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A new formula for electrical resonances prediction applied to the stratified resins in high frequency fields

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In this paper complex materials like stratified resins were studied. The resins where simulated using a high level computing program: High Frequency Structure Simulator (Ansoft Technologies), in order to determine their behavior at high frequencies, specific for the microwave fields. This task is imposed by the necessity of stratified materials usage at high frequencies and at higher temperatures. Simulation results, punctually confirmed by the experiments, are the basis for developing a new formula for the effective permittivity of the stratified materials, which indicates the dependence of this quantity of the geometrical and physical parameters of the samples and also indicates its frequency dependence. Conclusions are available, consecrated to the work strategy for mixed structures simulations in microwave field and to the strategy of formula synthesizing for material properties determination in this frequency range.

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1. Introduction

Material studies in the high frequency and microwave fields are seriously obstructed by the experimental limitations. Consequently, simulation strategies of mixed structures are developed more and more today. Complex simulation programs are used, like MPSim (Massively Parallel General Simulation Program), HFSS (High Frequency Structure Simulator) or more elaborated variants like: DIATOM (calculates synthetic absorption and stimulated emission spectra of diatomic molecules), VASE (Visual Atomic Simulation Environment), DISCUS (program for diffuse scattering and defect-structure simulation), Docking Study with HyperChem (the protein- and ligand-flexible docking program) and many others. Simulation results, punctually confirmed by experiments even for lower frequency range, lead to strategies of formula synthesizing for material properties determination.

The material class selected for analysis is represented by a group of stratified resins whose mixed structures are similar with different prototypes described before by some consecrated theoretical models.

For the analyzed material selection we had to consider that starting with the 1st of July 2006, the UE countries have to put on the market new electrical and electronic equipment which does not contain lead, mercury, cadmium, hexavalent chromium, polybrominated, biphenyls (PBB) or polybrominated diphenyl ethers (PBDE) [11]. Consequently, if we consider the board materials in the new lead-free soldering processes, these have to be halogen free and halogenated flame retardants have to be phase out. New board materials have to be able to withstand the requisite higher temperatures, imposed by higher melting temperature of the lead-free soldering paste, with no delamination, warpage, resin recession and voids [12], [15], [16].

Following this idea, the common halogen-free laminate materials were considered [11], [16]: CEM[1]-1; CEM-3; FR[2]1; FR2; FR3; FR4 and FR5.

The most used are the CEM-3 and, of course, the FR4 laminates.

The fire retardant **FR4** is the most commonly used insulating base material for circuit boards. The FR4 is a glass fiber epoxy laminate (epoxy resin with woven E glass like material reinforcement). The FR4 is transparent (the green color is given by the solder resist). There are used now the non-brominated versions of FR4. Their glass transition temperature (T_g) is between 130 and 150 ºC. We have also to consider some material properties for FR4: loss tangent, which is of 0.02 to 0.03 (would not be used in digital circuits above 1 GHz); relative permittivity, which takes values between 4.1 and 4.8, at 1 MHz (lower permittivity values allow faster signal propagation and permit thinner impedance control.)

The **CEM-3** material is a glass reinforced epoxy laminate, special designed as a cost effective alternative for FR4 material. The CEM-3 consists of an epoxy glass-mat core with woven epoxy glass face sheets engineered to performance characteristics similar to FR4. The epoxy resin system provides thermal reliability similar to FR4.

[1] the CEM are composite epoxy materials;
[2] the FR are fire retardant materials.

The other laminates were considered for variety and comparison. We will give a few details about them, too. The CEM-1 is a cellulose paper based laminate with one layer of (7628) woven glass fabric in the epoxy resin (it is not suitable for PTH). FR1 is a paper material with phenolic binder. FR1 and FR2 are basically the same, but with different T_g (130 ºC for FR1 and 105 ºC for FR2). FR3 is basically FR2 but instead of phenolic resin is used an epoxy resin as binder. The basic layer is paper. FR5 laminate uses multiple layers of woven glass doth impregnated with a poly-functional epoxy resin system to provide greater mechanical stability. Its T_g is typically between 150 and 160 ºC.

Material analysis was performed for these materials by simulations considering their components structure, using the HFSS program in microwave range. Results were punctually confirmed by measurements. Frequency behavior of the electric effective permittivity was determined, in order to elaborate a formula for theoretical prediction of resonances. We have used as starting point for our theoretical contributions the Dukhin and Shilov theory and the Kraszewski theory, applied to stratified layered samples. Other theories applying to mixed materials were also considered, given by Landau-Lifshitz, Garland and Tanner, etc.

2. Simulation, experimental and theoretical methods for board materials analysis

The electrical resonances of the considered board materials were determined using a simulation strategy. Experimental determinations for the confirmation of results were also performed for the CEM-3 and FR4 up to 18 GHz.

2.1. Simulation details

Samples of different nature and thicknesses were reconstructed with help of the HFSS program. We have considered parallelepipedic samples, with the cross-section of 22.86 x 10.16 mm and standard thicknesses of 0.8, 1.6, 2.4, respectively 3.2 mm. Samples dimensions were chosen to fit in the experimental setup. The simulated exposure field reproduces the disturbing field in which the printed circuit board (PCB) works. The real field is external generated or determined by PCB currents themselves [2], [6], [9]. We have used the resonances determination module of the HFSS for the board material resonances determination in microwave range (over 0.3 GHz) [5], [7], [14]. We have exploited this module up to 45 GHz.

