

A novel diagnosis method for hydropower unit rotor with coupling faults

XIANGQIAN FU*, YATAO LIN, HAO XU, ZHIHUI XIAO
School of Power and Mechanical Engineering, Wuhan University, China

According to the coupling faults of mass eccentricity and misalignment, the paper presents a novel fault diagnosis method for hydropower unit rotor based on the multi-scale morphology spectrum and morphological spectrum entropy. Such a method can be used to analyze the tested signals, which are divided into the stable periodic motion, periodic rubbing motion and chaotic motion. It is demonstrated that the morphology spectrum form and entropy value are different, and the characteristics of vibration efficiency under different condition can be identified very well. The present study therefore offers a methodology for the improvements of the safety, reliability and economical efficiency of hydropower unit rotors.

(Received August 23, 2013; accepted July 10, 2014)

Keywords: Hydropower unit, Nonlinear dynamics of rotor, Multi-scale morphology spectrum, Fault diagnosis

1. Introductions

Due to the coupling of hydraulic factor, mechanical factor and electrical factor, the vibration of hydraulic generator unit is rather complicated, especially for shaft system of unit. The complex structure of shaft system increases the dynamic variety of vibration, with induced by the deficiency in design, manufacture and installation of shaft system and the accident excitation. The long-term vibration may results in the efficiency less and fatigue damage of structure, and it would shorten the life of unit, even the resonance would be caused when the interference frequency is close to natural vibration frequency.

The research on shaft vibration of hydraulic generator unit is caused widely attention in the world [1-7]. At present, the research field about shaft vibration of hydropower unit generally can be divided into two parts as follow: the first part is research of the signal processes, it is tried to extrude the characteristics of vibration signal, and evaluate the state of the unit, achieve condition monitoring and faults diagnosis finally. For example, Tiwari[1] evaluated the unbalance motion of rotor system by mathematical model; Mohammad [2] explores the relationship beyond the peak vibration value of rotor, rotating rate and crack depth by simulation method; Jawaid [3] analyze the dynamic characteristics of rigid rotor under variable load; Lee[4]simulate the vibration of turbine shaft based on finite element method to analyze the error cause. The second part simplified the structure of hydropower unit to establish the mathematical model of unit shaft system, and then simulate the rotor motion with faults, such as unbalance and misalignment of rotor, to research the vibration mechanism. For example, Gravier [5] used the EMD method to analyze characteristics of blade

vibration to diagnose the fault; Rai [6] acquires the IMF of bearing vibration signal based on EMD method to analyze the frequency of signal; Jia [7] proposed a novel method to diagnose the fault of hydropower unit which is combined support vector machine and Hilbert Huang transfer.

However, because of the poor expert experiences, incomplete fault set, strong interferences and lack of coupling fault research, the fault diagnosis results is not precise enough, while the latter is limited by poor precise of mathematical model, the research of mathematical model is still stay in the theoretical research stage. Therefore, this paper combines the theory study and experiment testing, and propose a novel vibration signal processing method based on multi-scale mathematical morphology spectrum and morphological spectrum entropy to analyze the vibration signal of hydropower unit shaft, though exploring the vibration cause and vibration type to identify the characteristics of vibration efficiency under different condition, the research is to improve the safety, reliability and economical efficiency which has important academic significance and engineering application value.

The theory of mathematical morphology belongs to interdisciplinary sciences, which includes the multidisciplinary knowledge such as integral geometry, random set and functional analysis. The signal processing concept of mathematical morphology is to modify the shape of a signal, by transforming it through its interaction with another object, called the structure element. Because the morphology is a more perfect scientific system and the information of original signal is retained in the signal processing, the morphology has been widely application in microbiology, radiation medicine, industrial control and image processing and so on[8-9].

1.1 Theory of multi-scale morphology spectrum

The basic operators of morphology include dilation and erosion, which are defined as follow. Let $f(n)$ be the discrete one-dimensional signal over a domain $F=\{0,1,2,\dots,N-1\}$, and let $g(m)$ be the structure element over a domain $G=\{0,1,2,\dots,M-1\}$. The dilation and erosion operators are defined as

$$(f \oplus g)(n) = \max_{m \in G} \{f(n-m) + g(m)\} \tag{1}$$

$$(f \ominus g)(n) = \min_{m \in G} \{f(n-m) - g(m)\} \tag{2}$$

where \oplus represents the operator of dilation and \ominus represents the operator of erosion. More details about morphology can be found in [10].

