan-shaped plane is the optical lens parameters and slit parameters of

the receiving optical path system. When the optical lens parameters and the size of slit diaphragm are determined, the thickness of the fan-shaped plane is also determined. The thickness of the fan-shaped plane is related to the detection capability of the system, mainly because the ratio of the area occupied by the projectile changes when the projectile passes through different detection position. From Fig.3, we can know that when the projectile is located at different altitude positions, the area ratio of the projectile in the laser detection screen is different. Based on the principle of cross-section attenuation coefficient on the projectile surface reflected laser power, the longer the detection distance, the smaller the echo energy reflected from the surface of the projectile, so the thickness of laser detection screen affects the detection sensitivity of the system, and the sensitivity is reduced, and the test accuracy is also affected.

4. The characterization function of the output signal of laser-screen

When the projectile passes through the effective area of the laser detection screen, the projectile surface is illuminated by the laser and forms the reflected laser energy. According to the reflection characteristics of the projectile surface and the principle of specular reflection and diffuse reflection, the reflected laser energy on the projectile surface is a changeable luminous flux, which is detected by the photoelectric detector to form a changeable projectile signal. The number of reflected laser photons received by the photoelectric detector in the period can be obtained by formula (10).

$$n_p = \frac{P_r \cdot (b+l)}{v E_e} \tag{10}$$

where E_e is the energy of a single photon, $E_e = hc$, h is Planck constant, c is the velocity of light and v is the velocity of projectile.

The quantum efficiency of the photoelectric detector is denoted by η, the number of charges that were generated by the reflected laser of the projectile surface can be calculated with formula (11).

$$Q_e = \eta P_r \frac{b+l}{v} \frac{1}{hc} \tag{11}$$

The detection system receives both the reflected laser information from the projectile surface and the environmental noise. The environmental noise is the main influencing factor of the laser-screen, which comes from radiation sources such as the sun and the ground. In laser-screen, the noise of the detection system in generated from the inherent noise of the photoelectric detector and the environmental noise. To improve the detection ability of laser-screen, it is necessary to establish a signal-to-noise ratio function of the detection system. The noise of the photoelectric detector can be expressed by formula (12).

$$N_s = \sqrt{n_1^2 + n_2^2 + n_3^2 + n_4^2} \tag{12}$$

where n_1 is the root mean square value of shot noise, n_2 is the root mean square value of thermal noise, n_3 is the root mean square value of the generated composite noise, and n_4 is the root mean square value of temperature noise[17].

Environmental noise is mainly caused by the change of background brightness when the radiation of various sources enters the effective field of view of the laser detection screen, with its core influencer being solar radiation. The solar radiation power can be expressed by formula (13).

$$P_{sun} = \frac{\pi \theta_0^2}{4} M_{sun} \cdot A_0 \cdot \Delta\lambda \cdot \tau_0 \tag{13}$$

where M_{sun} is the total solar radiance, $M_{sun} = \int_{\lambda_1}^{\lambda_2} \varepsilon \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$, ε is the reflectivity of projectile surface[18-19], T is the temperature of the projectile surface, λ_1 and λ_2 are the upper and lower wavelengths of the solar spectral band received by the detection system, respectively, τ_0 is the lens transmittance, $\Delta\lambda$ is the bandwidth of narrow-band filter and θ_0 is the effective receiving field of view of the laser detection screen. According to formula (9), the equivalent photoelectron quantity that is generated by environmental noise can be obtained by formula (14).

$$Q_n = \eta P_{sun} \frac{b+l}{v} \frac{1}{hc} = \eta \frac{\pi \theta_0^2}{4} M_{sun} \cdot S_0 \cdot \Delta\lambda \cdot \tau_0 \cdot \frac{b+l}{v} \frac{1}{hc} \tag{14}$$

According to the definition of signal-to-noise ratio[20-21], the photoelectron quantity is used to measure signal and noise, and the signal-to-noise ratio is expressed by formula (15).

$$SNR = \frac{Q_e}{\sqrt{Q_e + Q_n + N_s^2}} \tag{15}$$

The signal-to-noise ratio can be used to evaluate the detection ability of the laser-screen. For more intuitive evaluation and analysis, the number of charges that are generated by the reflected laser energy by the projectile surface is converted into a voltage function, and the output voltage function of the projectile can be gained by formula (16).

$$V_o = R_i A_v \eta \frac{1}{hc} \frac{P_t \varepsilon \sigma(R) e^{-2\delta R} f_r A_0}{\pi R^2} = R_i A_v \eta \frac{1}{hc} \frac{P_t \varepsilon e^{-2\delta R} f_r A_0}{\pi R^2} \frac{S}{a_1 a_2 R^2 / f^2} \qquad (16)$$

where R_i is the equivalent resistance of the detection circuit and A_v is the gain of the detection circuit.

