Band structure and transmission characteristics of onedimensional photonic crystal composed with singlenegative materials

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Band structure and transmittance characteristics of one-dimensional photonic crystal composed with single negative materials (MNG and ENG) are studied by using simple transfer matrix method and Bloch theorem. The band structure and the transmittance for TE and TM-modes are studied for the structure with MNG and different values of electric permittivity (ϵ) when ENG and magnetic permeability (μ) is constant and vice-versa. To obtain a large band gap, the structure should have high ϵ and MNG with large thickness, and high μ and ENG with small thickness. The study of the transmittance shows the tunneling property of electromagnetic wave due to presence of zero- ϵ and zero- μ . The existence of zero- ϵ and zero- μ are revealed due to the band gap is not found in the same frequency range for the transmittance. Besides this, we have studied the transmittance of the structure of MNG-ENG with different values of ϵ and μ , angles of incidence, and thicknesses. By choosing proper thick ness of the structure and angle of incidence, a large omni-directional band gap can be achieved, which can be used as a filter.

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1. Introduction

In 1968, Veselago [1] first time proposed a peculiar medium possessing a negative refractive index (NIM), which contains simultaneously the negative permittivity (- ε) and the negative permeability (- μ) at certain frequency, and NIM is also known as the lefthanded material (LHM) [2, 3]. Experimentally it has been observed that the negative permittivity $(-\varepsilon)$ and negative permeability $(-\mu)$ of the NIM are frequency dependent. Other kind of materials is also existed, which has either negative permittivity $(-\varepsilon)$ with constant μ or negative permeability (- μ) with constant ε , called the single-negative (SNG) material. The possible materials of SNG are: (i) $\varepsilon < 0$ (ENG) and $\mu > 0$ (ii) $\varepsilon > 0$ and $\mu < 0$ (MNG). The single negative materials have unique properties as compared with the double negative in NIM. We know that the photonic crystals (PCs) are the artificial positive index materials with periodic modulation of the dielectric constants [4]. In periodic structure, the band-gap structures are determined by the symmetry, dielectric constant of material, and the scale length of crystal lattice of the PCs [4]. But the band structure can also be changed by changing (ϵ) and (μ) because $n_i^2 = \epsilon_i \mu_i$. The transmission properties of photonic crystals consisting of

mu-negative material (MNG) and positive-index material (PIM) showed that there can be existed a transmission band inside a single-negative gap. The width of the transmission band is only dependent on the thickness of the MNG layers and width of the transmission band controllable by thickness. The tunneling mode is localized strongly inside the PIM layers with the increase in the thickness of the mu-negative layers. Transmittance can be increased by decreasing the thickness of the MNG layers [5].

The heterostructure can be achieved a tunable zerophase-shift for omnidirectional filter. By simply adjusting the thickness of the defect layer of air, one can achieved adjustability of the tunneling mode. The omnidirectional tunneling modes in the heterostructure consisting of layered single-negative materials and one layer of defect has been discussed by Tong et al. [6]. The electromagnetic properties of one-dimensional photonic crystals with two kinds of single negative (SNG) materials have been investigated and it was observed that the photonic crystals have zero-effective phase gap as well as angular gap. The zero-effective phase gap is invariant with the incident angle and lattice scaling for different polarizations. The gap can be adjusted by varying the ratio between the two single negative material's thicknesses. The angular gap is highly dependent on the incident angle and polarization, but it is insensitive to the ratio between the two single

negative material's thickness and lattice scale length [7]. The propagation of light waves in one-dimensional photonic crystals (1DPCs) composed of alternating layers of two kinds of single-negative materials have discussed that the phase velocity is negative when the frequency of the light wave is smaller than the certain critical frequency ω_{cr} , while the Poynting vector is always positive [8].

The thickness-dependent photonic bandgap for a onedimensional photonic crystal consisting of two different single-negative (SNG) materials was studied theoretically by Yeh and Wu [9]. The two SNG materials include one with a single-negative permittivity (ε <0, μ >0) and the other with a single-negative permeability (ε >0, μ <0) have been considered to calculate the size of the bandgap and the positions of the band edges, which are strongly dependent on the thickness ratio of the two constituent SNG layers. By using the composite right/left-hand transmission line model, they have qualitatively explained the bandgap's shifting behaviors due to the variation of the thickness.