The HFSS program determines the resonant frequencies of a sample using an algorithm based on solving the matrix equation:

$$\boldsymbol{Sx} + k_0^{\,2}\boldsymbol{Tx} = \boldsymbol{b}\,, \qquad (1)$$

where $\boldsymbol{S}$ and $\boldsymbol{T}$ are matrices that depend on the geometry and the structural defined mesh; $\boldsymbol{x}$ is the electric field solution (vector); k_0 is the free-space wave number corresponding to that $\boldsymbol{x}$ mode and $\boldsymbol{b}$ is the value of the source (testing field) defined for the problem. For resonances determination the eigenmode solver of the HFSS sets $\boldsymbol{b}$ to zero. Equation is solved for sets of (k_0, $\boldsymbol{x}$), one for every $\boldsymbol{x}$.

All quantities in equation are frequency dependent. The wave number k_0 is related to the frequency of the resonant modes as follows:

$$f_{res,\,i} = \frac{k_0 c}{2\pi}\,, \qquad (2)$$

where c is the light speed in the free space.

2.2. Experimental details

Resonances determination by measurement was done using the experimental setup in Fig. 1. Electrical resonances of the board material samples were selected analyzing the effective permittivity evolution at frequency sweeping in microwave domain, from 0.3 to 18 GHz.

The main part of the installation is represented by the multi-function block HP Agilent 4396B, which includes the variable frequency microwave generator and the module for effective permittivity resonance determination. Before every set of sample measurements, the calibration of the installation was done with help of a calibration kit, available with the 43961A module, and with APC-7 connectors. For the effective permittivity determination, the S parameters were used, calculated when samples were exposed inside a waveguide with the same cross-section as the samples [5], [7]. Sample dimensions were given in the simulation case. Finally the PC has generated the effective permittivity versus frequency curves, on which the resonances have appeared.

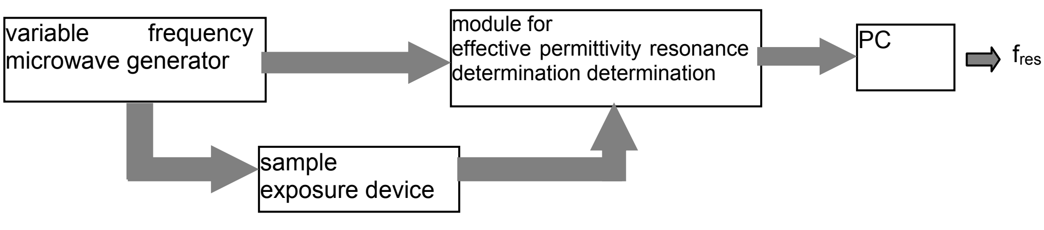

Fig. 1. Block diagram of the experimental setup for sample resonance determination.

2.3. Theoretical considerations

Experimental work generally indicates an electrical resonant behavior of composed material samples. The material electrical parameters (permittivity, permeability and conductivity) evolve resonantly with frequency, presenting magnitude peaks and valleys, in agreement with the sample geometry and the nature of the constituents [3], [9]. The great majorities of the theories which realize material parameters prediction for mixed materials do not illustrate the real frequency evolution of the analyzed parameter (do not indicate the resonances). We have proceeded to find a more complex formula for effective permittivity of the mixed structure, which to describe the frequency dependence of this quantity and to predict the resonances. A preliminary analysis was done, considering the most known and used theories in the field. For mixed stratified materials, the following theories can be applied:

▪ Dukhin and Sihlov theory

The mixed stratified material is represented by a binary dielectric mixture, consisting of alternate parallel layers, with permittivities ε_1, respectively ε_2, and thickness $2\eta L$, respectively $2(1-\eta)L$, with $\eta < 1$ (see Fig. 2)

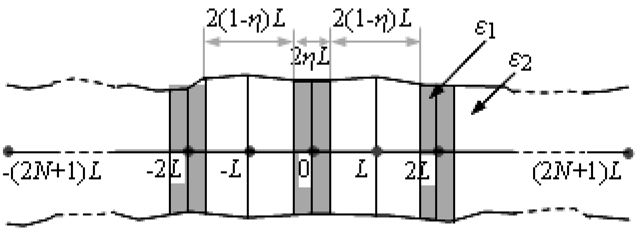

Fig. 2. Mixed stratified material, with alternative parallel layers, considered by Dukhin and Sihlov theory (after B. P. Scaife [8]).

This composed material has the effective permittivity given by [8]:

$$\frac{1}{\varepsilon_{ef}(k,\omega)} = \frac{1}{\varepsilon_1(\omega)} \cdot \frac{\sin(\eta kL)}{\sin(kL)} + \frac{1}{\varepsilon_2(\omega)} \left[1 - \frac{\sin(\eta kL)}{\sin(kL)} \right] \quad (3)$$

where k is the wavenumber corresponding to the testing field propagation through the mixed stratified material and $2N$ is an integer which denotes the number of pairs of alternate layers in the structure (see Fig. 2).

When we apply this model for the stratified board materials, we have to consider that the alternative layers are not homogeneous and not strictly parallel. At the same time, the thicknesses of successive layers of the same type are not strictly equal, like in this model. It is not necessary for the two successive layers that repeat in the model to have different nature, so the model can be applied from this point of view. Consequently, one can not expect a very precise determination of the effective permittivity using the expression (3). One observes that frequency dependence of the effective permittivity is illustrated and some resonances can be predicted. But only resonant valleys (where ε_{ef} converges to 0, corresponding to $\sin(kL) = 0$) are predicted by the theory. This means that there are given only a geometrical type of resonances, corresponding to a single geometrical parameter which characterizes the structure: L.