When no projectile passes through the laser detection screen, the outputs only noise signals in detection circuit, which can calculated by formula (17).

$$V_n = R_i A_v (\eta \frac{\pi \theta_0^2}{4} M_{sun} \cdot A_0 \cdot \Delta\lambda \cdot \tau_0 \cdot \frac{1}{hc} + N_s) \quad (17)$$

5. Detection probability calculation model of laser-screen

The detection performance of laser-screen is inevitably affected by the environmental noise and the photoelectric detector noise. To characterize the detection performance of the laser-screen, it is necessary to establish the detection probability and false alarm probability models. The noise signal is described as a Gaussian distribution, and the probability density function of the noise signal can be expressed by formula (18).

$$\rho(V_n) = \frac{1}{\sqrt{2\pi}V_n'} \exp[-V_n^2 / 2(V_n')^2] \qquad (18)$$

where V_n is the noise voltage amplitude and V_n' is the root mean square of the noise voltage. When a projectile passes through the laser detection screen, the output signal is the superposition of the reflected laser on the projectile surface and the noise signal. The output signal is $V_{out} = V_o + V_n$. The probability density function of the output signal can be expressed by formula (19).

$$\psi(V_n) = \frac{1}{\sqrt{2\pi}V_n''} \exp[-(V - V_o)^2 / 2(V_n'')^2] \qquad (19)$$

where V_n'' is the root mean square of noise voltage when the reflected laser beam enters the photosensitive area of photoelectric detector.

The detection probability is defined as the probability that the peak voltage of the signal superimposed with noise exceeds the threshold voltage when the laser is reflected by projectile surface. The probability that the peak voltage of the noise signal exceeds the threshold voltage when there is no projectile passing through the laser detection screen was used to denote the false alarm probability. Assuming V_T is the threshold voltage, the detection probability and false alarm probability of the laser-screen can be calculated by formulas (20) and (21).

$$P_d = \int_{V_T}^{\infty} \psi(V_n) dV_n = \frac{1}{2} \mathrm{e}\, rfc(V_T / \sqrt{2}V_n'') \qquad (20)$$

$$P_{false} = \int_{V_T}^{\infty} \rho(V_n) dV_n = \frac{1}{2} \mathrm{e}\, rfc[(V_T - V_o) / \sqrt{2}V_n''] \quad (21)$$

where $erfc(\cdot)$ and $erfc[\cdot]$ are complementary error functions[22] and $V_o = V_{Peak}(R)$, $V_{Peak}(R)$ are the projectile peak voltage outputs by the detection system at the detection distance R.

6. Application of laser-screen in projectile's coordinate parameters measurement system

Based on the design of laser-screen, we apply this design method to the projectile's coordinate parameters measurement system which use six laser screen to form an intersection test system. The spatial geometric relation of projectile's coordinate parameters measurement is shown in Fig.4, in which $G_1 - G_6$ are the laser detection screens, $o_1 o_2$ is the direction of flight of the projectile.
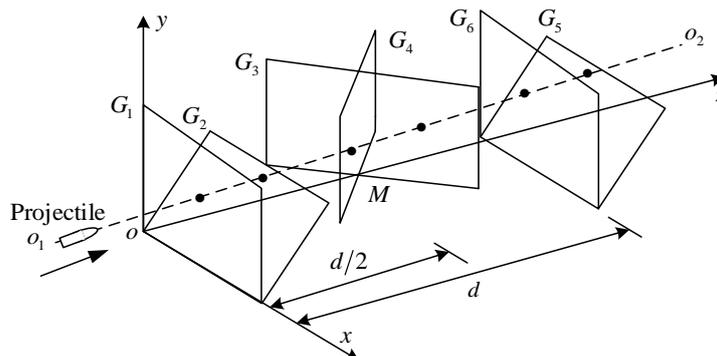
Fig. 4. The spatial geometric relation of projectile's coordinate parameters measurement system

When the projectile passes through each laser detection screen, each laser screen will obtain the reflected laser energy by the projectile, and through the signal identification and processing circuit, we gain the time value when the projectile passes through the laser detection screens $G_1 - G_6$, which denote as $t_1 - t_6$. In Fig.4, G_1 located in plane xoy, G_1 and G_6 are parallel to each other and they're both perpendicular to the plane xoz, and he distance between G_1 and G_6 is d. The intersection angles of G_2 and G_5 with plane xoy are α, while the intersection angles of G_3 and G_4 with plane yoz are β, and G_3 and G_4 are

perpendicular to plane xoz, at the same time, G_3 and G_4 intersect at the point M, $oM = d / 2$.