If the structure element $g(m)$ is to do telescopic changes by scale λ ($\lambda > 0$), which λ is at different value, and the multi-scale mathematical morphology structure of $f(x)$ is defined as follow:

$$F_\lambda(f) = \lambda [F(f_\lambda(x) / \lambda)]_{1/\lambda} \tag{3}$$

where the $f_\lambda(x)$ represents the multi-scale mathematical morphology analysis result about λ . The erosion operator of multi-scale mathematical morphology is defined as follow:

$$(f \ominus g)_\lambda = f \ominus \lambda g_{1/\lambda} \tag{4}$$

The dilation operator of multi-scale mathematical morphology is defined as follow:

$$(f \oplus g)_\lambda = f \oplus \lambda g_{1/\lambda} \tag{5}$$

The signal processing results of mathematical morphology is closely related to not only the operator but also the structural elements. Generally, more structural elements, more complex calculation, and the shape of structure elements is more similar to the objective function, the effect of processing results is more obvious, and ability of signal recognition is better. [11]. So, by considering the calculation and the noise characteristics of hydropower units vibration signal, the structural elements is chosen the triangle, and the size of scale from one to ten.

1.2 Theory of multi-scale morphology spectrum and morphological spectrum entropy

The multi-scale morphology spectrum of objective function $f(x)$ about $g(x)$ is defined as follow:

$$PS_f(f, g) = \frac{dA(f \circ \lambda g)}{d\lambda} \quad \lambda \geq 0 \tag{6}$$

$$PS_f(f, g) = -\frac{dA(f \square (-\lambda)g)}{d\lambda} \quad \lambda < 0 \tag{7}$$

where the $A(f) = \int_{R^m} f(x)dx$ represents the bounded area of $f(x)$ in characterization domain, and \circ represents the open operator, and \square represents the close operator. If $\lambda \geq 0$, the open operator is used in morphology spectrum, conversely, the close operator is used.

$U(f(x))$ is called the projection space of objective function $f(x)$, and it is defined as follow:

$$U(f(x)) = \{(x, t) | x \in R^m, t \in R, t \leq f(x), x \in \text{sup}(f)\} \tag{8}$$

In order to extrude the characteristics of hydropower unit vibration signal easily, besides the method of morphology spectrum, the concept of entropy is introduced to characterize the testing signal, and the numerical results of morphological spectrum entropy analysis is employed to represent the characteristics of vibration signal, which is different from the form results of morphological spectrum. And the numerical characteristics of morphology spectrum entropy are applied in the on-line monitoring and on-line fault diagnosis.

The morphology spectrum entropy $PSE(f / g)$ is defined as follow:

$$PSE(f / g) = -\sum_{n=k}^N q(n) \lg(q(n)) \tag{9}$$

where $q(n) = PS_f(n, g) / A(f \square kg)$, and if the $PSE(f / g)$ is divided by $\lg(N + k + 1)$, the normalized pattern spectrum entropy is acquired.

2. Dynamic analysis on hydraulic generator rotors with coupling faults based on rotor dynamics

2.1 Model of rotor dynamics

Since the unit rotors vibration induced by oil film oscillation was firstly investigated in the early nineteen twenties[12], the dynamic behaviors of hydraulic unit rotors have been researched for decades and some progress has made. For example, Wan [13-14] established the mathematical model to simulate the oil film oscillation by nonlinear dynamics, through comparing the axis orbit, power spectrum and Poincare map to analyze the bifurcation phenomena of rotors. Gwo-Chung [15] has studied the vibration phenomena of hydraulic turbines which have different number of blades to analyze the dynamic characteristics of the rotors.

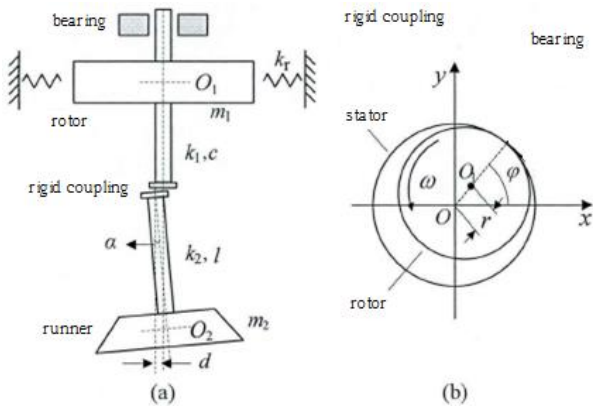


Fig. 1. Simplified model of hydropower unit rotor system.

