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A robust controller for synchronization of chaotic system

LI-DONG LIU[*], JI-FENG HU, ZI-SHU HE, CHUN-LIN HAN
School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China.

A new synchronization control method for chaotic system, which only needs one scalar time series of the master system to reach the synchronization, is present in this paper. It is robust to noise and the synchronization can be obtained in a high level noise. This method may have practical application in secure communications or optical communication because it only needs one scalar time series and it can be controlled in the background of noise. Finally, numerical examples are given for the typical Lorenz chaotic system to illustrate the effect of the proposed method.

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1. Introduction

Chaos phenomenon has attracted the attention of many researchers, and it has been used in electronics [1], optical communication [2-6] and radar [7-10]. The practical applications of chaos are heavily depended on chaotic synchronization. However, chaotic synchronization is greatly limited by its sensitivity to noise. Even small amounts of noise added to the synchronizing signal can degrade synchronization quality and the chaotic signal may not be recovered. Thus designing a robust controller is critical for chaotic synchronization.

Robust synchronization method for chaos has been researched by many researchers [11-15]. In [11], a robust synchronization method based on open-plus-close-loop method is proposed, which is efficient for a large level noise. In [12], another synchronization method is proposed, which adjusts the master and slave circuit and make the synchronization error arbitrarily close to the error caused by circuit mismatch for any amount of added noise. In [15], a synchronization method which is less sensitive to added noise based on coupling a shift map to a digital filter is proposed. However, in [11-15], the synchronization methods need to know more than one scalar time series of the master chaotic system. For example, in Lorenz system, there are three scalar time series (X,Y,Z) , and the existing robust synchronization methods such as [11-15] need to know all three scalar time series (X,Y,Z) of the master system. Since in practice, such as in secure communication or optical communication, the information signal or the transmitted signal is one scalar time series of the chaotic signal or the signal embedded in one scalar time series, thus the existing robust synchronization methods may not be suitable in these fields. Therefore, it is not only of theoretical interest but also of practical value to design a robust synchronization method which only needs one scalar time series.

In order to get a robust synchronization method which can be applied in communication and optical communication, a new control method which only needs one scalar time series to reach the robust synchronization is proposed in this paper. Our controller is designed by a fundamental concept from stability and control theory based on Lyapunov function. For illustration, a typical chaotic system, Lorenz system is used as an example. It is shown that the proposed method is robust to noise and easy to be accomplished. Though in this paper we focus on the Lorenz chaotic system, the analysis and the thought still can be extended to other chaotic systems.

This paper is organized as follows. In section 2, a new robust control method which only needs one scalar time series is present. In section 3, simulations on the Lorenz system is given to illustrate the effect of the proposed method. Brief conclusion of this paper is drawn in section 4.

2. Robust synchronization method

In this section, a new robust control method is proposed. It is illustrated by the well known chaotic system, Lorenz system.

The Lorenz chaotic system is given by

$$\begin{cases} \dot{x}_m = s(y_m - x_m) \\ \dot{y}_m = rx_m - y_m - x_m z_m \\ \dot{z}_m = x_m y_m - bz_m \end{cases} \tag{1}$$

where s , r and b are the parameters. We assume that the system (1) freely "moves" about, so it is called the master system. The synchronization problem is to design a controller (u_1, u_2, u_3) such that the trajectories of the slave system (2) asymptotically follow those of the master system.

$$\begin{cases} \dot{x}_s = s(y_s - x_s) + u_1 \\ \dot{y}_s = rx_s - y_s - x_s z_s + u_2 \\ \dot{z}_s = x_s y_s - bz_s + u_3 \end{cases} \quad (2)$$

In other words, it is desired that

$$\begin{aligned} &\lim_{t\to\infty} |x_s - x_m| \to 0 \\ &\lim_{t\to\infty} |y_s - y_m| \to 0 \\ &\lim_{t\to\infty} |z_s - z_m| \to 0 \end{aligned} \quad (3)$$

This problem was solved via Lyapunov based nonlinear control in [12]. It is shown that when full scalar time series are used, the robust synchronization can be made. However, considering the practical application, such as in secure communication and optical communication, only one scalar time series can be obtained. Thus in this paper, we design a new controller which only needs to know one scalar time series of the master chaotic system. Moreover, the controller is designed by exploiting and not dismantling the system's natural structure.

