

Applications of resonant piezoelectric devices to the measurement of physical quantities

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The most common applications of piezoelectric resonators as sensors are summarised. After a short presentation of piezoelectric resonator, the principle of each kind of device and the main relation between the measured quantity and the obtained frequency shift is given. The devices have been classified in two categories according to the fact that the variation of frequency involves, or not, the non linear properties of the material of the resonator.

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1. Introduction

Piezoelectric resonators, and more specially quartz resonators, are well known for their stability of resonant frequency. This stability comes from their low sensitivity to external perturbations. Despite this fact, they can be used to make sensors, because the frequency shift arising from the physical quantities to be measured is great in comparison with the intrinsic fluctuations of frequency of the resonator. In most applications of piezoelectric resonators as sensors, the output signal is a variation of frequency. One speaks of ‘frequency output sensor’. One of the advantages of this kind of device is that it is very easy to ‘count’ the frequency of a signal with a high accuracy.

The basic study of a piezoelectric resonator is made by using the linear relations, in tensorial notation, [1,2]

$$\begin{aligned} T_{ij} &= c_{ijkl} S_{kl} - e_{kij} E_k \\ S_{ij} &= s_{ijkl} T_{kl} + d_{kij} D_k \\ D_i &= \varepsilon_{ij} E_j + e_{ikl} S_{kl} \end{aligned} \quad (1)$$

and the equilibrium equations

$$\begin{aligned} \rho \frac{\partial^2 u_i}{\partial t^2} &= \frac{\partial T_{ij}}{\partial x_j} \\ \frac{\partial D_i}{\partial x_i} &= 0 \end{aligned} \quad (2)$$

with the proper boundaries conditions. It can be proof [2] that the addition of the piezoelectric effect doesn’t create new modes of resonance of the purely mechanical case, but changes only the resonant frequency.

In the vicinity of a resonance, a piezoelectric resonator is equivalent, in the electrical point of view, to the Butterworth-Van Dike circuit (Fig. 1).

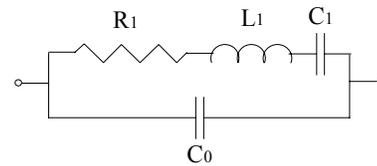


Fig. 1. Butterworth-Van Dike equivalent circuit.

R_1 , L_1 , C_1 are motional parameters. C_0 is the dielectric capacitance. By using expansion of the strain S_{ij} and electric field E_i unto the set of eigenfunctions, Lewis [3] has established the very general expression for the ratio $\frac{C_1}{C_0}$ (equal to the square of the electromechanical coupling factor).

$$\frac{C_1}{C_0} = \frac{\left(\int_B e_{kij} E_k^s S_{ij}^n dB \right)^2}{\left(\int_B \varepsilon_{ij} E_i^s E_j^s dB \right) \left(\int_B c_{ijkl} S_{ij}^n S_{kl}^n dB \right)} \quad (3)$$

The superscript n stands for the strain of the nth mode of vibration and the superscript s for the static electric field. B is the volume of the resonator. In addition we have the following relations for the resonant frequency ω_n and the quality factor Q :

$$L_1 C_1 \omega_n^2 = 1; \quad R_1 = \frac{1}{Q C_1 \omega_n} \quad (4)$$

From [3], we can express the amplitude of vibration if the device is driven by a voltage $V = V_0 e^{j\omega t}$ between the electrodes:

$$u_i(x_j, t) = a_n u_i^n(x_j) e^{j\omega t} \quad (5)$$

$$a_n = \frac{\omega^2 V_0 \int_B e_{kij} S_{ij}^n E_k^s dB}{\rho \omega_n^2 (\omega^2 - \omega_n^2) \int_B u_i^n u_i^n dB} \quad (6)$$

where E^s is the static electric field when a unit voltage is applied between the electrodes. Conversely, if the electrodes are short circuited when the device vibrates on the n th mode, the short circuit current is given by

$$I = -\omega \int_B e_{kij} E_k^s S_{ij}^n dB \quad (7)$$

2. Sensors based on modification of boundary conditions

2.1 Mass measurement

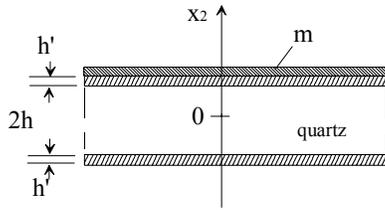


Fig. 2. Quartz crystal microbalance.