Because the distance between the six laser detection screens is relatively short, the projectile passes through the six laser detection screens can be regarded as a uniform linear motion. Let v be the flight velocity of the projectile, (v_x, v_y, v_z) is their velocity components in the coordinate system, and record the coordinates of the point of projectile is $P_{G_1}(x_1, y_1, z_1)$ in G_1, when $t_1 - t_6$ are determined, and $P_{G_1}(x_1, y_1, z_1)$ can gain by formula(22).

$$
\begin{cases}
z_1 = 0 \\
-y_1 \cos\alpha + z_1 \sin\alpha - v_y(t_2 - t_1)\cos\alpha + v_z(t_2 - t_1)\sin\alpha = 0 \\
-x_1 \cos\beta + z_1 \sin\beta + v_x(t_3 - t_1)\cos\beta + v_z(t_3 - t_1)\cos\beta = 0.5d\cos\beta \\
x_1 \cos\beta + z_1 \sin\beta + v_x(t_4 - t_1)\cos\beta + v_z(t_4 - t_1)\cos\beta = 0.5d\cos\beta \\
-y_1 \cos\alpha + z_1 \sin\alpha - v_y(t_5 - t_1)\cos\alpha + v_z(t_5 - t_1)\sin\alpha = d\sin\alpha \\
z_1 + v_z(t_6 - t_1) = d
\end{cases}
\tag{22}
$$

7. Calculation and analysis

7.1. Influence of detection distance on detection ability of laser-screen

The detection ability of the laser-screen is related to the detection distance, that is, the position where the projectile passes through the laser detection screen. Increase in the detection distance for the laser-screen leads to increase in the thickness of the laser detection screen. When the size of the slit diaphragm and the lens are constant, the thickness of the laser detection screen is linearly proportional to the detection distance, which is determined by the cross-sectional attenuation coefficient. It can also be seen that the cross-sectional attenuation coefficient decreases with the increase of the detection distance, that is, when the detection distance is bigger, the ratio of the area occupied by the projectile in the entire cross-sectional detection screen to the cross-sectional area of the projectile itself decreases. Fig. 5 shows the calculation result on the relationship between the cross-sectional attenuation coefficient and the detection distance, where $l1$ is the size that the diameter of projectile is 7.62mm, and $l2$ is the size that the diameter of projectile is 12.7mm.

It is also reflected from Fig. 5 that the cross-sectional attenuation coefficient is not only related to the detection distance, but also to the reflected laser power on the projectile surface at a certain detection distance. In addition, a larger projectile size causes a larger cross-sectional attenuation coefficient. To capture and recognize the projectile information the reflected laser

power that is detected by the photoelectric detector must be greater than the minimum detection power. Formula (17) shows that the noise signal of the detection system is mainly determined by the environmental noise and the inherent noise of the detection circuit when there is no projectile passing through the laser detection screen. The photoelectric detector can sense the laser reflection information, which is the difference between the laser reflection energy and the system noise. When this difference is greater than a certain threshold voltage, it can be considered that the laser-screen can capture or sense the information of projectile. Therefore, the noise voltage of the system can be defined as the minimum detection power of the system when there is no projectile passing through the laser detection screen.
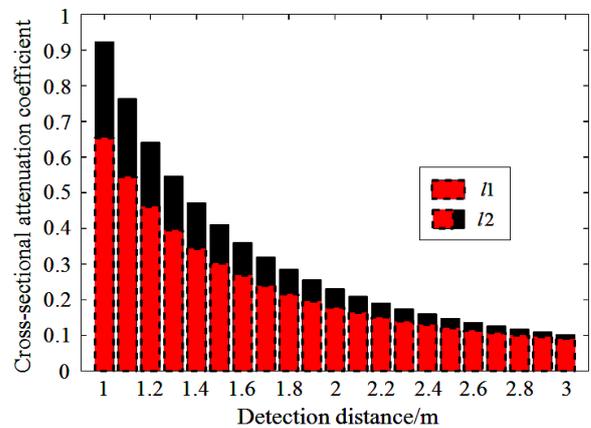
Fig. 5. The relationship between the cross-sectional attenuation coefficient and the detection distance (color online)