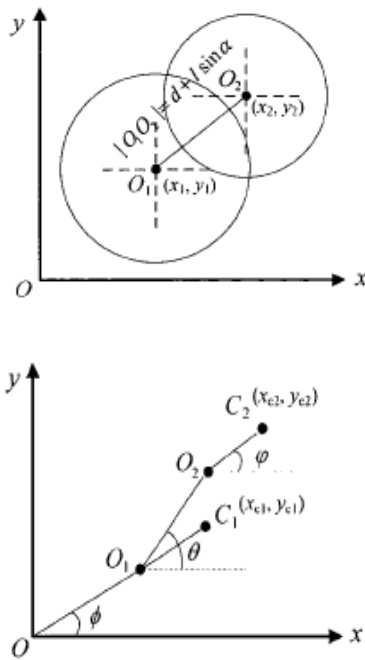


Fig. 2. Schematic diagram of rotor moving with coupling faults.

The simplified model of hydraulic units, including a turbine, a generator, and a shaft with two rotors and one bearing, is shown in fig1. Various faults types, such as unbalance, misalignment and rubbing, were coupled by this model. In fiugre1, O is treated as the design geometric center of unit, $o_1(x_1, y_1)$ and $o_2(x_2, y_2)$ are represented the initial state of geometric center of generator rotor and turbine rotor respectively. m_1 and e_1 represent the mass and eccentricity of generator, respectively, as well as m_2 and e_2 are represented similar meaning of turbine. In fig. 2, $c_1(x_{c1}, y_{c1})$ and $c_2(x_{c2}, y_{c2})$ a are represented the equivalent mass center of generator rotor and turbine rotor respectively.

We can derive the relationship between $o_1(x_1, y_1)$ and $o_2(x_2, y_2)$ as follow:

$$\begin{cases} x_2 = x_1 + |o_1o_2| \cos \theta = x_1 + (d - l \sin \alpha) \cos \theta \\ y_2 = y_1 + |o_1o_2| \sin \theta = y_1 + (d - l \sin \alpha) \sin \theta \end{cases} \quad (10)$$

And the rotor moving energy(T) is mainly composed of translational energy(T_G) and rotational energy(T_r):

$$\begin{aligned} T_G &= \frac{1}{2} m_1 v_{c1}^2 + \frac{1}{2} m_2 v_{c2}^2 \\ &= \frac{1}{2} m_1 (\dot{x}_{c1}^2 + \dot{y}_{c1}^2) + \frac{1}{2} m_2 (\dot{x}_{c2}^2 + \dot{y}_{c2}^2) \end{aligned} \quad (11)$$

where v_{c1} 、 v_{c2} are represented the linear velocity of generator and turbine respectively, and $\dot{\chi}$ is treated as the time derivative of χ .

At the initial state, the total potential energy of the rotor system can be expressed as:

$$U = \frac{1}{2} k_1 |oo_1|^2 + \frac{1}{2} k_2 |oo_2|^2 = \frac{1}{2} k_1 (x_1^2 + y_1^2) + \frac{1}{2} k_2 (x_2^2 + y_2^2) \quad (12)$$

Besides, dynamic behaviors of hydraulic unit with coupling faults are not only considered the impact of unbalance and misalignment, but also needed to consider the rubbing force and damping force. So the rubbing force can be defined as:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -H(r - \delta) \frac{(r - \delta)k_r}{r} \begin{bmatrix} 1 & -\mu \\ \mu & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} \quad (13)$$

where F_x 、 F_y are represented as the rubbing force with X direction and Y direction, r is treated as the axis orbit, δ is treated as the gap between the stators and rotors of unit, and $H(r - \delta)$ is treated as switching function, namely:

$$\begin{cases} H(r - \delta) = 0 & r < \delta \\ H(r - \delta) = 1 & r \geq \delta \end{cases} \quad (14)$$

If the damping coefficients with X direction and Y direction have the same value, the force on the generator rotor can be defined as follow without considering other interferences:

$$\begin{cases} Q_x = -c\dot{x}_1 + F_x \\ Q_y = -c\dot{y}_1 + F_y \end{cases} \quad (15)$$