Quartz crystal microbalance (QCM) use high frequency plates resonators vibrating in thickness shear modes. The mode is the slow shear mode (C mode) for which there exist cuts that are temperature compensated. The most commonly used cut is the singly rotated AT-Cut ($Y+35^\circ 20'$) and the doubly rotated SC-Cut ($\Phi = 22^\circ$, $\theta = 34^\circ$) [4].

Both cuts possess a cubic frequency-temperature curve (i.e. nulls first and second order temperature coefficients). The SC-Cut [5] has been introduced to reduce the frequency shift arising from the stress due to the difference of thermal expansion between the quartz crystal and the deposited layer on it.

The computation of the sensitivity of the QCM is made by considering the case of the infinite parallel plate resonator (Fig. 2). Both sides are covered by electrodes of thickness h' and mass per unit volume ρ' . The added mass m per unit surface to be measured is deposited on the upper face.

The mechanical displacement u , not necessary in the plane of the resonator, is of the form:

$$u(x_2) = A \sin(kx_2) + B \cos(kx_2) \quad (8)$$

The resonant frequency ω is related to k by

$$\omega = k \sqrt{\frac{C_e}{\rho}} \quad (9)$$

where C_e is the effective elastic coefficient given by the eigenvalue of the Christoffel matrix [1,2], associated to the mode of vibration, and ρ is the mass density of the crystal. In case of thin of the electrodes and the deposited layer, the wave propagation in it is negligible and the mechanical boundaries conditions are:

$$\begin{cases} kC_e (A \cos(kh) - B \sin(kh)) = (\rho' h' + m) \omega^2 (A \sin(kh) + B \cos(kh)) \\ kC_e (A \cos(kh) + B \sin(kh)) = \rho' h' \omega^2 (A \sin(kh) - B \cos(kh)) \end{cases} \quad (10)$$

The exact values of kh are obtained by finding the roots of the determinant of the above system.

By putting, for the n^{th} root

$$k_n h = n \frac{\pi}{2} - \delta_n \quad (11)$$

where δ_n is a small quantity, the above system can be linearized in δ_n . One obtains

$$\delta_n = \frac{\rho' h'}{\sqrt{\rho C_e}} \omega_n + \frac{m}{2\sqrt{\rho C_e}} \omega_n \quad (12)$$

Finally by putting this result in (9), we can get the relation between the added mass m and the variation Δf of frequency of the resonator:

$$m = -\frac{n}{2} \sqrt{\rho C_e} \frac{\Delta f}{f_n^2} \quad (13)$$

This is known as the Sauerbrey effect. Since f_n is proportional to the rank of overtone n , the best sensitivity of the QCM is obtained for $n = 1$ (fundamental mode).

2.2 Force frequency transducer

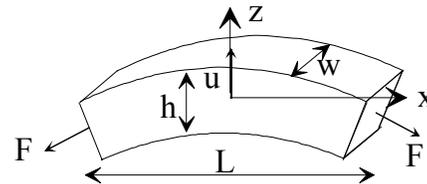


Fig. 3. Geometry of beam.

Low frequency force to frequency transducers use resonator vibrating in flexure mode. The vibration of a beam, when a static force F is exerted on it (Fig. 3), is governed by the equation [6]:

$$\frac{\partial^4 u}{\partial x^4} - \frac{12F}{\rho h^3 Y} \frac{\partial^2 u}{\partial x^2} + \frac{12\rho\omega^2}{h^2 Y} = 0 \quad (14)$$

h is the height of the beam and $Y = \frac{1}{s_{2222}}$ is the Young modulus in the x_2 direction. If L is the length of the beam, the frequency of resonance is, if no force is exerted:

$$\omega_0 = a_n \frac{h}{L^2} \sqrt{\frac{Y}{\rho}} \quad (15)$$

a_n is a constant which, for the fundamental mode of a clamped-clamped beam is: $a_1 = 6.45$

If a force F is applied, one gets an angular frequency deviation $\Delta\omega$ which is not strictly a linear function of F [7]. To the first order:

$$\Delta\omega = 0.948 \sqrt{\frac{1}{\rho Y} \frac{F}{wh^2}} \quad (16)$$

To be usable, this kind of transducer must be made with a resonator having a first order temperature coefficient equal to zero.

$$\omega(T) = \omega(T_0) (1 + \beta(T - T_0)^2) \quad (17)$$

For quartz, this is achieved by using an Y+0 cut [4]. The length lies in the YZ plane, making an angle θ (about 1.5°) with respect to the Y crystallographic axis. β is then about equal to $-70.10^{-9}/^\circ\text{C}$ [8].