Let us define the synchronization error dynamics

$$e_1 = x_s - x_m,\ e_2 = y_s - y_m,\ e_3 = z_s - z_m \quad (4)$$

Then subtracting Eq.(1) from Eq.(2), we obtain

$$\begin{cases} \dot{e}_1 = s(e_2 - e_1) + u_1 \\ \dot{e}_2 = re_1 - e_2 - x_s e_3 - z_m e_1 + u_2 \\ \dot{e}_3 = -be_3 + y_s e_1 + x_m e_2 + u_3 \end{cases} \quad (5)$$

The synchronization problem is solved if it is ensured that $\lim_{t\to\infty} e_i(t) = 0$ under no noise condition or $\lim_{t\to\infty} e_i(t) \le C$ (C is a small constant) when added noise.

In this paper, the slave system is then given by

$$\begin{cases} \dot{x}_s = s(y_s - x_s) \\ \dot{y}_s = rx_s - y_s - x_s z_s + (z_s + p)e_1 \\ \dot{z}_s = x_s y_s - bz_s - y_s e_1 \end{cases} \quad (6)$$

where p is a parameter which needs to define. We can see from Eq.(6) and Eq.(2) to obtain that

$$u_1 = 0; \qquad u_2 = (z_s + p)e_1; \qquad u_3 = -y_s e_1 \quad (7)$$

This means in the slave system we only need to know one time series ($x_m(t)$) of the master system. Next we will proof why the slave system in Eq.(6) can be synchronized to the master system in Eq.(1) and we also give how to choose the parameter p in Eq.(6).

Substituting Eq.(6) into Eq.(5) and regrouping terms yield

$$\begin{cases} \dot{e}_1 = s(e_2 - e_1) \\ \dot{e}_2 = (r + p)e_1 - e_2 - e_1 e_3 \\ \dot{e}_3 = -be_3 \end{cases} \quad (8)$$

Choosing the Lyapunov function

$$L = (1/2)(e_1^2 + e_2^2 + e_3^2) \quad (9)$$

we have

$$\dot{L} = -se_1^2 - e_2^2 - be_3^2 + (r + p + s)e_1 e_2 - e_1 e_2 e_3 \quad (10)$$

When the slave system synchronize to the master system, $e_1 e_2 e_3$ is much smaller than other terms in Eq.(10) and Eq.(10) could be simplified

$$\dot{L} \approx -se_1^2 - e_2^2 - be_3^2 + (r + p + s)e_1 e_2 \quad (11)$$

In order to make slave system (6) synchronize with the master system (1), $\dot{L} < 0$ should be satisfied.

We let $A = \begin{bmatrix} -s & \frac{r+p+s}{2} & 0 \\ \frac{r+p+s}{2} & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}$. Then $\dot{L} < 0$ is equal to make A be positive matrix. This gives us

$$-2\sqrt{s} - r - s < p < 2\sqrt{s} - r - s \quad (12)$$

So for any p given by Eq.(12) and any suitable initial condition of Eq.(1) and Eq.(6), $\lim_{t\to\infty} |\mathbf{e}| \to 0$, which means that $(x_s, y_s, z_s) \to (x_m, y_m, z_m)$.

3. Simulations and analysis

In order to illustrate the effect of the proposed robust synchronization method, in this section, we do simulations on the Lorenz system. For comparing, the performance of the traditional chaotic synchronization method [16] which also only needs to know one time series, is given.