The transverse-electric wave propagation through lossless trilayer stacks containing single-negative (SNG) materials in which only one of the two material constants, negatively permittivity (epsilon) or negatively permeability (mu), is also analyzed by considering the ENG/MNG/ENG, following combinations: ENG/DPS/MNG, DPS/ENG/DPS, and ENG/DPS/ENG, where ENG refers to epsilon-negative, MNG to munegative, and DPS to double-positive media by ref. [10]. The reflection phase difference between TE and TM waves in one-dimensional photonic crystals composed of single-negative (SNG) materials (permittivity- or permeability-negative) have been discussed within two omni-directional gaps, where the reflection phase difference is changed smoothly and increased with the increasing of the incident angle [11].

Recently, Castaldi et al. [12] have shown that resonant tunneling of electromagnetic fields can be occurred through a three-layer structure composed of a single-negative (i.e., either negative permittivity or negative permeability) slab paired with a bilayer made of (i.e., positive double-positive permittivity and permeability) media. The study results demonstrated that the counterintuitive tunneling phenomenon is also the possibility of synthesizing double-positive slabs, which is effectively exhibited single-negative like wave-impedance properties within a moderately wide frequency range. The band structure and bandgaps of one-dimensional Fibonacci quasicrystals composed of epsilon-negative materials and mu-negative materials are also studied to show an omnidirectional bandgap (OBG) existing in the Fibonacci structure. In contrast to the Bragg gaps, such an OBG is insensitive to the incident angle and the polarization of light, and the width and location of the OBG cease to change with increasing Fibonacci order, but vary with the thickness ratio of both components, and the OBG closes when the thickness ratio is equal to the golden ratio [13].

The transmission properties of structures with one or two kinds of lossy single-negative (permittivity-negative and permeability-negative) materials are analyzed to show the transmission of the structure, which is dependent on the material absorption and reflection. In sharp contrast to lossy dielectrics, the reflection of the lossy single-negative material(s) can be decreased as the dissipation coefficient increases. As a result, the transmission of the lossy singlenegative material(s) will be nonmonotonic as the dissipation coefficient varies. In particular, the transmission can be enhanced even when the dissipation coefficient increases [14].

The SNG has unique properties compared to doublenegative materials. We have calculated the band structure and transmittance characteristics of onedimensional photonic crystal composed with singlenegative materials (MNG and ENG) by using simple transfer matrix method and Bloch theorem. The band structure and the transmittance of the structure for TE and TM-modes are studied by varying the angle of incidence and thicknesses when the μ of the ENG and ϵ of the MNG are increased. Our result revel that the transmittance and band structure of SNG material are not found at same band edges due to presence zero- ϵ and zero- μ .

2. Methodology and physical model

We know the square of the refractive index of the material is equal to the multiple products of electric permittivity (ϵ) and magnetic permeability (μ) of the material. The refractive index is written as

$$\mathbf{n_i}^2 = \boldsymbol{\varepsilon}_i \,\boldsymbol{\mu}_i \,\,, \tag{1}$$

where ε_i and μ_i are the electric permittivity and magnetic permeability, respectively.

We have considered a one-dimensional photonic crystal (1DPC): (AB)^N consisting of alternating layers of MNG and ENG with N=16, as shown in the figure 1, where A and B are MNG and ENG, respectively, and N is the period number. The permittivity and the permeability take the forms of two different single-negative (SNG) materials: (i) MNG with a single-negative permittivity $(\epsilon_A > 0, \mu_A < 0)$ and (ii) ENG with single-negative permeability ($\epsilon_B < 0$, $\mu_B > 0$) [15]. The thicknesses of MNG and ENG layers are d_A and d_B, respectively, and the thickness of the period structure is $d=d_A+d_B$. The electric permittivity and the magnetic- permeability of the material A (MNG) are ε_A =constant and μ_A =1-(ω_{mp}/ω)², where ω_{mp} ($\omega < \omega_{mp}$) is magnetic plasma frequency, and similarly electric permittivity and the magnetic permeability of the material B (ENG) are $\varepsilon_{\rm B}=1-(\omega_{\rm ep}/\omega)^2$ and μ_B =constant, where ω_{ep} ($\omega < \omega_{ep}$) is electric plasma frequency. The permittivity and permeability of the MNG take in the form

$$\varepsilon_{\rm A} = \alpha \text{ and } \mu_{\rm A} = 1 - (\omega_{\rm mp}/\omega)^2$$
, (2)

where α =1, 2, and ω_{mp} is magnetic plasma frequency, and incident frequency $\omega < \omega_{mp}$.

