Analysis of optical focused electromagnetic field by a parabolic reflector coated with a plasma layer under normal incidence

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A theoretical investigation has been carried out to analyze the focusing of electromagnetic field by a parabolic reflector coated with a plasma layer under normal angle of incidence. The reflection and transmission coefficients at dielectric-plasma and plasma-perfect electric conductor interfaces are derived analytically. The focal region electromagnetic field expressions have been obtained using Maslov's method. The derived analytical field expressions at caustic or focal point of plasma coated parabolic reflector have been solved numerically using MATHEMATICA. The effects of some physical parameters such as the plasma and wave frequencies and the thickness of plasma layer on the focal regions reflected and transmitted field from plasma layer are studied. The comparisons of the computed results of the presented formulations with the published results of some special cases confirm the accuracy of the presented analysis.

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1. Introduction

In recent years the study of analysis of electromagnetic waves focused in the focal region by plasma as absorbers or reflector [1,2] is imperative in the current arena of advanced technologies for microwave, millimeter-wave, and optical device applications. The analysis of focal region filed is useful for optical spectroscopy, medical treatment and hyperthermia. The image field may be also useful to generate images of the human body with the help of radio frequency. Such type of analysis is very significant to find out the suitable parameters of the plasma which affect the reflection, absorption, and transmission of the electromagnetic energy. When cylindrical and spherical metallic structures, as reflector antennas for example, enter into the Earth's atmosphere with high velocities, plasma layer forms on their antennas surfaces [3]. Thus, usually antennas of space vehicles are on the surface in contact with the plasma layer and this layer affects the radiation characteristics of antennas [4]. Also, the existence of the plasma layer on a metallic target changes the reflected wave energy, especially in the study of the interaction of intense electromagnetic waves with a metallic surface.

Many types of reflectors have been investigated in the open literature recently for focused electromagnetic wave in the focal region by using different techniques, for example, elliptical reflectors, cylindrical reflectors, hyperbolic reflector, etc [5-7]. These reflectors are the reflective devices used to collect or project the power of the electromagnetic waves. The opposite is also true; an electromagnetic wave source placed at the focus produces a parallel beam of electromagnetic waves. Parabolic reflector is also used as electromagnetic field reflector. The main advantage of this reflector is that it focuses a parallel beam which is incident on it, travelling along the optical axis, at single point [8]. The resulting reflected image is free of aberrations in geometrical optics approximation. The parabolic reflectors have a very high gain, low cross polarization, and reasonable bandwidth. That is why parabolic reflector can be termed as ideal focusing systems.

The geometrical optics (GO) approximation is well known technique. However, in many problems, such as describing fields in the vicinity of caustic, ART or geometrical optics (GO) does not provide satisfactory results [9]. To overcome the defect of GO Maslov's method is used. Maslov's method is a combination of asymptotic ray theory (ART) and Fourier transform method [10-12]. This technique has been used to study analysis of high frequency field in focal region succesfully by many authors [13-18]. In this paper, we consider a long metallic parabolic reflector coated with a cold collision plasma layer on its surface. The reflection and transmission coefficients at free space-plasma and plasma-perfect electric conductor interfaces have been derived analytically. The electromagnetic field intensity in the caustic or focal region of this parabolic reflector has been derived using Maslov's method. The effects of some physical parameters such as the plasma and wave frequencies and the thickness of plasma layer on the focal regions field transmitted and reflected from plasma layer have been studied. The results of the presented formulations have been compared with the published results of some special cases which confirm the accuracy of the presented analysis.

2. Methods and formulation

Let us consider geometry of a parabolic reflector made of perfect metal in the presence of a plasma layer as shown in Fig 1,equation of surface of metallic parabolic reflector is given as

$$\zeta = f - \frac{\xi^2}{4f} \tag{1}$$

where f is the focal length of the metallic parabolic reflector. Then, the equation of the parabolic plasma layer placed on the metallic parabolic reflector is defined as follows:

$$\zeta = p - \frac{\xi^2}{4p} \tag{2}$$

where $p = f - d$ and d are focal length of the parabolic plasma layer and the thickness of the plasma layer, respectively. According to the mathematical concept, coordinates of a point on the parabolic surface of the plasma layer $P(\xi_0, \zeta_0)$ in terms of a point on the metallic parabolic reflector $Q(\xi, \zeta)$ are defined as [7]

$$\xi_0 = \xi - d\cos\alpha_2 \tag{3}$$
$$\zeta_0 = \zeta + d\sin\alpha_2 \tag{4}$$

where $\alpha_2 = tan^{-1}(\xi/2f)$.