Formula (16) shows that when the design parameters of the laser-screen and the laser emission power are constant, the reflected laser energy by the projectile surface is both related to the laser emission power and to the cross-sectional attenuation coefficient, which shows that the size of the laser reflection cross-sectional of the projectile in the laser detection screen is an important factor. Fig. 6 shows that the detection distance is proportionally related to the cross-sectional attenuation coefficient. Therefore, the laser-screen is a sensor related to the detection distance, and the detection ability of the detection system can be characterized by the detection distance. Fig. 6 shows the relationship between the output signal of the laser-screen and the detection distance, according to formula (16), it shows clearly that the output signal amplitude of laser-screen is inversely proportional to the detection distance.
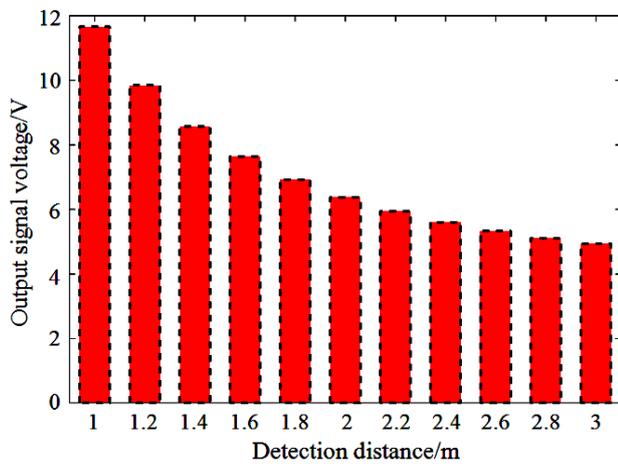
Fig. 6. The output signal under the different detection distances (color online)

7.2. Influence of detection distance on reliability of the detection system

According to the detection probability model of the laser-screen, its output signal is the superposition of the reflected laser energy on the projectile surface and the noise signal. At the same time, the reflected laser energy of the projectile surface is directly affected by the detection distance. Based on formulas (20) and (21), the detection probability and false alarm probability of the detection system are determined by V_o and V_n. These formulas are the main functions characterizing the reliability of the detection system. Formulas (9) and (16) show that the reflected laser energy of the projectile surface is related to the detection distance. The detection reliability of the laser-screen is determined by the detection and false alarm probabilities. Generally, the reliability of the detection system can be ensured only when the false alarm probability does not exceed 10^{-3} and the detection probability is not lower than 98%. When $V_T/V_n" \geq 1.5$ and $(V_{o-peak}(t)-V_T)/V_n" \leq 1.5$ are satisfied, the detection performance of the detection

system is reliable, and $V_{o-peak}(t)$ is the peak voltage of the projectile. Fig. 7 shows the relationship between the detection probability and the false alarm probability at different thresholds.

It can be seen from Fig. 7 that the detection probability and false alarm probability are related to the laser energy reflected by the projectile surface and the system noise, while the peak voltage of the projectile is related to the detection distance. Therefore, the detection reliability of the laser-screen is determined by the detection probability and the false alarm probability. When the threshold voltage is different, the detection probability and false alarm probability also change, and the higher the threshold is, the smaller the detection probability and the false alarm probability.
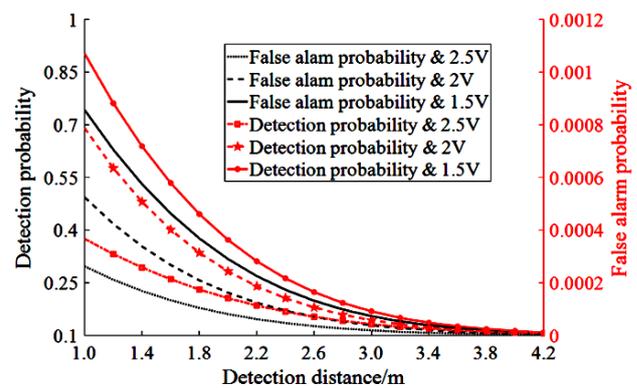
Fig. 7. The relationship between detection probability and false alarm probability at different thresholds (color online)

7.3. Influence of cross-section attenuation coefficient on the reflected laser energy of the projectile surface

The cross-sectional attenuation coefficient is an important factor affecting the laser reflection energy of the projectile surface, the cross-sectional attenuation coefficient is related to the detection distance and the cross-sectional area of the laser detection screen at the given detection distance. The cross-sectional area of the laser detection screen can be determined from the size of the slit diaphragm and the focal length of the lens. In the design of the system, $a_1 = 42mm$, $a_2 = 0.25mm$, the focal length of the lens was $50mm$, its effective detection field of view was about $41°$ and the luminous aperture was 1:1.8. When the spectral band of the photoelectric detector ranged between $480nm$-$1100nm$, the transmittance of the lens was 95%. According to the relationship between the detection distance and the thickness of the laser detection screen, and in combination with formula (16), the relationship between the cross-sectional attenuation coefficient and the laser reflected energy of the projectile surface can be calculated from formula (16). Fig.8 shows the calculation result.
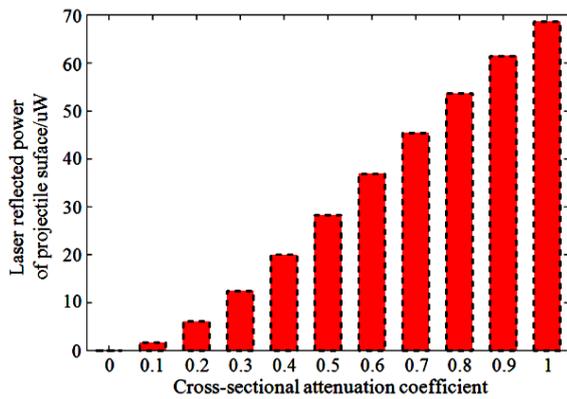
Fig. 8. The relationship between the cross-sectional attenuation coefficient and the reflected laser energy (color online)