Here, $\varepsilon_B = 1 - (\omega_{ep}/\omega)^2$ and $\mu_B = \beta$ (3) where $\beta = 1$, 2, and ω_{ep} is electric plasma frequency, and incident frequency (ω)< ω_{ep} .



Fig. 1. Schematic diagram of one-dimensional PC containing single negative materials.

For study the band structure and transmittance of the one-dimensional periodic structure containing MNG-ENG, we have used the simple transfer matrix formulation and Bloch theorem. A plane wave is injected from a vacuum into the SNG materials with an incident angle θ with respect to +z direction. In general, the electric and magnetic fields at two arbitrary position z and z+ Δz in the same layer can be related via a transfer matrix

[1,15,16,18] that gives

$$M_{j}(\Delta z, \omega) = \begin{pmatrix} \cos[k_{z}^{j}\Delta z] & i\frac{1}{q_{j}(\omega)}\sin[k_{z}^{j}\Delta z] \\ iq_{j}\sin[k_{z}^{j}\Delta z] & \cos[k_{z}^{j}\Delta z] \end{pmatrix},$$
(4)

where $k_z^j = \frac{\omega}{c} \sqrt{\varepsilon_j} \sqrt{\mu_j} \sqrt{1 - (\sin^2 \theta / n_j^2)}$ is the z

component of the wave vector k_z in the jth layer. For TE

and TM polarized waves, we have

$$q_j = \sqrt{\varepsilon_j} / \sqrt{\mu_j} \sqrt{1 - (\sin^2 \theta / n_j^2)}$$
 and

 $q_j = \sqrt{\mu_j} / \sqrt{\varepsilon_j} \sqrt{1 - (\sin^2 \theta / n_j^2)}$, respectively. Therefore the emplitudes of the transmitted and reflected

Therefore, the amplitudes of the transmitted and reflected waves can be connected with that of the incident wave by the matrix

$$X_N = \prod_{j=1}^N M_j(\boldsymbol{d}_j, \boldsymbol{\omega}).$$
 (5)

The transmission coefficient of the wave passing through this MNG-ENG into vacuum can be written as

$$t(\omega) = \frac{2\cos\theta}{\cos\theta [x_{11}(\omega) + x_{22}(\omega)] + i [\cos^{2\theta} x_{12}(\omega) - x_{21}(\omega)]}$$
(6)

where $x_{i,j}$ are the elements of the matrix X_N . The

transitivity of MNG-ENG with air substrate is $T = |t|^2$. The transverse magnetic (TM) wave can be treated in a similar way.

For an infinite periodic structure, according to Bloch's theorem, the dispersion at any incident angle obeys the relation [15]

$$K_{z}(\omega) = \frac{1}{(d_{A}+d_{B})} \cos^{-1} \left[\cos\left(k_{A}^{z}d_{A}\right) \cos\left(k_{B}^{z}d_{B}\right) - \frac{1}{2} \left(\frac{q_{B}}{q_{A}} + \frac{q_{A}}{q_{B}}\right) \sin\left(k_{A}^{z}d_{A}\right) \sin\left(k_{B}^{z}d_{B}\right) \right]$$

$$\tag{7}$$

where K_z is the z- component of the Bloch wave vector, and d_A and d_B are the thicknesses of layers A and B, respectively. The condition of $|\cos[K_z(d_A + d_B)]\rangle$ corresponds to the band gap of 1DPC and is known as the Bragg condition. We have used the equations (6) and (7) to calculate the transmittance and band structure of the periodic structure containing MNG-ENG materials, respectively.