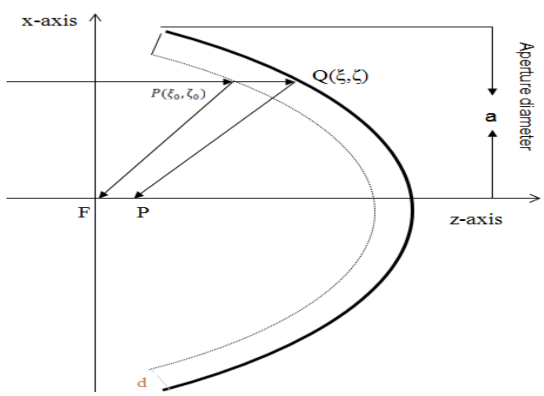


Fig. 1. Plasma coated parabolic reflector

Now, we consider a monochromatic electromagnetic wave incident on the parabolic plasma layer, parallel to its symmetry axis as [6,15]

$$\boldsymbol{E_{0i}} = E_i \hat{e}_x \exp(-jk_i z) \tag{5}$$
$$\boldsymbol{H_{0i}} = \eta_0^{-1} E_i \hat{e}_y \exp(-jk_i z) \tag{6}$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is intrinsic impedance of free space. As a plane wave is made incident on the plasma coated parabolic reflector, the wave vector of the reflected wave can be worked out by using Snell's law whose mathematical expression is described in the following form

$$\mathbf{P^r} = \mathbf{P^i} - 2(\mathbf{P^i}.\mathbf{n})\mathbf{n} \tag{7}$$

where $\mathbf{n}$ is unit normal to the surface and $\mathbf{P^i}$ is wave number of incident wave. Let unit vector normal to the surface of the metallic parabolic reflector and parabolic plasma layer are $\mathbf{n_1}$ and $\mathbf{n_2}$ respectively can be written

$$\mathbf{n_1} = \hat{e}_x \cos\alpha_1 + \hat{e}_z \sin\alpha_1 \tag{8}$$
$$\mathbf{n_2} = \hat{e}_x \cos\alpha_2 + \hat{e}_z \sin\alpha_2 \tag{9}$$

The reflected electric and magnetic fields from parabolic plasma layer can be written as

$$\boldsymbol{E_{0r}} = E_r \hat{e}_x \exp(jk_i z) \tag{10}$$
$$\boldsymbol{H_{0r}} = -\eta_1^{-1} E_r \exp(jk_i z) \tag{11}$$

The electric and magnetic fields of the wave that propagating towards the interface $z = d$ and reflected towards the interface $z = 0$ inside the plasma layer can be written

$$\boldsymbol{E_1^p} = E_{01}^p \hat{e}_x \exp(-jk_p z) \tag{12}$$
$$\boldsymbol{H_1^p} = Z^{-1} E_{01}^p \hat{e}_x \exp(-jk_p z) \tag{13}$$
$$\boldsymbol{E_2^p} = E_{02}^p \hat{e}_y \exp(jk_p z) \tag{14}$$
$$\boldsymbol{H_2^p} = -Z^{-1} E_{02}^p \hat{e}_y \exp(jk_p z) \tag{15}$$

The reflection and transmission coefficients at free space- plasma layer interface and at plasma layer-metal interface using boundary conditions can be obtained. The wave after reflection from the parabolic reflector will meet the border of the plasma layer, so part of it reflects through the layer and the other part transmits out of the layer, toward the parabolic reflector symmetry axis.