Fig. 9. The relationship between the laser emission power and the laser reflection energy of projectile's surface (color online)

Fig. 8 shows that the laser-screen is different from the conventional photoelectric detection sensor. The detection area that is formed is a fan-shaped detection plane. When the length of the projectile is not equal to the thickness of the laser detection screen, the cross-sectional attenuation coefficient is not linearly related to the detection distance. Therefore, when the length of the projectile is less than the thickness of the laser detection screen, the laser reflection energy of the projectile surface does not follow a linear relationship with the cross-sectional attenuation coefficient.

7.4. Influence of laser emission power on reflected laser energy of the projectile surface

In the above analysis, it can be seen that the reflected laser energy of the projectile surface is both related to the detection distance and cross-sectional attenuation coefficient, in addition to the laser emission power. These influence factors restrict the detection and false alarm probabilities of laser-screen. Changes in the laser emission power cause slight changes in the inherent noise of the detection system. The detection reliability of the detection system for the laser-screen is directly related to the threshold voltage, which is determined by the equivalent photoelectron number generated by the environmental noise. Formulas (14) and (17) show that the laser-screen considers the detection ability of low environmental illumination, in addition to the detection ability of high environmental illumination. Under a given level of environmental illumination, the inherent noise of the detection system mainly comes from the total solar radiation M_{sun}. When the noise is basically constant, the detection ability of the laser-screen is directly related to the laser emission power. The change relationship between the laser emission power and the laser reflection energy of the projectile surface can be calculated by formula (18). Fig.9 shows the calculation result on the relationship between the laser emission power and the laser reflection energy of projectile surface. The result shows that when the laser emission power increases, the laser reflected power of the projectile surface also increases when the detection distance is the same.
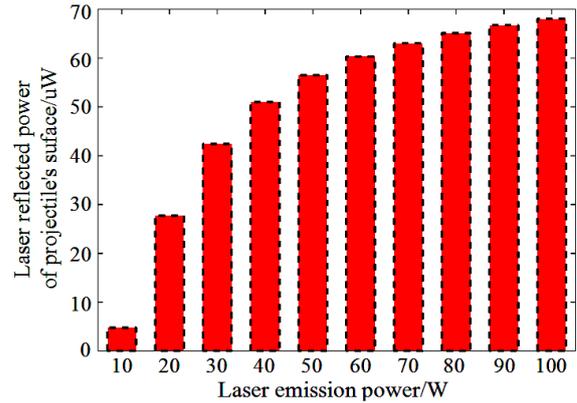
7.5. Comparative test and analysis

The focal length of the lens was $50mm$, the relative aperture was 1:1.8, the laser emission power was $40W$, and the size of slit diaphragm was $40mm \times 0.25mm$ in the designed laser-screen. According to the optical structure and imaging principle, for a detection distance of $5m$, the thickness of the laser detection screen was $30mm$. The thickness of the laser plane was also consistent with that of the laser detection screen. The air gun was used as the shooting weapon. The diameter and length of the projectile were $4.5mm$ and $5.6mm$, respectively, and the average velocity was $140m/s$. The acquisition card was used to gather the output signal of the projectile. The comparative test was performed experimentally by using the traditional photoelectric detection target [20] and the designed laser-screen under different environmental illuminations. The laser-screen was closest to the firing position of the weapon, and the traditional photoelectric detection target was placed between the laser-screen and the wooden target, with a distance of about $0.3m$. The distance between the traditional photoelectric detection target and the wooden target was about $0.3m$. The vertical detection screen was used to shoot the weapon. Moreover, the wooden target was close to the shooting position of the weapon, and was considered to be in a straight flight state, therefore, the position where the wooden target was hit by the projectile can be considered as the position where the projectile passes through the detection screen.