Letting $x_1 = x$, $y_1 = y$, and the results of using the Lagrange formula to collate the equation(11-15) as follow:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \quad (16)$$

The dynamics differential equations of hydraulic unit with

coupling faults could be obtained by Simultaneous equations (9-13):

$$[M]\ddot{Z} + [C]\dot{z} + Kz = F \tag{17}$$

namely:

$$\begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_1+m_2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k_1+k_2 & 0 \\ 0 & k_1+k_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m_1e_1\omega^2 \cos\phi + m_2e_2\omega^2 \cos\alpha \cos\varphi + m_2(d-l \sin\alpha)\omega^2 \cos\theta - k(d-l \sin\alpha)\cos\theta + F_x \\ m_1e_1\omega^2 \sin\phi + m_2e_2\omega^2 \cos\alpha \sin\varphi + m_2(d-l \sin\alpha)\omega^2 \sin\theta - k(d-l \sin\alpha)\sin\theta + F_y \end{bmatrix} \tag{18}$$

2.2 Case analysis

The parameter values of simulation model are same as the experiment test bench, the equivalent parameters are as follows: $m_1=5.0\text{kg}, m_2=5.2\text{kg}, c=2100\text{N}\cdot\text{s}/\text{m}, k_1=2.5\cdot 10^6\text{N}/\text{m}, k_2=2.5\cdot 10^6\text{N}/\text{m}, k_3=2.5\cdot 10^7\text{N}/\text{m}, l=0.5\text{m}, e_1=0.05\text{mm}, e_2=0.03\text{mm}, \delta=1\text{mm}, \mu=0.01$. If the rotation rate changes from 0 rad/s to 50 rad/s, the model is solved by the Runge-Kutta model, and the results of generator rotor vibration bifurcation diagram are shown in fig. 3.

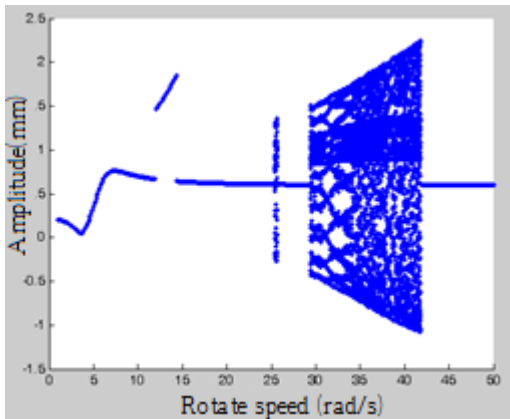


Fig. 3. The vibration bifurcation diagram of generator rotor.

With the increasing of rotation rate, the vibration type can be classified into stable periodic motion, periodic rubbing motion and complicated chaotic motion. If $\omega \in [1, 12], \omega \in [14, 25], \omega \in [26, 29]$ and $\omega \in [42, 50]$, the unit is in a stable state and the range of maximum rotor vibration within $[0, 0.08]\text{mm}$; when $\omega \in [12, 14]$, the maximum rotor vibration is exceeded the bearing clearance, which induces the periodic rubbing phenomenon strongly, so it means the rotor is in periodic rubbing motion. When $\omega \in [25, 26]$ and $\omega \in [29, 42]$ the operation of rotor is extremely unstable and the rubbing is irregular, so the rotor is in chaotic motion.

3. Experiments

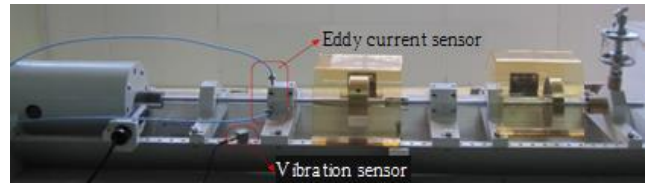


Fig. 4. Hydraulic unit vibration test bench.

The hydraulic unit vibration test bench in Fig. 4, is a simplification of hydraulic generator unit, which is used to simulate the dynamic features of rotor with coupling faults, such as eccentricity, misalignment and rubbing. And because of small mass and high rigidity of rotor, the influence of rotor gravity is ignored. The test bench is mainly composed of a generator rotor, turbine rotor and shaft, the eccentricity of generator rotor and turbine rotor is controlled by changing the mass of screws in rotor, and the misalignment faults is made by the customized rotor which keep the generator rotor and turbine rotor in misalignment state.