By applying Snell's law of reflection and refraction the wave vector $\mathbf{P_1}$ of the wave reflected the wave vector $\mathbf{Q_1}$ of the refracted wave by plasma parabolic layer are given by [11]

$$\mathbf{P_1} = -\hat{e}_x \sin 2\alpha_1 - \hat{e}_z \cos 2\alpha_1 \tag{16}$$
$$\mathbf{Q_1} = \hat{e}_x \mathrm{K} \sin\alpha_1 + \hat{e}_z (1 + \mathrm{K}\cos\alpha_1) \tag{17}$$

where $\mathrm{K} = \sqrt{\mathrm{n}^2 - 1 + \cos^2\alpha_1} - \cos\alpha_1$. Again by applying Snell's law of reflection the wave vector $\mathbf{P_2}$ of the wave reflected by the metallic parabolic surface is given by

$$\mathbf{P_2} = \hat{e}_x\left(\mathrm{K}_1 \sin(\alpha_1 - 2\alpha_2) - \cos(\alpha_1 - 2\alpha_2)\sin\alpha_1\right) + \\ + ez\mathrm{K}1\cos\alpha 1 - 2\alpha 2 - \sin\alpha 1 - 2\alpha 2\sin\alpha 1 \tag{18}$$

$$\mathbf{Q_2} = \hat{e}_x\left(\frac{1}{2} n\cos\alpha_1(\sin(\alpha_1 - 2\alpha_2) - \sin(3\alpha_1 - 2\alpha_2) + \\ 2\mathrm{K}2\sin 2\alpha 1 - 2\alpha 2 - \mathrm{K}2\sin\alpha 1 + \\ \hat{e}_z\left(\frac{1}{2} n\sin\alpha_1(\sin(\alpha_1 - 2\alpha_2) - \sin(3\alpha_1 - 2\alpha_2) + \\ 2\mathrm{K}2\sin 2\alpha 1 - 2\alpha 2 - \mathrm{K}2\cos\alpha 1 \tag{19}$$

where $\mathrm{K}_1 = \sqrt{\left(-1 + 2n^2 + \cos(2\alpha_1)\right)/2}$

$$\mathrm{K}_2 = \sqrt{1 - n^2 + \frac{n(\cos(\alpha_1 - 2\alpha_2) - \cos(3\alpha_1 - 2\alpha_2) + 2\mathrm{K}_1 \cos(2\alpha_1 - 2\alpha_2))^2}{4}}$$

The Jacobian associated with wave reflected by the parabolic plasma layer is obtained

$$\mathrm{J}_1(\tau) = \frac{\mathrm{D}_1(\tau)}{\mathrm{D}_1(0)} = 1 - \frac{\cos^2\alpha_1}{P}\tau \tag{20}$$

The fields expressions for the reflected and transmitted rays out of the plasma layer in geometric optics are obtained as under [13-18]

The Jacobian associated with wave reflected by the metallic parabolic layer is obtained

$$\mathrm{J}_2(\tau) = \frac{\mathrm{D}_2(\tau)}{\mathrm{D}_2(0)} = \frac{1}{4fK_2^2}(4fK_2^2 + \cos^3\alpha_1\, t\left(-\mathrm{K}_2(2K_2^2 + \right.$$
$n2+n-2K22+n\cos\alpha 1+n\mathrm{K}2+\cos\alpha 1\cos 2\alpha 1$

(21)

$$\boldsymbol{E_r}(x,z) = \frac{\eta_1 - jZ\tan k_p d}{\eta_1 + jZ\tan k_p d} E_i[\,\mathrm{J}_1(\tau)]^{-\frac{1}{2}} exp[-jk(\Psi_0 + t)] \tag{22}$$

$$\boldsymbol{E_t}(x,z) = -\frac{2\eta_1}{\eta_1 + Z}\frac{2Ze^{-j2k_p d}}{(\eta_1+Z)+(\eta_1-Z)e^{-j2k_p d}} E_i[\,\mathrm{J}_2(\tau)]^{-\frac{1}{2}} exp\left[-jk\left(\Psi_p + \tau + t\right)\right] \tag{23}$$

where $\Psi_0 = f\frac{\cos 2\alpha_1}{\cos{\alpha_1}^2} + dsin\psi$, $\Psi_p = P\frac{\cos 2\alpha_2}{\cos{\alpha_2}^2}$, t is parameter along the ray from coordinates of point on parabolic plasma layer to focal point and τ is the distance between the $P(\xi_0,\zeta_0)$ and $Q(\xi,\zeta)$. It is observed that the GO field the reflected and the transmitted wave becomes infinity at the Caustic points as is expected when $J(t) = 0$. We can derive the expression which is valid at the focal point using the Maslov's method. The exact location of focal or caustic point may also be obtained at $J(t) = 0$. By using Maslov's method we want to find valid field expressions in focal region. In the focal regions a valid field expression is given by [9-18].