▪ Landau-Lifshitz theory

This theory considers a stratified mixed dielectric structure, for which the absolute permittivity varies on the direction of stratification: $\varepsilon = \varepsilon(z)$. Variations are small and occurs around a volume mediated value, $\langle \varepsilon \rangle_{Vol}$. The effective permittivity of this structure is given by [8], [9]:

$$\sqrt[3]{\varepsilon_{ef}} \cong \left\langle \sqrt[3]{\varepsilon} \right\rangle_{Vol} = \left\langle \sqrt[3]{\langle \varepsilon \rangle_{Vol} + \delta\varepsilon} \right\rangle_{Vol} \quad (4)$$

For the considered board materials the permittivity variation occurs on all three directions Ox, Oy and Oz, not only on the Oz direction. In the same time, permittivity variations are not uniform, because of the heterogeneity of the material.

Another disadvantage of the formula consists of not describing the frequency dependence of the effective permittivity and not indicating the resonances.

▪ *Kraszewski* theory

This theory applies to biphasic mixed dielectrics, having different structures. We have chosen for our purposes a Kraszewski theory derived model of stratified mixed dielectrics, with an infinite number of thin parallel layers, of both components, not particular in an imposed order. One component is considered as basis material and the other component represents the inclusion material. Layers thicknesses are much smaller than the wavelength. The effective permittivity expression can be written as [8], [9]:

$$\varepsilon_{ef} = \left[\sqrt{\varepsilon_b} + f_{incl} \left(\sqrt{\varepsilon_{incl}} - \sqrt{\varepsilon_b} \right) \right]^2 , \quad (5)$$

where f_{incl} represents the volume fraction of inclusion material, with the permittivity ε_{incl}. The basis material (here the resin) has the permittivity ε_b.

We have to relieve again the limitations of the model: for the board materials the number of layers is finite and the layers are not strictly parallel. In the same time, the layers are not homogeneous. They can have the same nature, so this is not a limitation.

Unfortunately, one remarks that the Kraszewski formula does not describe the frequency dependence of the effective permittivity and does not illustrate resonances.

▪ *Other* theoretical models for stratified dielectrics

If we consider the structure in Fig. 3, the mixed material consists of N parallel dielectric homogeneous cylinders, with irregular cross-sections of area A_i and electric permittivities ε_i. The structure has finite dimensions, so the mixed dielectric thickness was denoted with g and the total cross-section was denoted with A.

The effective permittivity for the mixed dielectric in Fig. 3 is given by [8], [9]:

$$\varepsilon_{ef} = \sum_{i=1}^{N} w_i \varepsilon_i = \frac{1}{A} \sum_{i=1}^{N} A_i \varepsilon_i \,, \tag{6}$$

where w_i represents the relative volume of the constituents.

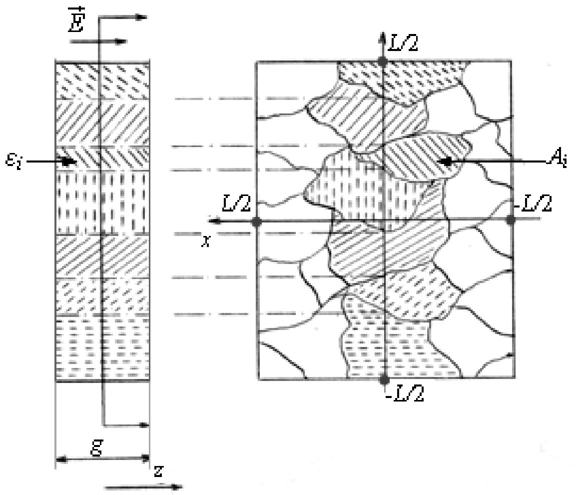

Fig. 3. Mixed dielectric material, consisting of parallel cylinders with irregular cross-sections (after B. P. Scaife [8]).

Similar limitations restrict the accuracy of the result: the formula (6) does not illustrate the frequency dependence of the effective permittivity and does not indicate the resonances. Otherwise, for the real stratified resins, the inside cylinders are not parallel and homogeneous.

Another variant of this theory analyses the structure in Fig. 4. The mixture consists of parallel wavy layers of homogeneous dielectrics, with the thickness g_i and permittivity ε_i.

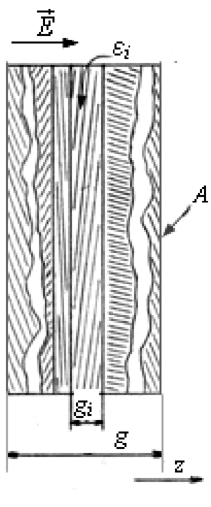

Fig. 4. Mixed dielectric material, consisting of parallel wavy layers of homogeneous dielectrics (after B. P. Scaife [8]).

The effective permittivity of the structure is given by [8]:

$$\frac{1}{\varepsilon_{ef}} = \sum_{i=1}^{N} \frac{w_i}{\varepsilon_i} = \frac{1}{g} \sum_{i=1}^{N} \frac{g_i}{\varepsilon_i} \tag{7}$$

If the constituent permittivities vary continuous after the $\vec{E}$ field direction (Oz), formula (7) can be replaced with [8]:

$$\frac{1}{\varepsilon_{ef}} = \frac{\int_0^g dz \cdot \frac{1}{\varepsilon(z)}}{\int_0^g dz} = \left\langle \frac{1}{\varepsilon} \right\rangle_{Vol} , \tag{8}$$

which characterizes an anisotropic mixture.

Formula limitations are the similar: does not illustrate the frequency dependence of the effective permittivity and does not indicate the resonances. Otherwise, for the real board materials, the inside layers are wavy, but not homogeneous.