Another condition is to have a good Q factor in order to have a reasonably low motional resistance. For that, the device must be balanced. In Fig. 4a, the displacements lie in the plane of the device. In Fig. 4b, the displacements are perpendicular to the plane; the width of the central beam is twice the one of other beams. Corresponding electrodes that can drive the vibrations are depicted on Fig. 5a and 5b respectively.

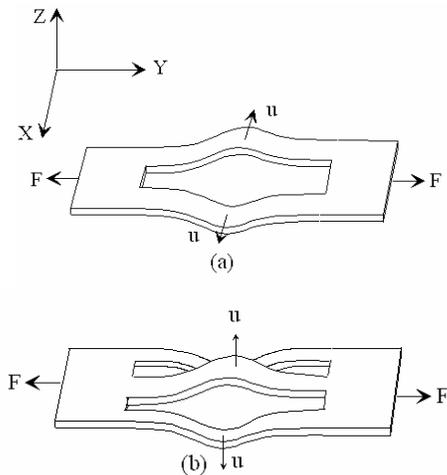


Fig. 4. Balanced transducers.

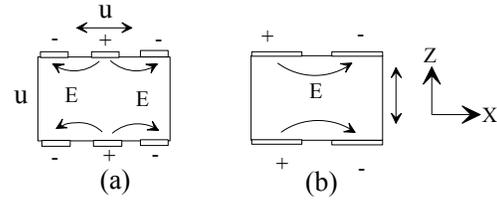


Fig. 5. Electrodes arrangement.

2.3 Angular rate sensor

The use of quartz resonators vibrating in flexure mode to measure an angular velocity was first introduced by Systron –Donner. This kind of sensors is based on the Coriolis effect.

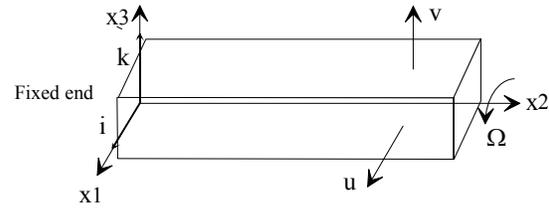


Fig. 6. Angular rate sensor.

Let us consider a tine of rectangular cross section, clamped at $x_2 = 0$ (fig.3).

We assume that the deformation of a section of the tine is negligible. The transverse displacement is

$$\vec{r} = u\vec{i} + v\vec{k} = q_1(t)\phi_1(x_2)\vec{i} + q_3(t)\phi_3(x_2)\vec{k} \quad (18)$$

$\phi_i(x_2)$ is the mode shape of the tine in the x_i direction and $q_i(t)$ is a time depending amplitude factor.

When the tine rotates, the absolute velocity of a point expressed in a fixed coordinate system is related to the velocity in the frame (x_i) by

$$\vec{v}_a(t) = \left(\frac{\partial \vec{r}}{\partial t} \right)_{/Rr} + \vec{\Omega} \times \vec{r} \quad (19)$$

The use of the Hamilton principle, or the Lagange's equations, conducts to the following system

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} \beta_1 & -2\Omega \\ 2\Omega & \beta_3 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} \omega_{01}^2 - \Omega^2 & 0 \\ 0 & \omega_{03}^2 - \Omega^2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (20)$$

ω_{01} and ω_{03} are the eigenfrequencies of the vibrations in x_1 and x_3 direction. β_1 , β_3 are damping coefficients. The time is driven in such a manner so that the amplitude of vibration in the x_1 (drive mode) is constant,

$$q_1(t) = a \sin(\omega t) \quad (21)$$

Under the rotation Ω , a vibration in the x_3 direction is induced.