The integrand and phase function of the integral of transmitted field from plasma layer is evaluated as under

$$\mathrm{Q} = \frac{D(\tau)}{D(0)}\frac{\partial(\mathrm{p_x})}{\partial(\mathrm{x})} = \frac{\cos^3\alpha_1(\mathrm{K}_2 + n\cos\alpha_1)(K_2^2 + n + 4\mathrm{K}_2 n\cos\alpha_1 + n\cos 2\alpha_1)\sin^2\alpha_1}{4PK_2^2} \tag{24}$$

$$\mathrm{S} = S_0 + \tau + \tau - z(\mathrm{x}, p_z)\, p_z + p_z x = \tau_1 + r\,cos(\,2\alpha_2 - \theta) \tag{25}$$

$$\tau_1 = \sqrt{(\xi_0 - \xi)^2 + (\zeta_0 - \zeta)^2} \tag{26}$$

Similarly the integrand and phase function for the integral of reflected field from plasma parabolic layer can be obtained easily. The final field expressions along the focal point of plasma parabolic layer and metallic parabolic reflector are obtained as

$$\frac{E(x,z)}{E_i} = 2\sqrt{\frac{2fk}{\pi}}\int_{-l/2}^{l/2}\frac{\eta_1 - jZ\tan k_p d}{\eta_1 + jZ\tan k_p d} sec\,\alpha_1 \times exp[-jk(r\,cos(\,2\alpha_1 - \theta) + d\sin\alpha_1)]d\alpha_1 \tag{27}$$

$$\frac{E(x,z)}{E_i} = 2\sqrt{\frac{2Pk}{\pi}}\int_{-l/2}^{l/2}\frac{2\eta_1}{\eta_1 + Z}\frac{Ze^{-j2k_1 d}}{\left((\eta_1+Z)+(\eta_1-Z)e^{-j2k_1 d}\right)}\sqrt{\frac{1}{\mathrm{Q}_{2x}}\frac{d\mathrm{Q}_{2z}}{d\alpha}\frac{\mathrm{K}_2}{\cos^3\alpha}} exp[-jk(\tau_1 + r\,cos(\,2\alpha_2 - \theta))]d\alpha_2 \tag{28}$$

where $l = tan^{-1}(a/2f)$ is the angle which subtends the aperture.

3. Numerical result and discussion

In this paper, optical focused electromagnetic fields reflected and transmitted from a plasma layer coated parabolic is observed at normal incident around the Caustic points F and P. Then the effects of thickness of coating, electron density in plasma and effective collision frequency of plasma on the optical focused fields are discussed. The incident frequency of electromagnetic wave is taken as $f = 1 \times 10^9$Hz. The numerical results are compared with the published literature to check the correctness of analytical calculations and also as well the working of software pack. By using MATHEMATICA software, the computations are made of equations (27) and (28). Figure 2 shows the comparison between previous study and our study. When plasma medium is replaced by the dielectric free space then the present work transforms into the published work as in [12] and good agreement is found between analytical and then numerical calculations.

Fig. 3a and Figure 3b show the comparison of the field intensity distribution of the reflected and transmitted field from plasma layer parabolic reflector around the Caustic points F and P along z-axis for the different values of thickness of plasma layer respectively. The solid, dashed, dot and thick dashed lines show the result at $d = 0.01$ m,$d = 0.02$ m, $d = 0.03$ m and $d = 0.1$ m for reflected field and $d = 0.010$ m,$d = 0.012$ m, $d = 0.014$ m and $d = 0.016$ m for transmitted field from plasma layer parabolic reflector respectively. From Fig. 3a and Fig. 3b it is observed that the field intensity shifts to a smaller value as we decrease the thickness of plasma layer. It is also observed that that ratio of decrease of field intensity of transmitted at focal point is 10 times less than the reflected field which shows absorption of field in the plasma layer.