3. Results and discussion

The one-dimensional photonic crystal (1DPC) has considered (AB)^N consisting of alternating layers of MNG and EMG materials with N=16. In the numerical investigations, we have chosen two cases, that are: (i) $\epsilon_A=2$, $\mu_B=1$,(ii) $\epsilon_A=1$, $\mu_B=2$, and also two thicknesses: (i) d_A =12mm and d_B =24mm (d_A/d_B =0.5) (ii) d_A =24mm and $(d_A/d_B=2)$ with plasma frequencies $d_{\rm B}=12$ mm $\omega_{ep} = \omega_{mp} = 12$ GHz. The optical constant of the ENG and MNG are taken from the equations (2) and (3). The scale length of the unit cell of the structure is d ($d=d_A+d_B$). The refractive indices of A (MNG) and B (ENG) materials with (i) $\varepsilon_A=2$, $\mu_B=1$, (ii) $\varepsilon_A=1$, $\mu_B=2$ are shown in the figures 2 and 3, respectively. The figure 2 predicts that, for large permittivity of MNG there is a large imaginary refractive index of the A material as compared to B, and for the large permeability of ENG, we have a large imaginary refractive index of the B material as compared to A, as shown in figure 3. In these cases, the refractive of the SNG of A and B materials has found zero.But due to existence of the single negative of MNG and $\varepsilon=2$ and single negative ENG and μ =2, the surface impedance of the periodic structure are changed. The impedance with zero-permittivity and zero-permeability in the MNG and ENG materials has also affected the band structure and the transmittance of the considered structure.

The Figs. 4 and 5 depict the transmittance and band structure for TE and TM mode for A layer (MNG and ϵ =2) and B layer (μ =1 and ENG) at the normal incident for the thickness ratio d_A/d_B=0.5 (d_A=12mm and d_B=24mm) and d_A/d_B=2.0 (d_A=24mm and d_B=12mm), respectively. For

 $d_A=12$ mm and $d_B=24$ mm, the band structure obtained are : 6.98-8.45 GHz and 7.36-8.82 GHz and transmittance are observed at 6.75-8.78GHz and 7.13-9.07GHz for TE and TM modes, respectively. Similarly, for d_A=24mm and d_B=12mm, the band structure obtained are 5.58-9.72GHz and 5.94-10.02GHz and transmittance are observed at 5.52-9.83GHz and 5.89-10.13GHz for TE and TM modes, respectively. The band structure and the transmittance have observed at different frequency ranges for TE and TM modes. This indicates that the structure of ENG-MNG materials has dominant property of the dielectric material for enlarged band gap. But the band structure and transmittance are not found at the same frequency. The studies of the optical properties reveal that the transmittance has tunneling property due to existence of zero- ε and zero- μ inside the structure.



Fig. 2. Refractive index of A (MNG and ε =2) and B (μ =1 and ENG) versus frequency



Fig. 3. Refractive index of A (MNG and $\varepsilon = 1$) and B ($\mu = 2$ and ENG) versus frequency



Fig. 4. Band structure and transmittance periodic structure containing A (MNG and ε =2) and B (μ =1 and ENG) versus frequency with thickness ratio 0.5



Fig. 5. Band structure and transmittance periodic structure containing A (MNG and ε =2) and B (μ =1 and ENG) versus frequency with thickness ratio 2.0



Fig. 6. Band structure and transmittance periodic structure containing A (MNG and ε =1) and B (μ =2 and ENG) versus frequency with thickness ratio 0.5



Fig. 7. Band structure and transmittance periodic structure containing A (MNG and ε =1) and B (μ =2 and ENG) versus frequency with thickness ratio 2.0



Fig. 8. Band structure and transmittance periodic structure containing A (MNG and $\varepsilon=1$) and B ($\mu=2$ and ENG) versus frequency with thickness ratio 0.5 for different angles of incidence



Fig. 9. Band structure and transmittance periodic structure containing A (MNG and ε =1) and B (μ =2 and ENG) versus frequency with thickness ratio 2.0 for different angles of incidence



Fig. 10. Band structure and transmittance periodic structure containing A (MNG and ε =2) and B (μ =1 and ENG) versus frequency with thickness ratio 0.5 for different angles of incidence



Fig. 11. Band structure and transmittance periodic structure containing A (MNG and $\varepsilon=2$) and B ($\mu=1$ and ENG) versus frequency with thickness ratio 2.0 for different angles of incidence.

