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Accurate prediction of effective core area and effective index of refraction of single mode graded index fiber by a simple technique

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The analytical expression for effective core area of single mode graded index fiber is prescribed and formulated using power series expression of the fundamental mode developed by Chebyshev technique. The step and parabolic index fibers are used as typical examples. Using the said formulation, we estimate the effective core areas for different values of normalized frequency V. We also evaluate the indices of refraction for the said fibers. The concerned evaluations require little computation. We also show the accuracy of our simple formalism by comparing our results with the available exact results. Thus the formalism will prove beneficial to the technologists in the field of optical technology.

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1. Introduction

The knowledge of effective core area [1-6] of the optical fiber is useful to predict the effects due to third order optical non-linearity such as self-phase modulation (SPM), cross phase modulation (XPM), soliton formation, four wave mixing (FWM), stimulated Raman and Brillouin scattering [7] etc. The fundamental study of optical communication system in simpler method is always a potential problem, where the estimation of effective core area and effective index of single mode graded index fiber has a great importance.

Further, the design of high power fiber laser requires minimization of non-linear effects so that output power is not reduced owing to the non-linear effects [8-10]. The effective core area can be increased by modifying the core radius and refractive index profile by suitable design, so that the operation can be restricted in the single-mode region. It deserves mentioning in this connection that increase of effective area reduces distortion due to non-linearity. So the prediction of effective area of single-mode graded index fiber has a great importance in all optical systems involving optical fiber, where accurate knowledge of fundamental mode of graded index fiber is a must.

In case of step index fiber, the analytical solutions for fundamental mode is readily available, but one has to resort to either numerical technique [11] or approximate method like the perturbation [12] or variational method [13-17] for derivation of fundamental mode in other kinds of fiber. Further, the variational method is found to be the most suitable one out of all the approximate methods since this method leads to prescription of closed form expression of modal field which gives analytical expressions for different propagation parameters. The variational technique involving two parameter Gaussian trial function for fundamental mode of graded index fiber is of great importance to predict the propagation characteristics over a long range [16, 17]. But, it is observed that in the low V region, the accuracy of this double parameter variational technique is not sufficient. At the same time, the variational technique involving Gaussian-exponential-Hankel function [18] is capable to provide accurate prediction of graded index fiber characteristics over the complete single-mode region including the low V region as well. However, these investigations require extensive computation. So, the formulation of a simple but accurate expression of the fundamental mode of graded index fiber is of great importance. A simple as well as accurate power series expression of fundamental mode of graded index fiber is available in literature [19-22] where little computation is sufficient for execution. The Appendix part is included in the paper to explain the computational resources. Presently, computational resources are very much simulator based and in this respect our study and proposal of simpler but accurate method of determination of basic parameters seem to be very user friendly with the system engineers.

The effective refractive index (n_{eff}) is also an important parameter which is dependent not only on the wavelength of the signal but also on the concerned propagating mode. That is why it is also known as modal index. Practically, it denotes the overall delay of light beam in a particular mode in which it propagates. The correct value of effective index of refraction is important for system modeling, optical device assembling, and selection of index matching gel to minimize joint losses and back reflection as well.

The effective index of refraction has its proven importance in case of Fiber Bragg grating Sensors [23]. Some basic properties of Photonic crystal fibers can be explored with the knowledge of effective index of refraction [24-25]. In order to predict the effective index of refraction of graded index fiber, the knowledge of accurate values of cladding decay parameter is required.

In this paper, the simple power series form of fundamental mode of graded index fiber is used to determine the analytic expression for effective fiber core area for single-mode graded index fiber. On the basis of said formulation, we have also reported the concerned evaluation and present the excellent match between our predictions and the available exact results. Moreover, based on the values of cladding decay parameter [26-28], we have estimated the effective index of refraction accurately in a simple fashion.