The simulations are operated as follows. Let the master system is defined by Eq.(1), the slave system by using the proposed method in this paper is defined by Eq.(6). For both systems, let $s = 16, r = 45.6, b = 4$. The initial condition for the master system is given by (-1,1,0) and that of the slave system is given by (2,3,5). For comparing, the master is the same, but the slave system is given by Eq.(13) as [16]. Four group simulations (SNR=40dB, SNR=20dB, SNR=0dB) have done. This is shown in Fig.1-Fig.4.

$$
\begin{cases}
\dot{x}_s = s(y_s - x_m) \\
\dot{y}_s = r x_m - y_s - x_m z_s \\
\dot{z}_s = x_m y_s - b z_s
\end{cases}
\tag{13}
$$

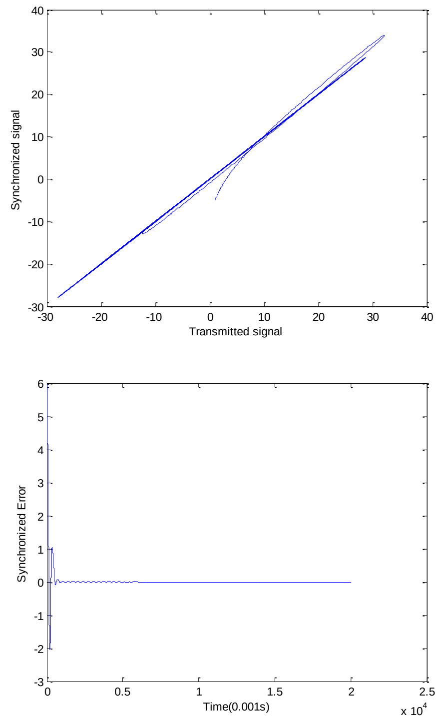

Fig.1 (a) The synchronization performance of the proposed method in this paper with no noise.

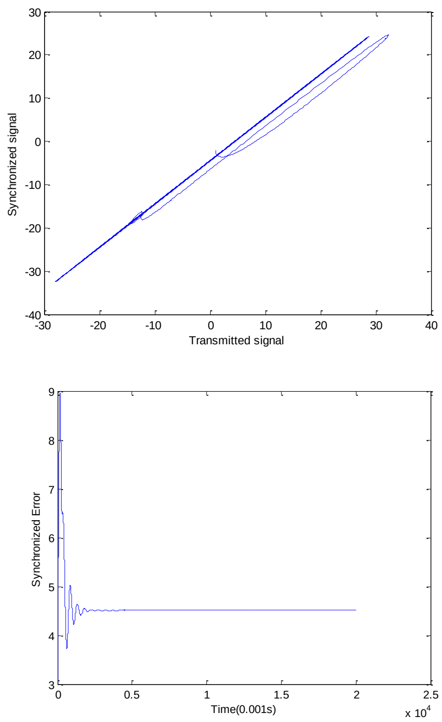

Fig.1 (b) The synchronization performance of the method in [16] with no noise.

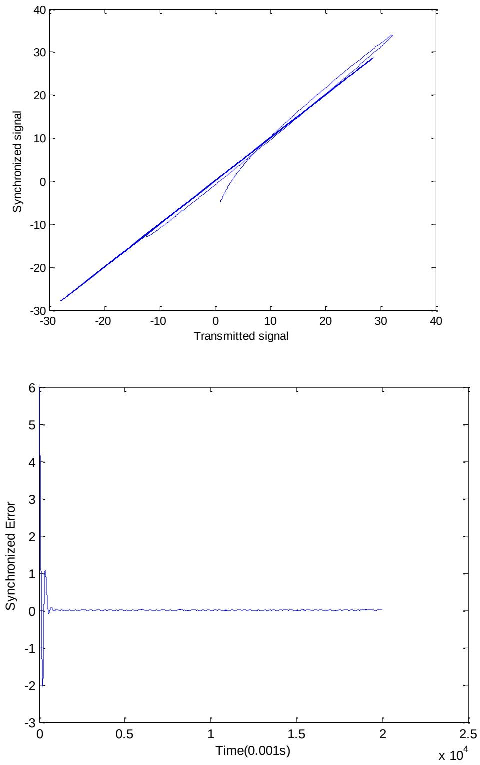

Fig.2 (a) The synchronization performance of the proposed method in this paper at SNR=40dB.