The wooden target was used to record the position that the projectile was shoot into, the center of the lens of the laser-screen was defined as the coordinate origin. When the environmental illumination was about $4500lx$ and the laser emission power was $40W$, five projectiles were fired, then the coordinate position of the five projectiles and the peak voltage of the projectiles were recorded. Table 2 shows the test data of the five projectiles, among, the maximum amplitude of projectile signal is $12V$. In Table 2, V_o and V_n are the peak voltage of projectile and the noise average voltage in laser-screen, respectively, and V_o' and V_n' are the peak voltage of projectile and noise average voltage in the traditional photoelectric

detection target, respectively, the unit of voltage is volt. "-" indicates that projectile information cannot be

identified.

Table 2. The test data at environmental illumination of 4500lx

No.	x /m	y /m	V_o /V	V_n /mV	V_o' /V	V_n' /mV
1	0.45	1.52	10.95	784	10.21	903
2	0.51	1.86	9.86	765	8.75	921
3	-0.36	2.21	8.57	802	7.48	908
4	0.31	2.63	7.14	816	5.21	996
5	0.49	3.02	5.08	738	3.87	915
6	-0.28	3.34	3.48	779	1.52	845
7	-0.15	3.55	2.16	783	-	982
8	0.22	3.71	-	763	-	927

Fig. 10 illustrates the output signal of the 4-*th* projectile of two the test devices at environmental illumination of 4500*lx*, CH1 is the projectile signal output by the laser-screen and CH2 is the projectile signal output by the traditional photoelectric detection target. The average voltage of noise was about 780*mV* in laser-screen, and the average voltage of noise was about 916*mV* in traditional photoelectric detection target (Table 2). When the detection distance increases, the peak voltage of the projectile decreases in the two detection devices. From the second to the fifth projectile, the height of the projectile hitting position basically increased linearly, but the peak voltage of the projectile did not decrease linearly, which shows that when the heights of the projectile are different, the thickness of the laser detection screen is also different.

When the height of projectile shooting was greater than 2.5*m*, the peak output voltage of the traditional photoelectric detection target was smaller, in comparison to the laser-screen. When the height was 3.34*m*, the peak voltage of the projectile of the traditional photoelectric detection target was less than 1.52*V*, while the voltage of the laser-screen remained at 3.48*V*. This also reflects that the detection ability of the designed laser-screen was considerably improved. When the height was 3.55*m*, the peak voltage of the projectile of the laser-screen was about 2.16*V*, and the signal-to-noise ratio about 2.78. When the height was 3.71*m*, no target was able to identify the projectile.
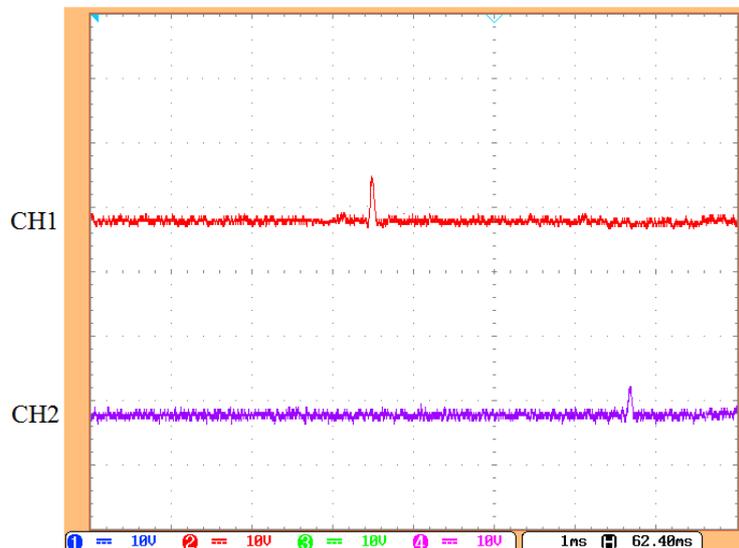
Fig. 10. The output signal of the 4-th projectile at environmental illumination of 4500lx (color online)

It can also be seen from the data in Table 2 that the peak voltage of the projectile in the laser-screen at different heights varies greatly, which shows that the detection performance of the sensor is related to the position of the laser detection screen where the projectile is located, and that the contribution of the reflected laser energy of the projectile surface to the photoelectric

detector is also different. From the second, third and fifth projectile, the position of *ox* direction remained almost the same, the positions of *oy* direction were 1.86*m*, 2.21*m* and 3.02*m*, respectively, and the peak values of the projectile signal were 9.86*V*, 8.57*V* and 5.08*V*, respectively. The peak value of the fifth projectile was half the peak value of the second projectile, while the second projectile only