2. Analysis

The expression of refractive index profile of a weakly guiding optical fiber is

$$n^2(R) = \begin{cases} n_1^2(1-2\delta f(R)), & R \le 1 \\ n_2^2\ , & R > 1 \end{cases} \tag{1}$$

Here, R = r/a and grading parameter $\delta = (n_1^2 - n_2^2)/2n_1^2$ where 'a' is the core radius with n_1 and n_2 representing the refractive indices of the core axis and cladding respectively. Further, the shape of the refractive index profile of the fiber is defined by $f(R)$ and in case of graded index fiber, it is given as

$$f(R) = R^q \tag{2}$$

where q stands for the profile exponent and its values for step and parabolic index fibers are ∞ and 2 respectively.

In case of graded index fiber, the normalized power series expression for fundamental mode is expressed as [19-22].

$$\Psi(R) = \begin{cases} 1 + A_2R^2 + A_4R^4 + A_6R^6 & R \le 1 \\ (1 + A_2 + A_4 + A_6)\dfrac{K_0(WR)}{K_0(W)} & R > 1 \end{cases} \tag{3}$$

Here, A_2, A_4 and A_6 are constants while W is the cladding decay parameter and they are found as described in [20].

The effective core area A_{eff} is given as [29]

$$A_{eff} = 2\pi a^2 \frac{\left(\int_0^\infty \psi^2(R)\,R\,dR\right)^2}{\int_0^\infty \psi^4(R)\,R\,dR} \tag{4}$$

Employing Eq. (3) in Eq. (4), one can easily find the effective core area

$$A_{eff} = 2\pi a^2 \frac{\left(\int_0^1 \left(1 + A_2R^2 + A_4R^4 + A_6R^6\right)^2 RdR + \int_1^\infty (1 + A_2 + A_4 + A_6)^2 \frac{K_0^{\,2}(WR)}{K_0^{\,2}(W)} RdR\right)^2}{\int_0^1 \left(1 + A_2R^2 + A_4R^4 + A_6R^6\right)^4 RdR + \int_1^\infty (1 + A_2 + A_4 + A_6)^4 \frac{K_0^{\,4}(WR)}{K_0^{\,4}(W)} RdR} \tag{5}$$

Following [30-32], one can evaluate Eq. (5) and thus A_{eff} is found as

$$A_{eff} = 2\pi a^2 \frac{[0.5\{T_1 + 2T_2 + T_3(T_4^2 - 1)\}]^2}{T_5 + T_3^2 T_6} \tag{6}$$

Where

$$T_1 = \left(1 + \frac{A_2^2}{3} + \frac{A_4^2}{5} + \frac{A_6^2}{7}\right)$$

$$T_2 = \frac{A_2}{2} + \frac{A_4}{3} + \frac{A_6}{4} + \frac{A_2A_4}{4} + \frac{A_2A_6}{5} + \frac{A_4A_6}{6}$$

$$T_3 = (1 + A_2 + A_4 + A_6)^2$$

$$T_4 = \alpha + \frac{\beta}{W}$$

$T_5 =$

$\frac{1}{2} + A_2 + \frac{1}{3}(3A_2^2 + 2A_4) + \frac{1}{2}(A_2^3 + 3A_2A_4 + A_6) +$
$\frac{1}{10}(6A_4^2 + 12A_2A_6 + 12A_2^2A_4 + A_2^4) + \frac{1}{3}(3A_2A_4^2 +$
3A22A6+A23A6+3A4A6+
$\frac{1}{7}(3A_6^2 + 12A_2A_4A_6 + 3A_2^2A_4^2 + 2A_2^3A_6 + 2A_4^3) +$
$\frac{1}{4}(3A_2A_6^2 + 3A_2^2A_4A_6 + 3A_4^2A_6 + A_2A_4^3) + \frac{1}{18}(6A_2^2A_6^2 +$
12A2A42A6+12A4A62+A44
$+\frac{1}{5}(3A_2A_4A_6^2 + A_6^3 + A_4^3A_6) + +\frac{1}{11}(3A_4^2A_6^2 + 2A_2A_6^3) +$
$\frac{1}{6}A_4A_6^3 + \frac{1}{26}A_6^4$

$$T_6 = \frac{1}{K_0^4(W)}\frac{1}{W^2}[\exp(0.2076W^2 - 5.0785W - 0.1185)]$$

The terms A_2, A_4, A_6 in the expressions after Eq. (6) can be understood from the significance of the terms presented in the Appendix