$$q_3(t) = b \sin(\omega t + \varphi) \quad (22)$$

The sensitivity of the gyrometer is obtained by putting (21) and (22) in (20) and by solving the resulting system for b . One gets

$$\delta = \frac{b}{a} = \frac{2\Omega}{\left[\left[1 - \left(\frac{\omega_{03}}{\omega} \right)^2 \right]^2 + \left(\frac{\omega_{03}}{Q\omega} \right)^2 \right]^{\frac{1}{2}}} \quad (23)$$

where we have introduced the Q factor of the sensing mode

$$Q = \frac{\omega_{03}}{\beta_3} \quad (24)$$

The usual way is to drive the sensor at the frequency $\omega = \omega_{01}$. The maximum of sensitivity is obtained if $\omega_{01} = \omega_{03}$. But in this case δ is proportional to the Q factor of the sensing mode. It is then difficult to have a good stability of it. In most practical applications, the frequencies of the driving and sensing modes are slightly different. The sensitivity is then practically independent of Q

$$\delta = \frac{2\Omega}{\left[1 - \left(\frac{\omega_{03}}{\omega_{01}} \right)^2 \right]} \quad (25)$$

If the sensor is made in a Z-cut plate, the driving and sensing electrodes can be similar to that of figure 5. The measure of the amplitude of the sensing mode is in most application made by using a charge amplifier to convert the current to a voltage. In this case the output current can be computed with the help of (7), and the amplitude of the drive mode by (6).

3. Sensors based on modification of material properties

Unlike the preceding cases, a second kind of sensors is based on the nonlinearity of the properties of the material. It means that the frequency shift is related to high

order coefficients. The order of the coefficient is defined by considering the polynomial expansion of the internal energy, or another thermodynamic function, with respect to the pertinent independent variables [9]. Usual elastic, dielectric and piezoelectric coefficients appear in quadratic terms, and are defined as second order coefficients (linear coefficients). In that sense, coefficients of first order thermal sensitivity of elastic coefficients are of third order.

This kind of sensors is made with high frequency plate resonators, vibrating in thickness mode. To have good performances, these resonators are of trapped energy type. That means the vibration is only located in the centre of the plate and has a non uniform distribution. This is obtained by using the mass loading effect [10] or by using plano-convex plates.

The two main kinds of sensors are thermal and force sensors.

3.1 Resonator submitted to a static deformation

To study resonators submitted to an external action that modify the velocity of acoustic wave, three states of the resonator are considered.

1. A natural state, in the reference configuration, without vibration. The body occupies the volume V_0 , bounded by the surface S_0 . The coordinates of a material point are a_i , the mass per unit volume is ρ_0 .

2. An initial state, also static, when the resonator is submitted to a change of temperature or to a static stress. V_1 is the occupied volume, bounded by the surface S_1 . ρ_1 denotes the mass per unit volume, and the coordinates of a point are X_i . $U_i = X_i - a_i$ is the static displacement.

3. A final state, when a small vibration, not considered as infinitesimal, is superimposed to the initial state.

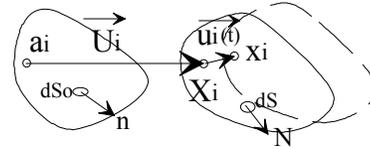


Fig. 7. The three states of a resonator.

They are two ways to treat the problem. The first one is to use the coordinates X_i in the initial state. In this case the size, mass per unit volume and orientation of the resonator with respect to the crystallographic axis, are submitted to changes when a static action is exerted on the resonator. The second one uses the coordinates a_i in the natural state as independent variables (Lagrange's formulation). In this case the size, mass per unit volume and orientation are constants. To do that, one must use the first Piola-Kirchoff stress tensor P_{ij} , related to the usual Cauchy stress tensor T_{kl} by the relation [9]

$$P_{ij} = J \frac{\partial a_i}{\partial X_k} T_{kj} \quad (26)$$

where J is the Jacobian

$$J = \det \left| \frac{\partial X_i}{\partial a_j} \right| \quad (27)$$

The fundamental law of mechanics is then

$$\rho_0 \ddot{u}_i = \frac{\partial P_{ij}}{\partial a_j} \quad (28)$$

and the boundaries condition are

$$f_j = P_{ij} n_i dS_0 \quad (29)$$

One can also defines material, or Lagrangian, electric field W and electric displacement Δ [9,11]:

$$\begin{aligned} W_i &= \frac{\partial X_k}{\partial a_i} E_k \\ \Delta_i &= J \frac{\partial a_i}{\partial X_j} D_j \end{aligned} \quad (30)$$

3.2 Force-frequency effect

In this section we consider a resonator submitted to an external static force. The resulting effect on the piezoelectric resonator is only a change of its resonant frequency due to the change of the apparent elastic coefficients. So we consider a purely mechanical problem. The strain of the body is described by the Green strain tensor.