For the obtained result verification, we have also used the Wiener inequality indicating the limits between which the relative effective permittivity, ε_{ef}, has to evolve, for a given volume distribution of the mixture constituents [8], [9]:

$$\frac{1}{\sum_{i=1}^{N} w_i \frac{1}{\varepsilon_i}} \le \varepsilon_{ef} \le \sum_{i=1}^{N} w_i \varepsilon_i \tag{9}$$

3. Results obtained for the stratified resins effective permittivity and resonances

We have considered for analysis board materials of different types, in order to obtain relevant results, useful in elaborating a general formula for theoretical prediction of resonances.

The analyzed materials were:

- old classical board materials:
  - Pertinax, based on phenol-formaldehyde resin with paper like reinforcement material;
  - a woven glass fabric, based on epoxy resin with glass fiber like reinforcement material;
- board materials with high toxicity level, which has to be eliminated:
  - a flame retardant based on bromine (a polybrominated biphenyl PBB);
  - a polybrominated dibenzodioxin (PBDD);
  - a polybrominated dibenzofuran (PBDF);
- halogen free board materials, agreed by the new lead-free technologies:
  - CEM-1, CEM-3, FR1, FR2, FR3, FR4, FR5.

Effective permittivity values were computed for the considered board materials, using the previous theories. Chemical structure and macromolecular structure (internal disposure of the material reinforcement in the resin) have to be considered for every material [1], [10], [11].

The obtained results are given in Table 1. All values of the effective permittivity given by theories are DC values (the Dukhin and Sihlov theory result was obtained by extrapolation, considering low frequency exposure fields [8], [10]).

For comparison, we have extract from literature the DC measured value for effective permittivity and also the value corresponding to a frequency of 1 MHz (the most indicated by the manufacturers).

Table 1. Effective permittivity for the considered board materials, calculated using proper theoretical models, like Dukhin and Sihlov, Kraszewski theory, etc.. The effective permittivity values, measured in DC, respectively at 1 MHz, were taken from literature and presented for comparison.

material / $\varepsilon_{r,ef}$	Pertinax	w. glass fabric	PBB	PBDD	PBDF	halogen-free materials						
						CEM-1	CEM-3	FR1	FR2	FR3	FR4	FR5
$\varepsilon_{r,ef}$ Dukhin-Sihlov	3.984	4.326	3.982	4.613	4.996	5.621	5.698	4.285	4.623	4.684	4.602	4.913
$\varepsilon_{r,ef}$ Kraszewski	4.162	4.426	4.175	4.762	5.104	5.798	5.843	4.366	4.785	4.791	4.748	5.173
$\varepsilon_{r,ef}$ Landau-Lifshitz	4.213	4.558	4.286	4.917	5.372	7.953	6.016	4.518	5.032	4.994	4.983	5.286
$\varepsilon_{r,ef}$ by relation (7)	3.162	3.516	3.211	3.715	4.241	4.602	4.522	3.346	3.442	3.837	3.756	4.172
$\varepsilon_{r,ef}$ by relation (6)	3.218	3.452	3.162	3.602	4.118	4.685	4.591	3.164	3.265	3.645	3.684	4.062
$\varepsilon_{r,ef}$ in DC (lit.)	3.824	4.216	3.926	4.468	4.973	5.381	5.287	4.032	4.186	4.577	4.442	4.831
$\varepsilon_{r,ef}$ at 1 MHz (lit.)	3.645	4.008	3.745	4.215	4.683	5.027	4.976	3.853	3.896	4.328	4.290	4.571

If we consider as etalon the measured DC values of the relative effective permittivity, extracted from literature, one observes that the most fair results are given by Dukhin and Sihlov theory, which is the most elaborated theory, making a complex characterization of the stratified materials. Good enough results are given by Kraszewski theory, derived from Landau-Lifshitz model. The effective permittivity values calculated with the dedicated formulas (6), respectively (7) present the lowest accuracy.

At 1 MHz, the permittivity values decrease in comparison with the DC values because it is known that the effective permittivity presents a general descendent evolution with frequency, determined by orientation difficulties of the molecular dipoles in the external field, when the frequency increases.

The relative errors, corresponding to each theory results, are given in percents in Table 2. We have considered as etalon the measured DC value, given by literature:

$$er(\%)=\frac{\Delta\varepsilon_{r,\,ef}}{\varepsilon_{r,\,ef\ etalon}}\cdot 100(\%)=\frac{\varepsilon_{r,\,ef\ thoer}-\varepsilon_{r,\,ef\ etalon}}{\varepsilon_{r,\,ef\ etalon}}\cdot 100(\%) \quad (10)$$

The relative errors were graphical represented in Fig. 5, for comparison.

Table 2. The relative errors of the relative effective permittivity values, obtained with the considered theoretical models, for the analyzed board materials. The measured CD values, taken from literature, were etalon considered.

material / er (%)	Pertinax	w. glass fabric	PBB	PBDD	PBDF	halogen-free materials						
						CEM-1	CEM-3	FR1	FR2	FR3	FR4	FR5
er Dukhin-Sihlov	4.184	2.609	1.426	3.245	0.462	4.460	7.774	6.275	10.440	2.338	3.602	1.697
er Kraszewski	8.839	4.981	6.342	6.580	2.634	7.749	10.516	8.284	14.310	4.676	6.889	7.079
er Landau-Lifshitz	10.173	8.112	9.170	10.049	8.023	47.798	13.789	12.054	20.210	9.111	12.179	9.418
er by relation (7)	-17.312	-16.603	-18.212	-16.853	-14.719	-14.447	-14.469	-17.014	-17.774	-16.168	-15.443	-13.641
er by relation (6)	-15.847	-18.121	-19.460	-19.382	-17.193	-12.934	-13.164	-21.528	-22.002	-20.363	-17.064	-15.918
$\varepsilon_{r,ef}$ in DC (lit.)	3.824	4.216	3.926	4.468	4.973	5.381	5.287	4.032	4.186	4.577	4.442	4.831

All these errors are systematical errors, introduced by the method. Relative errors are positive for Dukhin and Sihlov, Kraszewsk, respectively Landau-Lifshitz theories, corresponding to the obtained permittivity values bigger than the real ones. The effective permittivity determinations performed by the formula (6), respectively (7) generate negative relative errors, the obtained effective permittivity values being smaller than the real ones.