The effective Index of refraction of an optical fiber is defined as,

$$n_{eff} = \sqrt{(n_1^{\,2} - \frac{U^2}{a^2k_0^{\,2}})} = \sqrt{(n_1^{\,2} - n_2^{\,2})\frac{W^2}{V^2} + n_2^{\,2}} \tag{7}$$

Here, k_0 is the propagation constant in free space and defined as $k_0 = 2\pi/\lambda$

Further, U, V and W are the waveguide parameter, normalized frequency and cladding decay parameter respectively and those are related by the following expression [1]

$$U^2 = V^2 - W^2 \tag{8}$$

3. Results and discussions

Using the formulations of effective area of graded index fiber, we have studied the variation of effective area A_{eff} as well as effective index of refraction n_{eff} with respect to some typical V numbers. In this context, we choose step and parabolic index fibers as examples. Our investigation for step index fiber has been presented in Table 1 while that for parabolic index fiber has been presented in Table 2.

Table 1. Effective area (A_{eff}) and Effective index of refraction (n_{eff}) for Step Index fiber

V Number	A_{eff} (m^2)		n_{eff}	
	Our Result	Exact	Our Result	Exact
1.4	1.40542E-10	1.40612E-10	1.44881	1.44884
1.6	9.53936E-11	9.54101E-11	1.44998	1.45012
1.8	7.42103E-11	7.42283E-11	1.45105	1.45113
2	6.21755E-11	6.22341E-11	1.45199	1.45241
2.2	5.47460E-11	5.46952E-11	1.45283	1.45312
2.4	4.91552E-11	4.91792E-11	1.45355	1.45362

Table 2. Effective area (A_{eff}) and Effective index of refraction (n_{eff}) for Parabolic Index fiber

V Number	A_{eff} (m^2)		n_{eff}	
	Our Result	Exact	Our Result	Exact
2.1	7.27861E-11	7.27903E-11	1.44863	1.44872
2.5	4.72747E-11	4.73184E-11	1.44984	1.44991
2.8	3.81412E-11	3.81371E-11	1.45066	1.45075
3	3.43074E-11	3.43212E-11	1.45114	1.45127
3.2	3.11392E-11	3.11431E-11	1.45159	1.45174
3.5	2.74670E-11	2.75124E-11	1.45219	1.45225

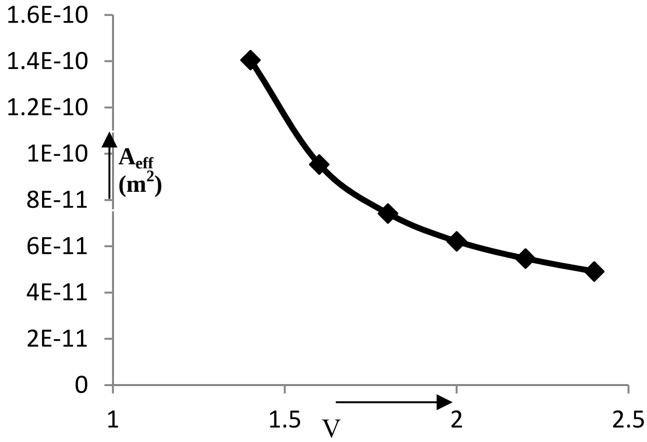

Fig. 1. Variation of Effective area A_{eff} with normalized frequency 'V' for step index fiber [— exact result: ■ results from our formulation]