$$G_{ij} = \frac{1}{2} \left(\frac{\partial x_k}{\partial a_i} \frac{\partial x_k}{\partial a_j} - \delta_{ij} \right) \quad (31)$$

According to the three states introduced in the above section, the Green and the Piola tensors can be splited in a static part plus a dynamic part

$$\begin{aligned} G_{ij} &= \bar{G}_{ij} + \tilde{G}_{ij}(t) \\ P_{ij} &= \bar{P}_{ij} + \tilde{P}_{ij}(t) \end{aligned} \quad (32)$$

It can be shown that the fundamental law of dynamic for the vibration is

$$\frac{\partial \tilde{P}_{kj}}{\partial a_k} = \rho_0 \frac{\partial^2 u_j}{\partial t^2} \quad (33)$$

where $u_i = x_i - X_i$ is the time depending displacement corresponding to the vibration.

To express the Piola tensor, it is usual to consider the following expression of the energy [9,11]

$$\rho_0 U = c_{ijkl} G_{kl} + \frac{1}{2} c_{ijklmn} G_{kl} G_{mn} + \dots \quad (34)$$

The elastic coefficients c_{ijkl} and c_{ijklmn} are the second (linear) and third (nonlinear) order coefficients. They are sometime labeled material or fundamental constants. The Piola tensor is obtained from this expression of the energy. For its dynamic part, one gets [12]:

$$\tilde{P}_{kj} = A_{kjli} \frac{\partial u_i}{\partial a_l} \quad (35)$$

The A_{kjli} coefficients are effective elastic coefficients

$$A_{kjli} = c_{kjli} + T_{kl} \delta_{ij} + c_{kjlimn} G_{mn}^1 + c_{kjml} \frac{\partial U_j}{\partial a_m} + c_{kmli} \frac{\partial U}{\partial a_m} \quad (36)$$

where

$$G_{mn}^1 = \frac{1}{2} \left(\frac{\partial X_k}{\partial a_i} \frac{\partial X_k}{\partial a_j} - \delta_{ij} \right) \quad (37)$$

is the static Green tensor and T_{kl} the static stress.

In a trapped energy resonator, the vibration u has a non uniform distribution. To compute the frequency shift arising from the external force exerted on the resonator, one can use a perturbation method [13]. To do that, the Piola tensor is separated into a linear part \tilde{P}_{kl}^l (containing the second order coefficients) and a non linear part \tilde{P}_{kl}^{nl} :

$$\tilde{P}_{kj} = \tilde{P}_{kj}^l + \tilde{P}_{kj}^{nl} \quad (38)$$

$$\tilde{P}_{kj}^l = c_{kjli} \frac{\partial u_i}{\partial a_l} \quad (39)$$

$$\tilde{P}_{kj}^{nl} = \left(T_{kl} \delta_{ij} + c_{kjlimn} G_{mn}^1 + c_{kjml} \frac{\partial U_i}{\partial a_m} + c_{kmli} \frac{\partial U_j}{\partial a_m} \right) \cdot \quad (40)$$

$$\frac{\partial u_i}{\partial a_l} = \hat{c}_{kjli} \frac{\partial u_i}{\partial a_l}$$

By expanding the displacement u on the set of the normalized eigenmodes u^n of the unperturbed problem, solution of

$$\frac{\partial \tilde{P}_{kj}^l}{\partial a_k} = -\rho_0 \omega_n^2 u_j^n \quad (41)$$

with the proper boundaries conditions and the normalizing condition

$$\int_{V_0} \rho_0 u_i^n u_j^n dV_0 = \delta_{ij} \quad (42)$$

and using the usual perturbation method, the frequency shift is finally given by :

$$\Delta \omega = \frac{\int_{V_0} \hat{c}_{kjli} \frac{\partial u_i^n}{\partial a_l} \frac{\partial u_j^n}{\partial a_k} dV_0}{2 \omega_n} \quad (43)$$

where ω_n is the unperturbed resonant frequency.

Even the relative frequency shift is small, it is possible to build a frequency output force sensor, based on a piezoelectric resonator because they exist cuts which are quite insensitive to the temperature.

In doubly rotated cut, like SC cut, it is also possible to use two modes, one sensitive to the force to be measured with a low temperature sensitivity and a second mode as thermal “built-in” sensor [14].