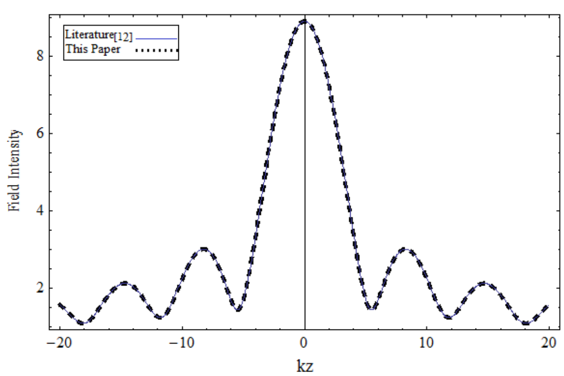
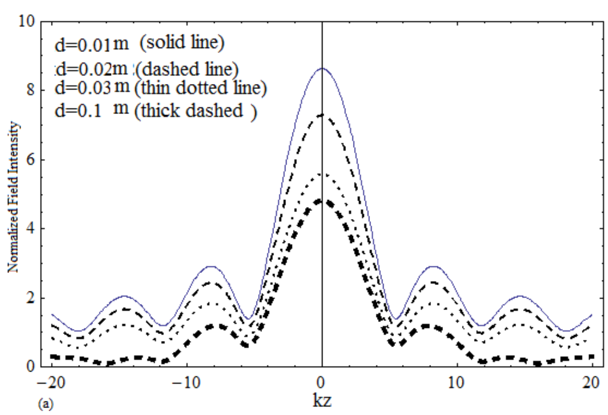


Fig. 2. Comparison of normalized field intensity of plasma layer coated parabolic reflector under special conditions (dashed line) and parabolic reflector (solid line)[12] at Caustic point.



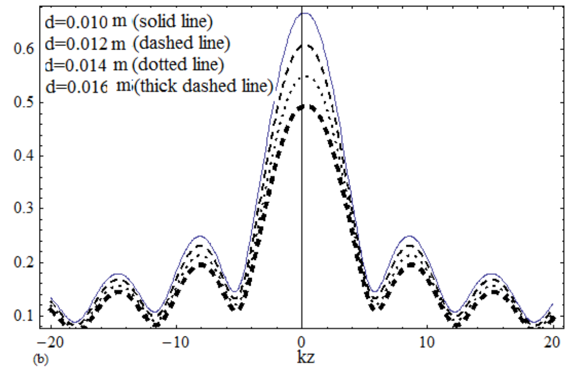


Fig. 3. Normalized field intensity distribution around focal points F and P of plasma layer coated parabolic reflector with respect to kz for different values of thickness of plasma layer (a) Reflected field (b) Transmitted field.

Fig. 4a and Figure 4b show the comparison of the field intensity distribution of the reflected and transmitted field from plasma layer parabolic reflector around the Caustic points F and P along z-axis at the different values of electron density of plasma layer. The solid, dashed, dot and thick dashed lines shows the result of $n = 1.0 \times 10^{19} m^{-3}$,$n = 5 \times 10^{18} m^{-3}$, $n = 1.0 \times 10^{18} m^{-3}$ and $n = 5.0 \times 10^{17} m^{-3}$ for reflected field and $n = 1.0 \times 10^{17} m^{-3}$,$n = 2.0 \times 10^{17} m^{-3}$, $n = 3.0 \times 10^{17} m^{-3}$ and $n = 4.0 \times 10^{17} m^{-3}$ for transmitted field from plasma layer parabolic reflector respectively. From Figure 4a it is observed that the field intensity shifts to a smaller value as we decrease the values of electron density of plasma layer. From Figure 4b it is observed that the field intensity shifts to a smaller value with minor increase in the values of electron density of plasma layer. These effects will helpful for increasing or decreasing image field at Caustic points.

Fig. 5a and Figure 5b show the comparison of the field intensity distribution of the reflected and transmitted field from plasma layer parabolic reflector around the Caustic points along z-axis at the different values of effective collision frequency of plasma layer. The solid, dashed, dot and thick dashed lines shows the result at $v = 5.0 \times 10^{10} Hz$, $v = 6.0 \times 10^{10} Hz$, $v = 7.0 \times 10^{10} Hz$and $v = 8.0 \times 10^{10} Hz$ for reflected field and $v = 0.5 \times 10^{10} Hz$, $v = 1.0 \times 10^{10} Hz$, $v = 1.5 \times 10^{10} Hz$and $v = 82.0 \times 10^{10} Hz$ for transmitted field from plasma layer parabolic reflector respectively. From Figure 5a it is observed that the field intensity shifts to a smaller value as we increase the values of effective collision frequency of plasma layer. From Figure 5b it is observed that the field intensity shifts to increase with minor increase in the values of effective collision frequency of plasma layer.