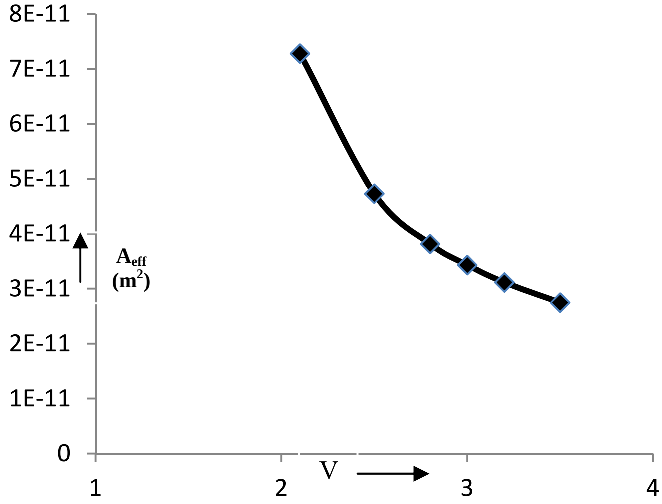

Fig. 2. Variation of Effective area A_{eff} with normalized frequency 'V' for parabolic index fiber [— exact result: ■ results from our formulation]

Side by side, we have presented the variation of A_{eff} versus the said V values for the Step index fiber ($q=\infty$) in Fig. 1. Here, values are represented by square dots while the exact values are represented by solid lines. It is seen that our values match excellently with the exact ones [18]. Similarly in Fig. 2, we represent the variation of effective area A_{eff} with normalized frequency 'V' for the parabolic index fiber ($q=2$). Here also, the square dots which represent our results match excellently with the solid lines, which correspond to exact values [18], Further, Figs. 3 and 4 are representing the variations of effective index with normalized frequency 'V' for step and parabolic index fibers respectively. As in other Figs. 1and 2, we also obtain excellent agreement between our values (square dots) and available exact values [18] (solid lines).

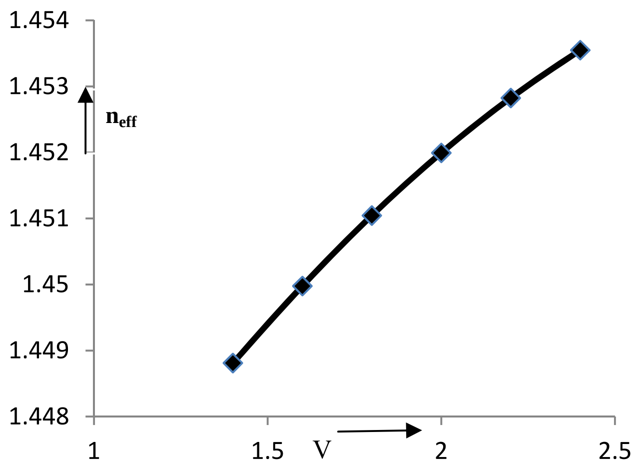

Fig. 3. Variation of Effective index of refraction n_{eff} with normalized frequency 'V' for step index fiber [— exact result: ■ results from our formulation]

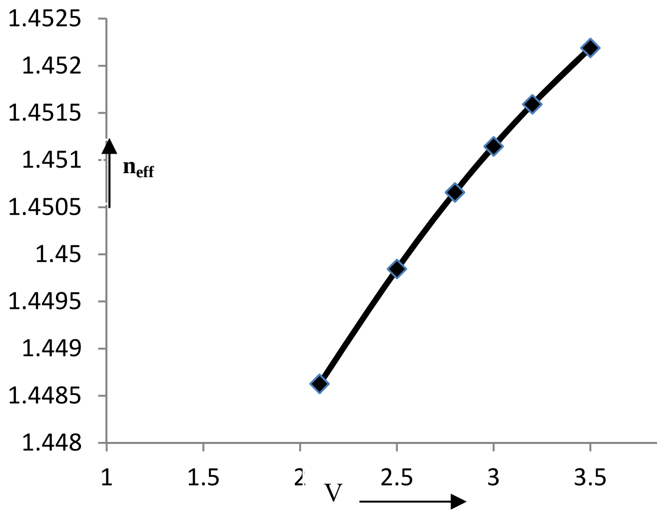

Fig. 4. Variation of Effective index of refraction n_{eff} with normalized frequency 'V' for parabolic index fiber. [— exact result: ■ results from our formulation]