Errors are the most consistent for the Landau-Lifshitz theory, which has to be avoided, at least for some specific materials (the theory appears to be not proper for materials with three-dimensional variation of the permittivity - especially materials with multiple components, for which the permittivity variation on neither direction can be neglected).

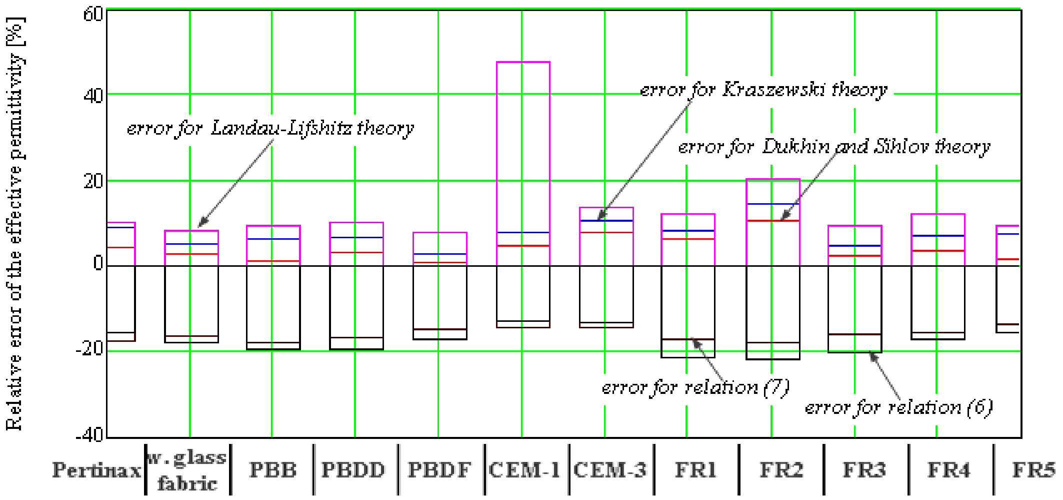

Fig. 5. The relative error of the relative effective permittivity values for the considered theories - graphical representation. One observes that the Dukhin and Sihlov theory gives the most precise results for the board materials.

Graphical representations from Fig. 5 suggest the fact that the modeling theory has to be chosen considering the material structure. It is clear from the figure that, if we impose an error maximum level for our determinations, then one, two or neither theory can be chosen, more or less precise, depending of the material. For example, if we observe the doted line in Fig. 5, a precision of 6 % of the result can be reached applying both Dukhin and Sihlov and Kraszewsk theories for PBDF, only Dukhin and Sihlov theory for FR4 and neither theory in the case of CEM-3 board material.

The considered board materials were simulated using the HFSS program and their electrical resonances were obtained, in microwave range. Results are given in Table 3. The resonances values obtained by measurements for verification were also given in the table. Resonance positions on frequency scale were illustrated in Figs. 6, 7 and 8, for each board material category.

One observes the good agreement between simulations and measurements. We have succeeded to find all the simulated resonances by measurements. A systematic negative error occurs, when we perform the measurements, due to the method [5], [7]. At the same time, measurements have illustrated a few more resonances, with lower magnitude, which have not a structural interpretation and are linked by the field discontinuities at the macroscopic edges of the samples.

Resonant frequency determinations are very important for a correct evaluation of the electric behavior of the board materials. Breakups of the electrical properties occur at these frequencies, at which an external field polarizes more easy or more hard the material, depending on resonance character: resonant peak or resonant valley [4], [9], [13], [16].

Table 3. Resonant frequencies for the considered board materials, in microwave range, obtained by simulations. The measured values for resonances, up to 18 GHz, obtained for confirmation, were also given in the table.

Res.No	Pertinax f_{res}[GHz] simulated/ measured	**w. glass fabric** f_{res}[GHz] simulated/ measured	PBB f_{res}[GHz] simulated/ measured	**PBDD** f_{res}[GHz] simulated	**PBDF** f_{res}[GHz] simulated	halogen-free materials **CEM-1** f_{res}[GHz] simulated	**CEM-3** f_{res}[GHz] simulated /measured	**FR1** f_{res}[GHz] simulated	**FR2** f_{res}[GHz] simulated /measured	**FR3** f_{res}[GHz] simulated	**FR4** f_{res}[GHz] simulated /measured	**FR5** f_{res}[GHz] simulated /measured
0.	6.103/ 6.083	5.843/ 5.826	5.747/ 5.717	3.862	3.057	2.286	2.972/ 2.884	4.886	4.754/ 4.596	4.062	4.124/ 3.986	1.986/ 1.443
1.	7.542/ 7.492	7.026/ 6.982	6.342/ 6.305	4.674	4.425	4.062	3.289/ 3.146	7.063	6.887/ 6.743	4.828	4.596/ 4.413	3.069/ 2.884