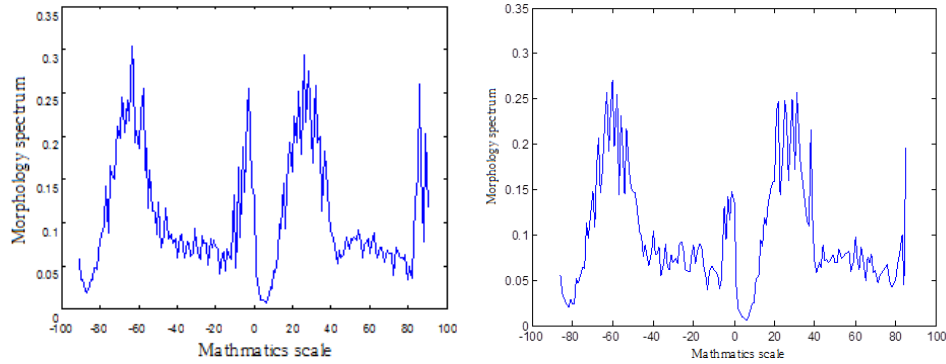
The purpose of experiment is to verify the credibility of simulated signal of hydraulic unit rotor by comparing the tested signal, and then lay the foundation for vibration pattern recognition and condition diagnosis with coupling faults. So the experiment is mainly done at the rotating rate of 5rad/s、 12rad/s、 30rad/s to test the shaft vibration signal of X direction and Y direction. At each test group, the vibration signal would be tested 100 times, and the 30 sets of tested signal is as the original vibration data of hydropower unit shaft to analyze the relationship between inter vibration mechanism and peak value and spectrum of tested signal based by multi-scale morphology spectrum and morphological spectrum entropy, and the others is used as the sample for fault diagnosis to verify the precision of the novel method which this paper proposes.

4. Signal analysis of hydropower unit shaft

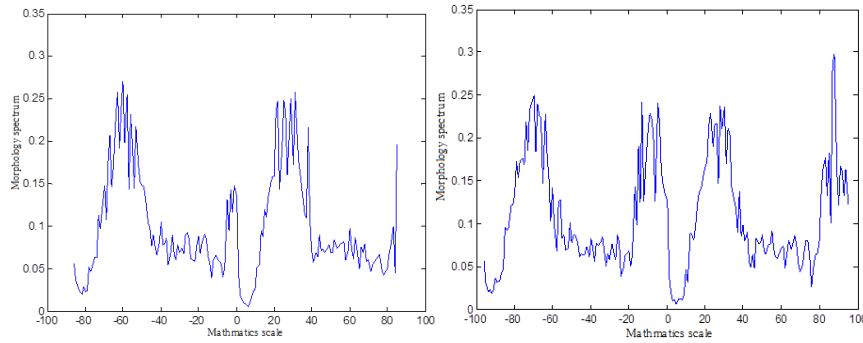
4.1. The characteristics of vibration signal at different rotating rate

By analyzing the simulation results in part 2, the

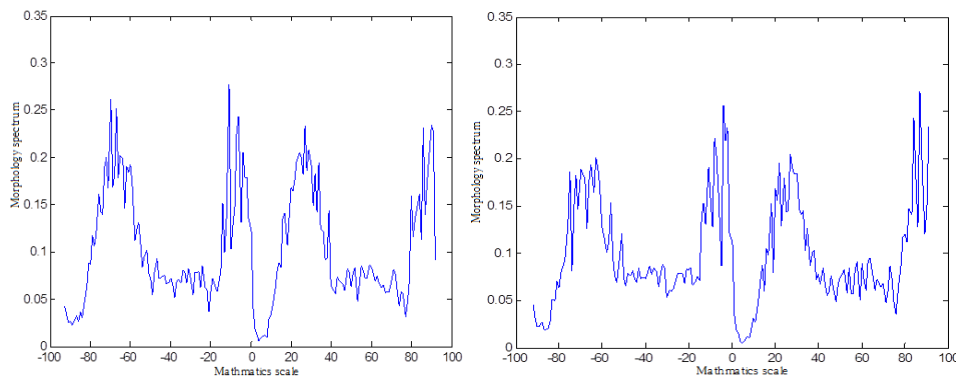
motion of rotor is in the same state when the rotating rate is increased for 30 rad/s to 35 rad/s step by step. So we would test the vibration of hydropower unit rotor at same condition, and then verify the morphology spectrum and entropy of tested signal from 30 rad/s to 35 rad/s step by step are similar, the results are shown in fig. 5 and table 1.