Here, we have considered n_1=1.46, numerical aperture =0.2 and a= 3.65×10^{-6} m. We have studied our results with variation of 'V' number taking numerical aperture and radius of the core as constants. So, the variation of 'V' is dependent on wavelength 'λ' only. It is also observed that the effective area is decreasing with increase of V number and this leads to the concept that fibers of large V number will promote more non-linear distortion. So, the results can guide one to choose appropriate V number so as to reduce four –wave mixing, self-phase modulation, stimulated Brillouin scattering and stimulated Raman scattering resulting from third order nonlinearity. Third order nonlinearity will be predominant around the first higher order cut off 'V' number as the effective area is less there. This is applicable for both types of the fiber. This has also been reported in ref. [33]. The relevant mathematical expression for fundamental modal field in case of graded index fiber has been found by iterative method based on Chebyshev formalism [33]. Once the values of A_2, A_4, A_6 and W are found in presence of Kerr-nonlinearity, the effective area and index of refraction in this context are found easily by using Eqs. 4 and 7 respectively. It has been found that in both kinds of fibers, the effective index of refraction increases with increase of V number and consequently higher V number implies less group velocity for the fundamental mode and as such more delay for it. Taking care of the fact that each spectral line has some width, however small it may be, and the medium of propagation is dispersive, we have considered here the group velocity in place of phase velocity. This is consistent with the results available in literature [34]. Finally, it can be mentioned that the excellent match between the available exact results [18] and the results estimated by our simple formalism establishes the accuracy of our method. Accordingly, it is expected that the prescribed user friendly method will be beneficial to the system engineers in the process of prediction of bending loss, laser performance, capture cross section, back scattering, fiber mechanical reliability etc.

4. Conclusion

We have formulated analytical expressions for effective core area for single-mode graded index fiber. Our formalism is based on accurate power series expression for fundamental mode of graded index fiber. We have estimated the said parameter in case of some typical graded index fibers, namely step and parabolic index fibers. Side by side, we have estimated index of refraction for the said fibers by using values of corresponding cladding decay parameter available in literature. The results found match excellently with the available exact results. The execution of our formalism involves little computation. Accordingly, the method prescribed is user friendly and it will, thus, benefit the engineers working in the field of technology concerned with various kinds of optical devices and sensors as well as WDM communication system.

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Appendix

For weakly guiding single-mode fiber, the fundamental modal field $\psi(R)$ in the fiber-core is expressed by the following scalar wave equation [1, 20].

$$\frac{d^2\psi}{dR^2}+\frac{1}{R}\frac{d\psi}{dR}+\left(V^2(1-f(R))-W^2\right)\psi=0, \qquad R\le 1 \qquad \text{(a1)}$$

along with the boundary condition

$$\left(\frac{1}{\psi}\frac{d\psi}{dR}\right)_{R=1} = -\frac{WK_1(W)}{K_0(W)} \qquad \text{(a2)}$$

Where
$V = [k_0 a (n_1{}^2 - n_2{}^2)^{1/2}]$ and $W \; [= a(\beta^2 - n_2^2 k_0^2)^{1/2}]$ are the normalized frequency and cladding decay parameter respectively with k_0 and β representing the free space wave number and propagation constant respectively.