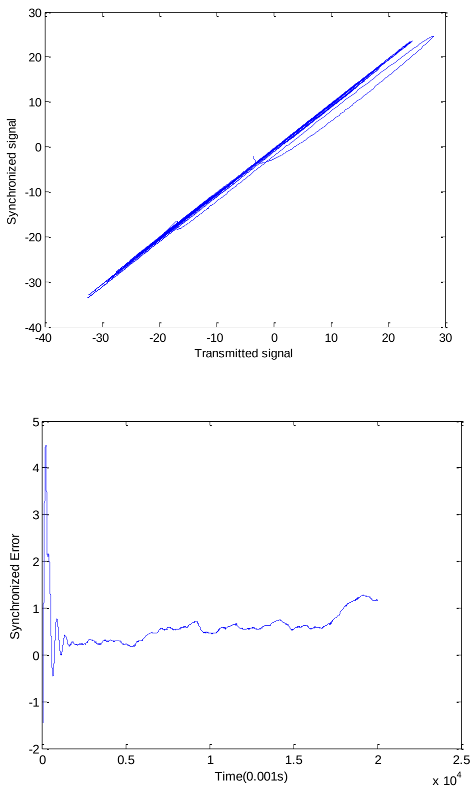

Fig.2 (b) The synchronization performance of the method in [16] at SNR=40dB.

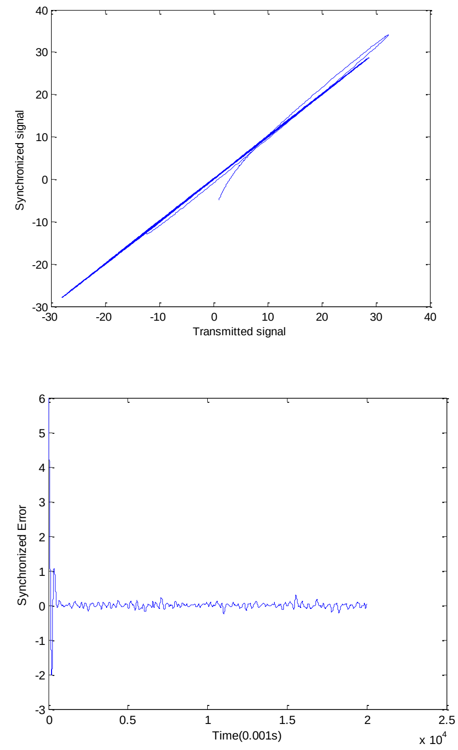

Fig.3 (a) The synchronization performance of the proposed method in this paper at SNR=20dB.

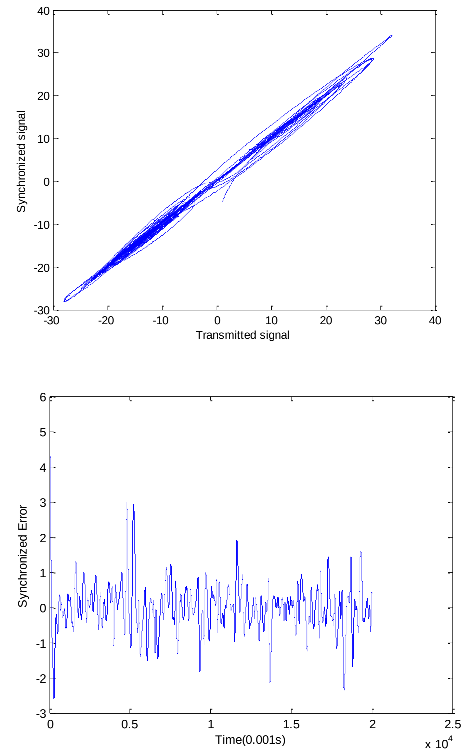

Fig.4 (a) The synchronization performance of the proposed method in this paper at SNR=0dB.