2.	8.773/ 8.713	8.153/ 8.095	8.008/ 7.953	5.028	5.066	5.137	3.571/ 3.489	8.126	7.584/ 7.692	6.021	7.108/ 6.945	3.750/ 3.594
3.	10.643/ 10.563	8.846/ 8.801	8.219/ 8.164	5.742	5.883	5.822	3.914/ 3.796	8.965	8.043/ 8.967	6.934	7.913/ 7.764	4.137/ 4.002
4.	10.686/ 10.598	9.274/ 9.224	8.814/ 8.762	6.228	6.394	6.486	4.550/ 4.483	9.684	8.675/ 8.582	7.863	8.783/ 8.340	4.736/ 4.615
5.	11.727/ 11.643	9.885/ 9.816	9.862/ 9.808	6.815	6.995	7.083	5.051/ 4.964	10.187	9.136/ 9.117	8.226	11.637/ 11.494	4.890/ 4.703
6.	12.017/ 11.937	10.172/ 10.845	10.882/ 10.813	7.226	7.428	7.581	5.237/ 5.076	10.742	9.662/ 9.552	8.976	12.752/ 12.613	5.177/ 5.124
7.	12.493/ 12.139	10.456/ 10.348	12.575/ 12.504	7.963	8.004	8.143	5.464/ 5.384	11.321	9.958/ 9.846	9.582	13.142/ 13.013	6.200/ 6.081
8.	12.813/ 12.702	10.851/ 10.743	13.158/ 13.085	8.045	8.625	8.662	5.788/ 5.665	11.856	10.427/ 10.313	10.118	13.501/ 13.398	6.409/ 6.294
9.	13.710/ 13.598	11.113/ 11.016	13.954/ 13.872	8.594	9.228	8.976	5.819/ 5.828	12.382	11.065/ 10.945	10.781	14.820/ 14.694	6.469/ 6.358
10.	14.334/ 14.197	11.478/ 11.352	14.553/ 14.468	9.158	9.946	9.425	6.014/ 5.986	12.946	11.729/ 11.604	11.143	15.866/ 15.723	6.622/ 6.516
11.	14.929/ 14.764	11.894/ 11.772	15.321/ 15.331	9.932	10.683	10.026	6.033/ 5.921	13.261	12.325/ 12.197	12.023	17.625/ 17.483	6.751/ 6.645
12.	15.680/ 15.523	12.271/ 12.154	15.681/ 15.582	10.662	11.324	10.843	6.422/ 6.382	13.586	12.824/ 12.682	12.964	19.884	6.978/ 6.868
13.	15.849/ 15.695	12.573/ 12.148	17.100/ 16.992	11.487	12.586	11.766	6.603/ 6.569	14.028	13.127/ 13.971	13.825	23.247	7.096/ 7.004
14.	16.420/ 16.256	12.961/ 12.843	17.610/ 17.489	11.962	13.843	12.548	6.748/ 6.683	14.528	13.646/ 13.502	15.238	24.104	7.296/ 7.202
15.	17.289/ 17.057	13.082/ 12.934	18.108	12.478	15.761	13.929	6.962/ 6.883	14.927	14.141/ 14.020	16.684	25.006	7.412/ 7.332
16.	17.357/ 17.221	13.823/ 13.691	18.348	13.065	16.828	15.167	7.308/ 7.214	15.491	14.958/ 14.803	17.984	25.456	7.553/ 7.548
17.	17.795/ 17.326	14.961/ 15.824	19.871	13.861	18.156	17.253	7.462/ 7.388	16.082	15.431/ 15.286	19.681	27.011	7.618/ 7.538
18.	19.089	15.724/ 15.573	20.370	14.763	19.487	19.401	7.549/ 7.418	16.824	15.986/ 15.832	23.184	28.866	7.723/ 7.616
19.	20.002	16.413/ 16.262	22.216	15.626	21.156	21.554	7.749/ 7.623	17.483	16.956/ 16.799	25.104	30.228	8.028/ 7.936

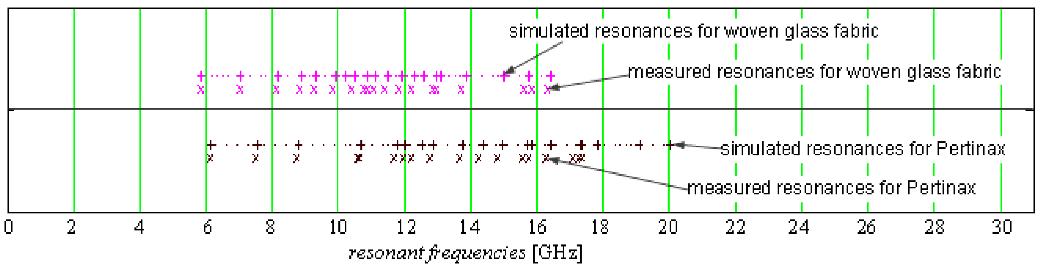

Fig. 6. Resonant frequencies for Pertinax and woven glass fabric, obtained by simulations and measurements.

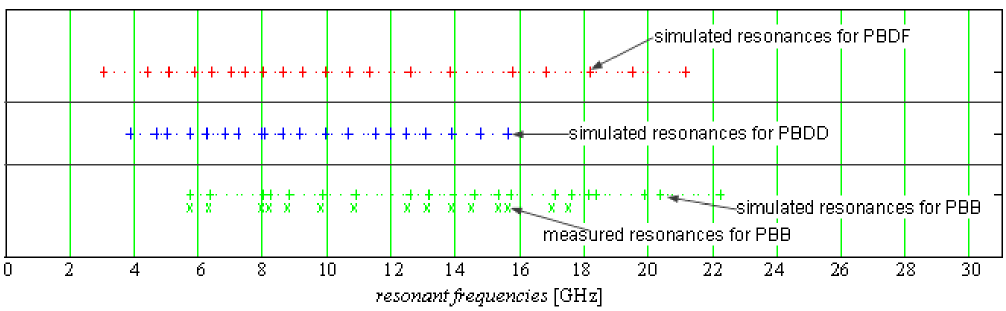

Fig. 7. Resonant frequencies for PBB, PBDD and PBDF, obtained by simulations and measurements.