(a) Morphology spectrum at rotating rate of 30rad/s; (b) Morphology spectrum at rotating rate of 31rad/s.



(a) Morphology spectrum at rotating rate of 32rad/s; (b) Morphology spectrum at rotating rate of 33rad/s.



(a) Morphology spectrum at rotating rate of 34rad/s; (b) Morphology spectrum at rotating rate of 35rad/s.

Fig. 5. Morphology spectrum at different rotating rateS.

Table 1. The morphology spectrum entropy of hydropower unit vibration signal at different rotating rate.

Rotating rate	30.0 rad/s	31.0 rad/s	32.0 rad/s	33.0 rad/s	34.0 rad/s	35.0 rad/s
Morphology spectrum entropy	3.67	3.64	3.80	3.74	3.78	3.75

In Fig. 5 and Table 1, the form of morphology spectrum of tested signal at different rotating rate are all very similar, namely, they have 3.5 peaks and 4 valleys. And the value of morphology spectrum entropy is growing with the rotating rate increasing, but the values are very closely to each other. The results show that, on the one

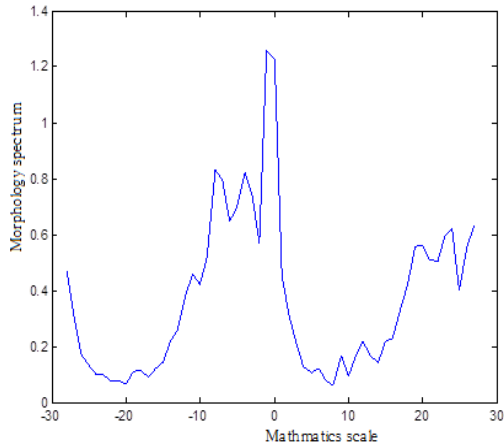


Fig. 6. Morphology spectrum at rotating rate of 5rad/s.

hand, the simulation results is verified and the actual motion of rotor is in the state of chaos, on the other hand, the novel method of morphology spectrum is able to extrude the characteristics of tested signal to diagnosis the motion of rotor.

4.2. Analysis on vibration signal of hydropower unit shaft

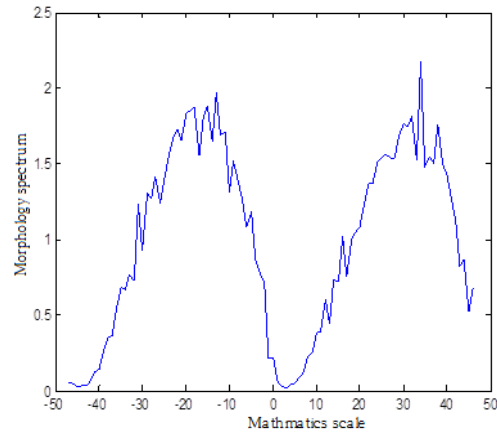


Fig. 7. Morphology spectrum at rotating rate of 12rad/s.

At the rotating rate of 5 rad/s、12 rad/s、30 rad/s, the morphology spectrum of tested signal by multi-scale morphology is shown in figs. 5-7.

The simulation results at the rotating rate of 5 rad/s、12 rad/s、30 rad/s show that the corresponding motion of rotor is belongs to three types: stable periodic motion, periodic rubbing motion and chaotic motion. And the analysis on tested signal has the same result with simulation. So, the tested signal is divided into three types too, and the form of morphology spectrum has the different pattern, namely, the form of 1.5 peaks and 2 valleys, the form of 2 peaks and 1.5 valleys, the form of 3.5 peaks and 4 valleys.

At the rotating rate of 5 rad/s、12 rad/s、30 rad/s, the 30 groups tested vibration signals with coupling faults is analyzed and the morphology spectrum entropy is calculated based on the concept of entropy, and the results is shown in table 2. The results show that, the change scale of morphology spectrum entropy is [2.71, 3.0] and the average is 2.81 at the rotating rate of 5 rad/s, and the change scale of morphology spectrum entropy is [3.10, 3.15] and the average is 3.12 at the rotating rate of

12 rad/s, and the change scale of morphology spectrum entropy is [3.62, 3.82] and the average is 3.69 at the rotating rate of 30 rad/s. So the different motion type of rotor lead to the different value of morphology spectrum entropy and different form of morphology spectrum, which is laid the foundation for on-line monitoring and fault diagnosis.