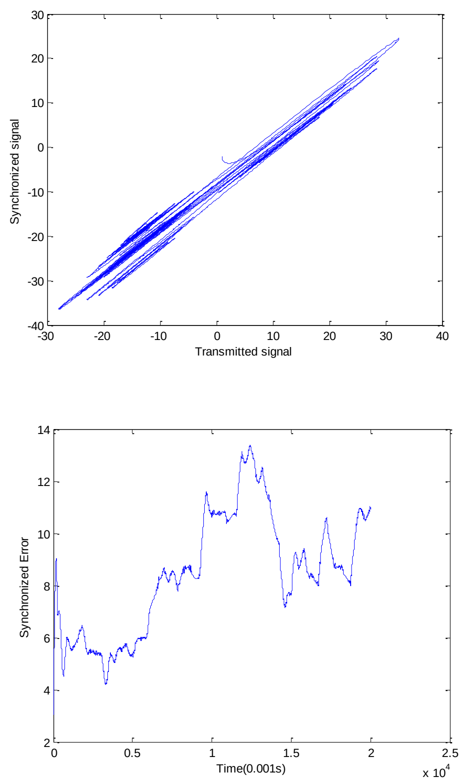

Fig.3 (b) The synchronization performance of the method in [16] at SNR=20dB.

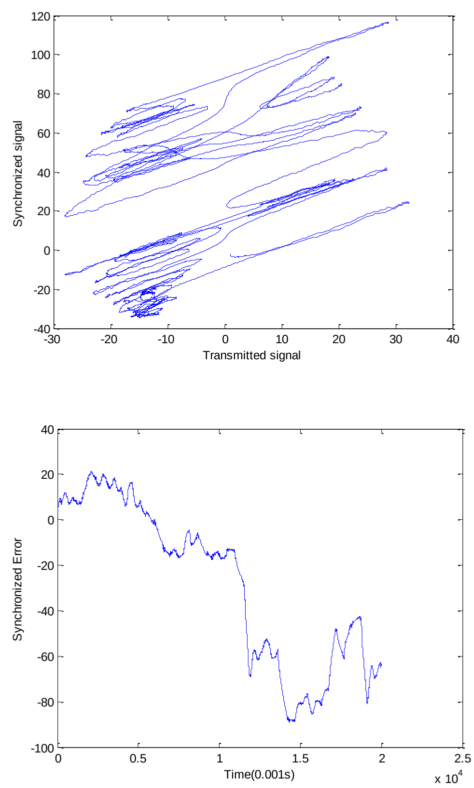

Fig.4 (b) The synchronization performance of the synchronization method in [16] at SNR=0dB.

We can see from Fig.1-Fig.4, for both methods, the transmitted signal (master signal) and the synchronized signal are almost in the diagonal, which means the two signals are almost the same when the SNR is high (noise level is low). But the synchronization error becomes much bigger when the SNR is low for the method in [16], while the synchronization error is not much affected by the noise for the proposed method in this paper and the transmitted signal and the synchronized signal are still nearly in the diagonal even at SNR=0dB. The synchronization error is bounded in a small interval. This means that the proposed synchronization method is robust to noise. Though the synchronization method in [11-15] can perform the similar noise robust feature, these method need to know more than one time series of master system.

4. Conclusions

In this paper, a new synchronization control method is performed. It achieves exponential synchronization with no noise and the synchronization is still obtained in a high level noise. Different to the existing robust synchronization methods, it only needs to know one times of the master system. This is could be more easily accomplished in the practical application.

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*Corresponding author: liulidong_1982@126.com