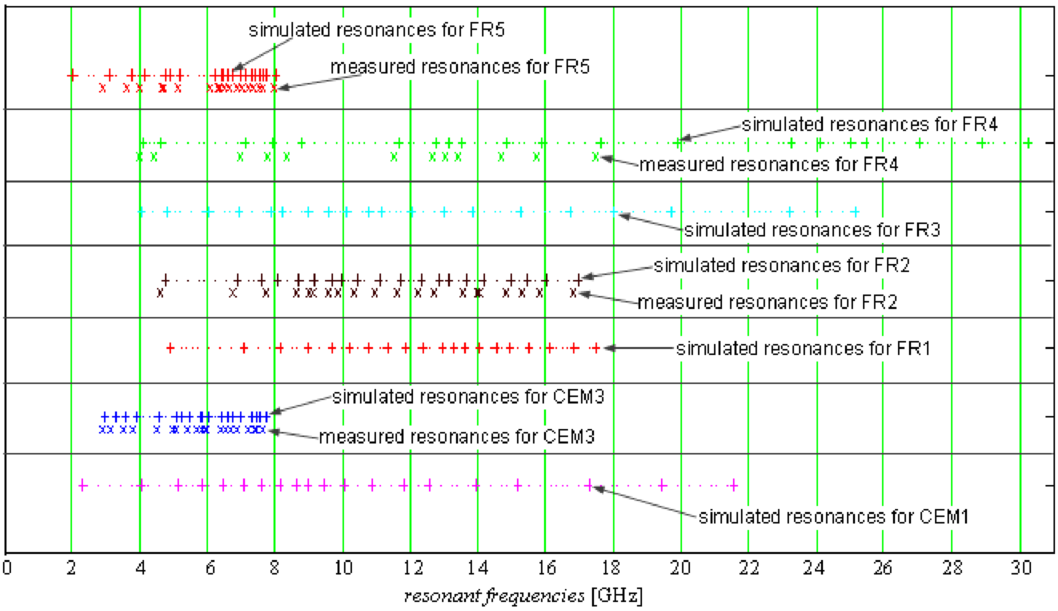

Fig. 8. Resonant frequencies for the halogen-free board materials CEM-1, CEM-3, FR1, FR2, FR3, FR4 and FR5, obtained by simulations and measurements.

4. A new formula for the effective permittivity of the stratified resins

Considering the limits of the previous theories, used for effective permittivity calculation, we have proposed a new formula for the effective permittivity of the stratified mixed materials exemplified in this paper for the analyzed board materials. The synthesized formula (11), valuable in microwave domain, considers the effective permittivity dependence on the geometrical and physical parameters of the mixed material and illustrates the frequency dependence of this quantity.

$$\varepsilon_{r,ef} = \sum_m ct_m \cdot (f - f_m) \cdot \sum_a \frac{1}{ct_a \cdot (f - f_a) \cdot \sum_{i=1}^{N} \frac{g_i}{g} \frac{1}{\left[\sqrt{\varepsilon_{base}} + f_{incl,i}\left(\sqrt{\varepsilon_{incl}} - \sqrt{\varepsilon_{base}}\right)\right]^2} \cdot \frac{\sin(kg_i)}{\sin(kg)}} \quad (11)$$

Formula (11) can be re-written more explicit like expression (12), which illustrates very clearly the resonances. The first sum in formula (11) corresponds to the resonant valleys (each term of the sum defines a valley) and the second sum corresponds to the resonant peaks. In expression (12) appears separately this term.

$$\varepsilon_{ef} = \left[ct_m \cdot (f - f_m) + ct_n \cdot (f - f_n) + \ldots\right] \cdot \left[\frac{1}{ct_a \cdot (f - f_a) \cdot M} + \frac{1}{ct_b \cdot (f - f_b) \cdot M} + \ldots\right], \quad (12)$$

where we have denoted:

$$M = \sum_{i=1}^{N} \frac{g_i}{g} \frac{1}{\left[\sqrt{\varepsilon_{base}} + f_{incl,i}\left(\sqrt{\varepsilon_{incl}} - \sqrt{\varepsilon_{base}}\right)\right]^2} \cdot \frac{\sin(kg_i)}{\sin(kg)} \quad (13)$$

- ε_{base} and ε_{incl} are: the permittivity of the basis material in each layer (here the board material resin), respectively the permittivity of the inclusion material (here the material reinforcement);
- N represents the number of layers in the stratified material (every layer is indicated by the i index);
- $f_{incl,\,i}$ is the volume fraction of inclusion material, in the layer number i;
- g_i is the medium thickness of the layer i; the material has the total thickness g;
- k represents the wavenumber corresponding to the field propagation through the mixed stratified material;
- f is the operating frequency;
- f_a, f_b, etc. represents the resonant frequencies, corresponding to the resonant peaks;
- f_m, f_n, etc. represents the resonant frequencies, corresponding to the resonant valleys;
- ct_a, ct_b, etc., respectively ct_m, ct_n, etc. are semi-empirical constants, inverse proportional with resonances magnitude; there are directly linked to physical characteristics of the samples (constituents nature) and depend on the external field strength.