The fundamental modal field in the cladding of the fiber is given by,

$$\psi(R) \sim K_0(WR), \qquad R > 1 \qquad \text{(a3)}$$

Taking care of the fact that the fundamental modal field $\psi(R)$ is an even function of R with $\psi'(0)$ being zero and $\psi(0)$ non-zero, the Chebyshev power series of $\psi(R)$ can be expressed as [35, 36]

$$\Psi(R) = \sum_{j=0}^{j=M-1} a_{2j} R^{2j} \tag{a4}$$

For the sake of simplicity and accuracy, it is sufficient to retain terms up to j = 3 in [19-22] whereby one gets

$$\psi(R) = a_0 + a_2 R^2 + a_4 R^4 + a_6 R^6 \tag{a5}$$

The Chebyshev points are given as follows [36]

$$R_m = \cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right); m = 1, 2\ldots\ldots, (M-1) \tag{a6}$$

It is seen from Eq. (a4) that j=3 corresponds to M=4 and thereby the corresponding Chebyshev points are found from Eq. (a6) and those are given below,

$R_1 = 0.9749$, $R_2 = 0.7818$ and $R_3 = 0.4338$

Using Eq. (a5) in Eq. (a1), one gets the following three equations corresponding to the values of R_1, R_2 and R_3 given above.

$$\begin{aligned} &a_0[(V^2(1-f(R_i))-W^2)] + a_2[4+R_i^2(V^2(1-f(R_i))-W^2)] \\ &+a_4[16R_i^2 + R_i^4(V^2(1-f(R_i))-W^2)] \\ &+a_6[36R_i^4 + R_i^6(V^2(1-f(R_i))-W^2)] = 0 \end{aligned} \tag{a7}$$

By applying least square fitting in the region $0.60 \le W \le 2.5$, one can formulate the following linear relationship

$$\frac{K_1(W)}{K_0(W)} = \alpha + \frac{\beta}{W} \tag{a8}$$

It deserves mentioning in this connection that over said range, the above linear relationship corresponds to $\alpha = 1.0364623$; $\beta = 0.3890323$.

Using Eq. (a8) and Eq. (a5) in Eq. (a2), we get

$$a_0(\alpha W + \beta) + a_2(\alpha W + 2 + \beta) + a_4(\alpha W + 4 + \beta) + a_6(\alpha W + 6 + \beta) = 0 \tag{a9}$$

Three equations given by Eq. (a7) and one equation given by Eq. (a9) will produce nontrivial solution for a_2, a_4, a_6 if

$$\begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 \\ \alpha_4 & \beta_4 & \gamma_4 & \delta_4 \end{vmatrix} = 0 \tag{a10}$$

where,

$$\alpha_i = V^2(1-f(R_i)) - W^2$$

$$\begin{aligned} \beta_i &= 4 + R_i^2(V^2(1-f(R_i)) - W^2) \\ \gamma_i &= 16R_i^2 + R_i^4(V^2(1-f(R_i)) - W^2) \\ \delta_i &= 36R_i^4 + R_i^6(V^2(1-f(R_i)) - W^2) \end{aligned} \tag{a11}$$

here, i = 1, 2, 3 and

$$\alpha_4 = \alpha W + \beta$$

$$\beta_4 = 2 + \alpha_4; \quad \gamma_4 = 4 + \alpha_4; \quad \delta_4 = 6 + \alpha_4$$

Using Eq. (a10), one can find *W* for a given value of *V*. Again, knowing *W* for a particular *V*, one can calculate a_2, a_4, a_6 in terms of a_0 by employing any three of four equations given by Eq. (a7) and Eq. (a9). Thus the normalized field for fundamental mode for each value of V is found by this simple formalism and those are as follows

$$\begin{aligned} \psi(R) &= 1 + A_2R^2 + A_4R^4 + A_6R^6, & R \le 1 \\ &= (1 + A_2 + A_4 + A_6)\frac{K_0(WR)}{K_0(W)}, & R > 1 \end{aligned} \tag{a12}$$

where, $A_{2j} = a_{2j} / a_0$; j = 1, 2, 3 with the value of W being found by the present method.

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