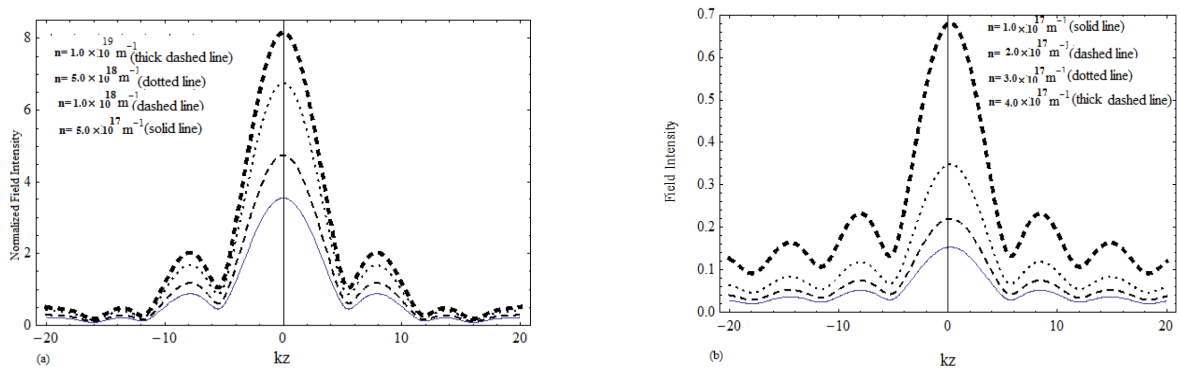


Fig. 4. Normalized field intensity distribution around focal points F and P of plasma layer coated parabolic reflector with respect to kz for different values of electron density of plasma layer (a) Reflected field (b) Transmitted field.

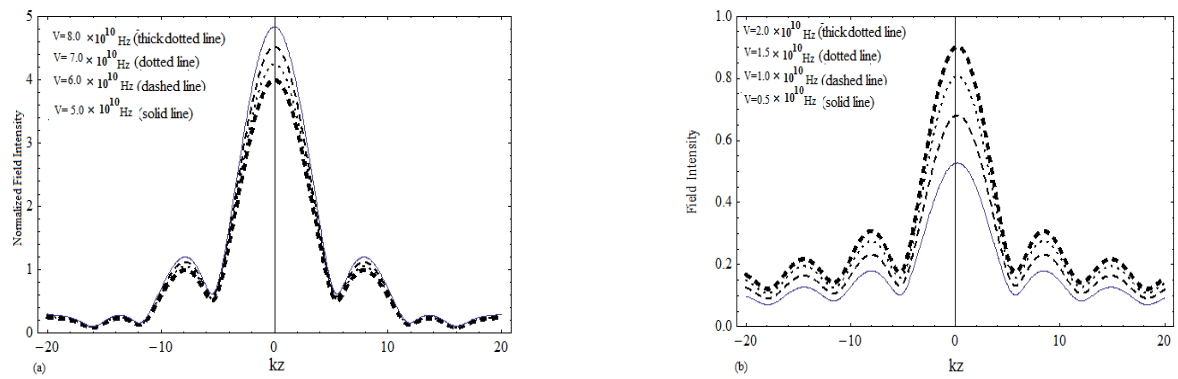


Fig. 5. Normalized field intensity distribution around focal points F and P of plasma layer coated parabolic reflector with respect to kz for different values of effective collision frequency of plasma layer (a) Reflected field (b) Transmitted field.

4. Conclusion

In this paper, the reflection and transmission of electromagnetic wave propagating through a plasma layer coated parabolic reflector for normal incidence is analyzed numerically by using Maslov's method. The reflection and transmission coefficients for free space-plasma and plasma-perfect electric conductor interfaces are derived analytically. The field distribution is presented in the form of numerical results for different parameters to clarify the focusing behavior of plasma layer coated parabolic reflector under normal angle of incidence. It is cleared that the normalized reflected and transmitted field intensity of plasma layer coated parabolic reflector around the Caustic points along the z-axis decreases by increasing the thickness of plasma layer. The normalized reflected field intensity decreases as the value of electron density decreases whereas normalized transmitted field decrease with the increase value of electron density around the Caustic point decreases along z-axis of plasma layer coated parabolic reflector . The normalized reflected field intensity decreases as the value of the collision effect frequency increases whereas normalized transmitted field decreases with the decrease in value of collision effect frequency. The absorption of field in the plasma layer is also observed. These results can be helpful in the study and designing of optical devices.

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