5. Fault diagnosis based on morphology spectrum and morphology spectrum entropy

Using the above research to analyze the rest 70 groups of tested signal and diagnose the fault type of rotor, the results are shown in Table 2. The results show that: the diagnosis rate of rotor in stable periodic motion is highest, and the rate is above 98.6%, and the diagnosis rate of rotor in chaotic motion is above 95.7%, and the diagnosis rate of rotor in periodic rubbing motion is lowest, but the value is also above 90%. And the diagnosis rate based on morphology spectrum reaches 100%.

Table 2. The success rate of fault diagnosis.

Rotating rate	$\omega=5\text{rad/s}$	$\omega=12\text{rad/s}$	$\omega=20\text{rad/s}$	$\omega=30\text{rad/s}$	$\omega=50\text{rad/s}$
Morphology spectrum entropy	100%	90%	100%	95.7%	98.6%
Morphology spectrum	100%	100%	100%	100%	100%

6. Conclusions

The tested vibration signal of hydropower unit rotor with coupling faults is analyzed based on morphology spectrum and morphology spectrum entropy, the corresponding research results as follow:

Based on morphology spectrum, the tested signal is divided into stable periodic motion periodic rubbing motion and chaotic motion, and the form of morphology spectrum has the different pattern, namely, the form of 1.5 peaks and 2 valleys, the form of 2 peaks and 1.5 valleys, the form of 3.5 peaks and 4 valleys.

Based on morphology spectrum entropy, the tested signal is divided into stable periodic motion periodic rubbing motion and chaotic motion, and the average value of morphology spectrum entropy are the different, namely, 2.81, 3.12 and 3.75.

The novel method can identify the characteristics of vibration efficiency under different condition, and the research is to improve the safety, reliability and economical efficiency and has important academic significance and engineering application value.

Acknowledgments

This study is funded by the National Natural Science Foundation of China (No. 51179135).

References

- [1] R. Tiwari, V. Chakravarthy, *Mechanism and Machine Theory*, **44**(4), 92 (2009).
- [2] A.A. Mohammad Shudeifat, A. Eric Butcher, R. Carl Stern, *International Journal of Engineering Science*, **48**(10), 921 (2010).
- [3] I. Jawaid Inayat-Hussain, *Mechanism and Machine Theory*, **45**(10), 1651 (2010).
- [4] An Sung Lee, Jin Woong Ha, *Journal of Sound and Vibration*, **283** (3-5), 507 (2005).
- [5] B. M. Gravier, N.N.J., J.A. Pelstring, *Proceedings of the International Offshore and Polar Engineering Conference*, p. 268, 2001.
- [6] V. K. Rai, A. R. M., *Mechanical Systems and Signal Processing*, **21**, 2607 (2007).
- [7] Jia Rong, Wang Xiaoyu, Cai Zhenhua, *Proceedings of the CSEE*, **26**(22), 128 (2006).
- [8] Jing Wang, GuanghuaXu, QingZhang, et al. *Mechanical Systems and Signal Processing*, **23**(1), 236 (2009).
- [9] Rujiang Hao, Fulei Chu, *Journal of Sound and Vibration*, **320**(4-5), 262 (2009).
- [10] P. Maragos, RW. Schafer, *IEEE Transactions on Acoustics, Speech, and Signal Processing*, **ASSP-35**(7), 1153 (1987).
- [11] M. Nachtgael, P. Sussner, T. Mélange, E. E. Kerre, *Information Sciences*, **181**(10), 1971 (2011).
- [12] B.L.Newkirk, H.D. Taylor, *General Electric Review*, **28**(8), 559 (1925).
- [13] Cai-Wan Chang-Jian, Chao-Kuang Chen, *Tribology International*, **42**(3), 426 (2009).
- [14] Cai-Wan Chang-Jian, Chao-Kuang Chen, *Mechanism and Machine Theory*, **42**(3), 312 (2007).
- [15] Gwo-Chung Tsai, *Journal of Sound and Vibration*, **271**(3-5), 547 (2004).

Corresponding author: hyqgxu@126.com