Considering the conclusions obtained from error analysis in Table 2, the formula proposed by us is based on all the used theories, but respects the syntax of the Dukhin and Shiloh formula, with terms ponderability suggested by Kraszewski theory.

We can mention that the radicals in formula (11) could be replaced with an integral expression, which to characterize more fair the volume distribution of the inclusion material in every layer. Such of formulas have been tested [1], [8], [9], but no significant precision improvement was obtained. On the contrary, the radical expression seems to give better results, so we have kept it.

Another advantage of the proposed effective permittivity formula is that formula can be applied to composed stratified materials having different inclusions in successive layers. Consequently, formula is useful for the new board materials, which are in study, with different layers succeeding in a particular order and having special properties.

The proposed formula is a semi-empirical one, the effective permittivity values at each operating frequency being calculated with help of some semi-empirical constants, depending of the experimental or simulation determined resonances. It appears again the utility of electrical resonances determination and the great facility offered by simulation procedure, especially at high frequencies where measurements are very difficult to be performed.

Formula (11) is valuable in microwave range, where the operating frequency is high enough and so are the resonant frequencies (of 10^{10} GHz order). This fact ensures us that, in the vicinity of a resonance, only its corresponding term in the second sum has to be kept, the other terms having to low values. (Only the corresponding $1/(f - f_i)$ is less then 10^2 times lower than the others.)

Depending on the material components geometry and nature, the effective permittivity for a mixed stratified material, calculated with formula (11) presents a frequency evolution similar with that illustrated in Fig. 9. We have exemplified here with our results obtained for a FR4 board material sample with parallelepipedic shape (22.6 x 10.16 x 1.6 mm), exposed to the electromagnetic field TE_{10} generated inside a rectangular waveguide.

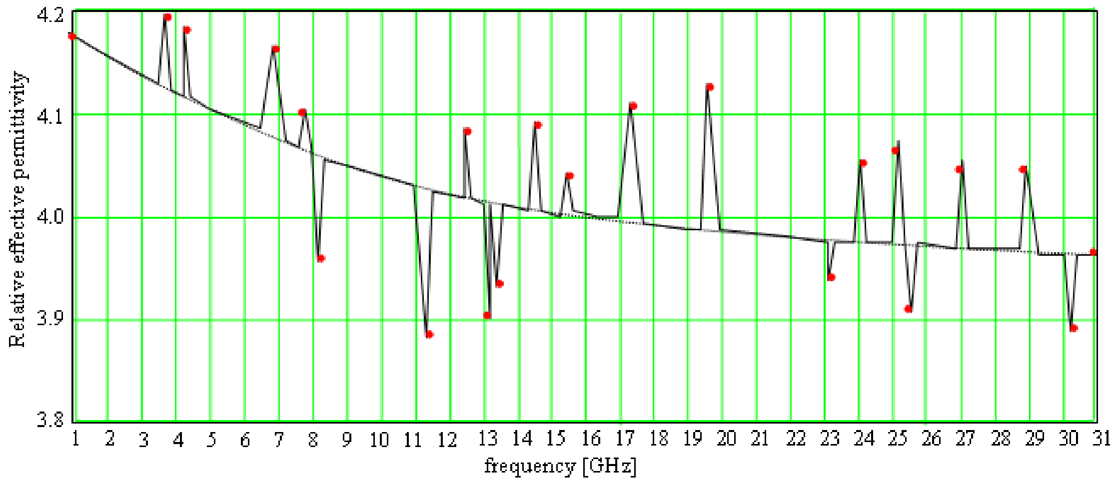

Fig. 9. The effective permittivity for the FR4 board material, calculated with our new formula for the effective permittivity of the stratified resins. All resonances obtained by simulation are illustrated on graph.

5. Conclusions

Mixed stratified materials were analyzed in this paper and a new formula for the electrical effective permittivity was proposed, which enable us theoretical prediction of the electrical resonances, in microwave range.

Board materials were considered in our study: classical type materials used until now (Pertinax, PBB, PBDD, etc.), but pollutant, respectively the halogen-free type, used for the new PCB lead-free design (FR4, CEM-3, etc.)

Electrical resonances of the board materials were determined by simulations, up to 45 GHz and punctually confirmed by measurements, up to 18 GHz.

Theoretical determination of the electric effective permittivity was performed, using consecrated theoretical models, applicable for the mixed stratified structures. Results were compared and relative errors were estimated. Limits of every theory/model were mentioned, to illustrate the necessity of a new better formula.

A new formula for the effective permittivity of the mixed stratified materials was proposed. The new formula characteristics are:

- Is based on the existing theories in the field, which gave the best results until now and are the most used in practice.
- Illustrates effective permittivity dependence on the geometrical and physical parameters of the samples.

▪ Illustrates effective permittivity dependence on frequency of the field propagating through the samples (resonances magnitude and resonances type: peak or valley depend directly on the field strength).

▪ Indicates electrical resonances of the mixed material samples.

▪ Is valuable in microwave range.

▪ Is a semi-empirical formula, the effective permittivity values at each operating frequency being calculated with help of some semi-empirical constants, depending of experimental or simulation determined resonances.

▪ Can be applied to the composed stratified materials having different inclusions in successive layers and also different oriented layers, considered by the modern technologies.

An example of the new formula application was given in case of the FR4 board material. Its effective permittivity was represented versus frequency, in microwave range. All resonances obtained by simulation for this material were illustrated on graph.

Based on our result viability, work strategies can be developed for solving the following problems:

▪ Determination of the resonances for the material parameters.

▪ Formula synthesizing for estimating the material parameters, in microwave range, which is a frequency domain difficult to be tackled by classical methods.

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