reduced by 1.3V compared with the third projectile. This results shows that as the detection distance increases, the thickness of the laser detection screen also increases, and the cross-sectional attenuation coefficient is variable. Therefore, the reflected laser energy of the projectile surface decreases with the increase of the detection distance. This result is consistent with the theoretical formulas (3), (9) and (16). According to the noise under environmental illumination, the selected threshold voltage was 1.5 times of the system noise voltage. In addition, as the detection distance increases to 3.0m-4.0m, the cross-sectional area of the projectile remains basically unchanged at the upper and lower limits of the detection distance, but the cross-sectional area of the laser detection screen changes at these limits, so the cross-sectional

attenuation coefficient was 0.02-0.05. This leads to the decrease of the cross-sectional area attenuation coefficient, which means that the detection ability of the system is reduced. The peak signal of the projectile has great attenuation, mainly because the change of cross-sectional area attenuation coefficient of the system is not linear, resulting in the non-linear reduction of laser energy reflected by the projectile surface (Table 3).

In order to further verify the detection ability of the laser-screen, we carried out another experiment when the environmental illumination was about 560lx, and shoot seven projectiles. Table 3 is the test data, Fig. 11 is the output signal of the 4-th projectile of two test devices at environmental illumination of 560lx.

Table 3. The test data at environmental illumination of 560lx

No.	x /m	y /m	V_o /V	V_n /mV	V_o' /V	V_n' /mV
1	-0.38	1.55	11.42	613	-	1021
2	-0.23	1.81	11.13	576	-	988
3	-0.16	2.58	10.26	559	-	967
4	0.28	2.87	9.58	542	-	1027
5	0.41	3.32	7.62	635	-	1018
6	-0.22	3.54	5.33	571	-	876
7	-0.18	3.82	3.29	608	-	915
8	-0.38	3.89	3.05	615	-	1021

Fig. 11 shows that for environmental illumination of about 560lx, the noise change of the two test devices was not very large, but for the traditional photoelectric detection target, the environmental illumination is far lower than 2000 lx, and its detection ability becomes very poor. For the laser-screen, when the detection distance was about 2.58m, the amplitude of the projectile signal increased by about 2V, compared with the projectile signal with a the detection distance of 2.63m in Table 2. At the same time, under the condition of environmental illumination mentioned in Table 3, the peak voltage of the projectile of laser-screen increased considerably at basically close positions, and traditional photoelectric detection target is basically unable to detect the

information of the projectile. Because the environmental illumination is under 560lx, the impact of the radiation formed by the sun is almost zero, so when no projectile passes through the laser detection screen, the inherent noise of laser-screen is only 580mV. The impact on the detection probability and false alarm probability is only the inherent noise of the system itself. This shows that with the help of the active light source formed by the high-power laser, the detection ability of the laser-screen was greatly improved under the low illumination condition, which is conducive to the night test.
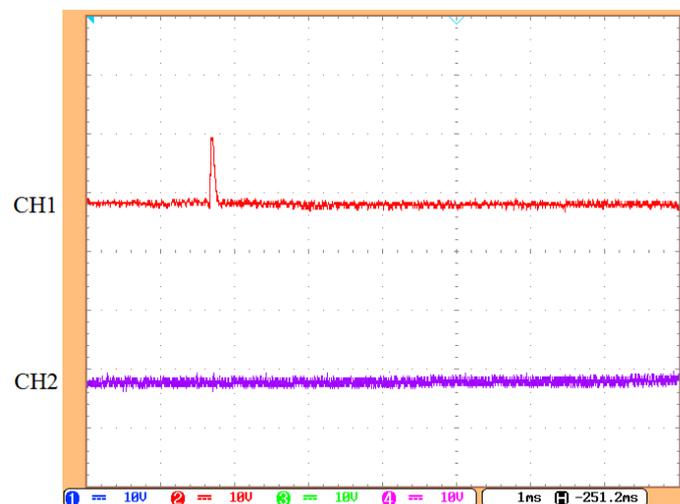
Fig. 11. The output signal of the 4-th projectile at environmental illumination of 560lx (color online)

In order to further verify the superiority of the design method and theoretical model in this paper, when the environmental illumination is about 1550*lx*, we conduct a group of comparative test to measure the coordinate parameters of 8 projectiles, and the diameter of the shot projectile is 4.5*mm*. We compared the test results without laser light source and with laser light source. The traditional six sky screen test system without laser light source adopts the calculation method in reference [23] and [24], which is called the traditional test system. Table 4 shows the test data of comparative test.