The Fig. 6 and 7 show the transmittance and band structure containing A layer (MNG and ɛ=1) and B layer (μ =2 and ENG) for TE and TM mode at the normal incident for the thickness ratio $d_A/d_B=0.5$ ($d_A=12mm$ and $d_B=24mm$) and $d_A/d_B=2.0$ ($d_A=24mm$ and $d_B=12mm$), respectively. For d_A=12mm and d_B=24mm, the band structure is obtained at 5.57-9.73GHz and transmittance is observed at 5.52-9.83GHz for both modes. Similarly for $d_A=24$ mm and $d_B=12$ mm, the band structure is obtained within the range 7.01-8.45GHz and transmittance is observed at 6.74-8.70GHz for both modes. The band structure and the transmittance have observed at same frequency range for TE and TM modes. This indicates that such structure has dominant property of the magnetic material. The studies of the optical properties reveal that the transmittance has tunneling property due to existence of zero- ε and zero- μ inside the structure and predicted the ultra thin film of the A layer (MNG and $\epsilon=1$) has shown large band gap.

For the existence of the tunneling properties of structure with MNG and ENG materials, the transmittance

is obtained 100%, where band gap is forbidden because the band structure and transmittance is same for PBG. To study the tunneling property of the structure with MNG and ENG, we have also studied the transmittance of the structure with varying the angles of incidence and thicknesses for both modes. The figures 8 and 9 show the transmittance and band structure with A layer (MNG and ϵ =1) and B layer (μ =2 and ENG) for TE and TM modes at different angles of incidence for the thickness ratio $d_A/d_B=0.5$ (d_A=12mm and d_B=24mm), and d_A/d_B=2.0 $(d_A=24$ mm and $d_B=12$ mm), respectively. For the ultra thin film of the layer with MNG and $\varepsilon=1$ has the large band gap as compared to the thick layer of the μ =2 and ENG. This shows that the MNG has negative permeability and this effect is seen dominant to produce the large band gap. The transmittance for TM mode is shifted towards the higher frequency as the angles of the incidence is increased due to positive μ ; however the transmittance of the TE mode is shifted towards the lower frequency due to zero- ε in the ENG. Similar effects have been observed for $d_A/d_B=2.0$, but the band gap is small in comparison to the previous case.

The transmittance and band structure with A layer (MNG and $\varepsilon=2$) and B layer ($\mu=1$ and ENG) for TE and TM mode at different angles of incidence for the thickness ratio $d_A/d_B=0.5$ ($d_A=12$ mm and $d_B=24$ mm), and $d_A/d_B=2.0$ (d_A=24mm and d_B=12mm), respectively are shown in figures 10 and 11. Figure 10 shows that the transmittance of the structure with A layer (MNG and $\varepsilon=2$) and B layer (μ =1 and ENG), where small band gap is found for $d_A/d_B=0.5$. The higher frequency's band gap is nearly constant for all angles of incidence. But the lower frequency's band gap is shifted towards the higher frequency for both modes. The reason to find such properties is due to the zero-µ of MNG material and the large electric permittivity. The band gap of the structure with thick $(d_A/d_B=2.0)$ material of A layer (MNG and $\varepsilon=2$) and B layer (μ =1 and ENG) has been found large. The higher frequency band edge is constant for TE mode due to zero-µ but lower frequency band edge is shifted towards the higher frequency due to the large electric permittivity. This result reveals that the optical properties with MNG and ENG materials have complementary to each other when the positive refractive index and thickness are changed.

4. Conclusion

We have studied band structure and transmittance characteristics of one-dimensional photonic crystal composed with single negative materials (MNG and ENG) by using simple transfer matrix method and Bloch theorem. The band structure and the transmittance for TE and TM modes are studied for structure containing A (MNG and ε =2) and B (μ =1 and ENG), and A (MNG and ε =1) and B (μ =2 and ENG) with different thicknesses ratio like d_A/d_B=0.5 (d_A=12mm and d_B=24mm) and d_A/d_B=2.0 (d_A=24mm and d_B=12mm). A large band gap is obtained when structure has high (ε) and MNG with large thickness of MNG, and high (μ) and ENG with small thickness of ENG. The studies of transmittance have shown the tunneling property of electromagnetic wave due to existence of zero- ε and zero- μ . The transmittance of structure of MNG-ENG with different values of (ϵ) and (μ), angles of incidence, and thicknesses have been studied. The variation of the angles of incidence predicted that the transmittance has reversed characteristics due the existence of the zero- ϵ and zero- μ inside the structure. The results revealed that a large omnidirectional band gap of one-dimensional photonic crystal can be used as a filter by choosing properly thickness and angle of incidence.

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