3.3 Thermal sensor

In a general manner, the n th order thermal sensitivity being defined by

$$T^{(n)}X = \frac{1}{n!} \frac{1}{X} \frac{dX}{dT} \quad (44)$$

The variation of the frequency of a resonator with respect of the temperature is written as

$$f_{nmp}(T) = f_{nmp}(T_0) \left(1 + \sum_{j=1}^3 T^{(j)} f(\Delta T)^j \right) \quad (45)$$

In this case the static deformation arises from the temperature change $\Delta T = T - T_0$ between the natural and the initial state. The usual way to compute the thermal coefficients $T^{(j)}$ is to use the expression of the frequency computed in the initial state and to derive it with respect of temperature. In this case, all quantities, elastic coefficients as well as size, mass per unit volume and even orientation of the resonator, are functions of T .

If, unlike to proceed by this way, one use the Lagrange’s formulation of the problem, it is possible to obtain an expression of the frequency of the resonator where all “geometric” quantities are those in the natural state, i.e. independent of T . To do that we use the material coordinates a_i as independent coordinates.

It can be established that the constitutive relations for the increment of Piola stress tensor and material electric displacement are [15]:

$$\begin{aligned} \tilde{P}_{ij} &= G_{ijkl} \frac{\partial u_l}{\partial a_k} - R_{mji} \tilde{W}_m \\ \tilde{\Delta}_i &= N_{ij} \tilde{W}_j + R_{ijk} \frac{\partial u_j}{\partial a_k} \end{aligned} \quad (46)$$

where \tilde{W}_m and $\tilde{\Delta}_i$ are the dynamic increment of material electric field and electric displacement.

The G_{ijkl} coefficients have a lower symmetry with respect to the indices than the usual elastic coefficients. In matrix notation the G_{ij} matrix is a 9 by 9 matrix. They are numerically equal to the usual elastic coefficients c_{ij} at the reference temperature T_0 , but possess different thermal sensitivities. They are related to the usual elastic

coefficients and thermal expansion coefficients [15]. The computation of the frequency of a resonator is quite similar to that of the classical treatment. One gets in the case of a resonator with flat parallel faces

$$f_n^2 = \frac{n^2}{4h_0^2} \frac{G_e}{\rho_0} \left(1 - 8 \frac{k^2}{n^2 \pi^2} \right) \quad (47)$$

G_e is the effective elastic coefficient associated to the propagation in the direction perpendicular to the plate [16] and k is the piezoelectric coupling factor. k involves the piezoelectric coefficients. In the case of quartz crystal, their thermal sensitivities are small and not well known. It is commonly accepted to neglect the thermal variation of k . Under this approximation, in Lagrange’s formulation, the thermal coefficients of the temperature are simply, but exactly

$$\begin{aligned} T^{(1)}f &= \frac{1}{2} T^{(1)} G_e \\ T^{(2)}f &= \frac{1}{2} T^{(2)} G_e - \frac{1}{8} [T^{(1)} G_e]^2 \\ T^{(3)}f &= \frac{1}{2} T^{(3)} G_e - \frac{1}{4} T^{(1)} G_e T^{(2)} G_e + \frac{1}{16} [T^{(1)} G_e]^3 \end{aligned} \quad (48)$$

For quartz resonators, it is possible to find cuts having a first, and sometime a second, order thermal coefficient that is equal to zero. The most know, and used cuts, are the singly rotated AT-cut ($\theta = 35^\circ 21'$) and the doubly rotated SC-cut ($\phi = 22^\circ$, $\theta = 34^\circ$). It is also possible to find cuts which exhibit a quite linear frequency to temperature behaviour. The non rotated Y cut is sometime used, but cuts like LC-cut introduced by Hewlett-Packard [17] and the singly rotated NLC-cut ($\phi = 0^\circ$, $\theta = -35.5^\circ$) [18] have a better linearity. All these cuts are working on the slow shear mode (C mode). It can be noticed that the fast shear mode (B mode) of the SC cut can also be used as ‘built in’ thermal sensor for this cut.

4. Conclusions

The most common applications of piezoelectric resonators as sensors are summarised. After a short presentation of piezoelectric resonator, the principle of each kind of device, the main relation between the measured quantity and the obtained frequency shift are given. The devices have been put in two categories according to the fact that the variation of frequency involves, or not, the non linear properties of the material of the resonator.

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