Table 4. Comparative test data

No.	The design method of this paper		The traditional test system	
	x /m	*y* /m	*x* /m	*y* /m
1	-0.22	1.28	-0.28	1.33
2	0.56	1.45	0.49	1.38
3	0.45	1.62	0.51	1.56
4	-0.78	1.75	-	-
5	-0.51	1.98	-	-
6	-0.22	2.79	-	-
7	-0.18	3.41	-	-
	-0.38	3.55	-	-

It can be seen from Table 4 that under the condition of environmental illumination is about 1550*lx*, when the detection distance is greater than 1.62*m*, the traditional test system can no longer recognize the information of the projectile. Compared with it, the laser-screen designed in this paper, which is taken as the detection sensor of the six sky screen intersection test system, has a great improvement in the detection ability. When the detection distance is 3.55*m*, the test system with laser-screen can also obtain the parameters of projectile. Fig.12 is the comparative waveform diagram of the third projectile collected in the comparative test, in which Fig.12 (a) is the waveform collected by the traditional test system, and Fig.12 (b) is the waveform collected by the projectile's coordinate parameters measurement system that formed by the six laser-screen, which is also called the six laser-screen test system, among, V_1-V_6 are the waveform output by G_1-G_6 respectively.
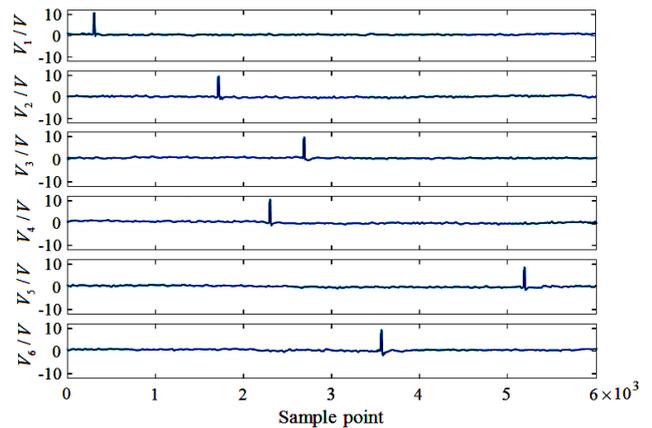
(b) Waveform collected by the six laser-screen

Fig. 12. Comparative waveform diagram of the third projectile collected in the comparative test (color online)

Obviously, the six laser-screen test system designed with laser assisted light source can solve the problem of weapon projectile parameter measurement in low illumination environment. Starting from the design and detection performance of laser-screen, this paper studies the calculation method of projectile laser reflection energy and detection probability modeling of laser-screen, and applies it to the six laser-screen test system. The theoretical method and design idea of this paper will provide new means and technology for the current development of weapon dynamic projectile parameter measurement.

Based on the experimental analysis, to improve the detection probability of laser-screen, we can use two measure, one is when the optical system parameters of the sensor are determined, the core factor that can affect the detection ability is the reflected echo energy of the projectile, and the power of the emitting laser is mainly

(a) Waveform collected by traditional six sky screen

considered to improve the reflected echo energy; and the other is we can increase the gain of the photoelectric detection circuit.

8. Conclusions

Based on a new design of a laser-screen with a high-power laser, a reflected laser energy model was established in this paper from the cross-sectional ratio of the projectile in the laser detection screen, by considering the influence of the cross-sectional attenuation coefficient, environmental illumination, laser emission power and other influencing factors. According to the cross-sectional attenuation coefficient and the reflected laser power of the projectile surface, the calculation functions of the detection probability and false alarm probability of the system were deduced and the influence of the detection distance was discussed on the detection ability, cross-section attenuation coefficient and laser emission power on the laser reflection energy of the projectile surface. Through experimental tests, the established model was consistent with the detection ability of the laser-screen. The theoretical model and analysis method presented in this paper not only provide an engineering basis for the research and establishment of the application design of photoelectric detection target, but also provide a scientific technical means for the development of multi-screen intersection with laser-screen to measure the flight parameters of weapon projectiles.

In this paper, we research a new design method and detection model of laser-screen, uses the high power line lasers as an active light source and one-dimensional multi-unit PIN detector as the core detection and receiving device in receiving optical path module to form laser detection screen with a fan-shaped detection plane, this design method solves the difficult problem of the detection of the traditional sky screen which only depends on the environmental illumination, and use the reflected laser energy by the projectile as the sensing information of the projectile, which avoids the deficiency of the low signal-to-noise ratio of the system when the shading amount of the projectile is small.

The laser-screen is an active detection sensor, which retains the traditional passive detection mode, and adds the active detection mode, which can be flexibly switched to solve the impact of environmental illuminance instability on the test system, so the recognition rate and capture rate can be improved. The research theoretical method and calculation model in this paper can provide a new detection method for the weapon test range in the plateau environment, and effectively improve the test ability of weapon equipment.

Acknowledgments

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