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Breast tumour detection using the numerical analysis of the thermal inverse problem

F. HANTILA, O. DROSU, M. MARICARU
Politechnica University of Bucharest, Romania

The thermographical detection of breast tumour is an inverse and bad-conditioned problem for thermal field. Solving this problem needs, mainly, to solve an enormous number of direct problems. This paper analyses Pennes equation to model the thermal distribution within a healthy or malignant breast and proposes an efficient procedure for solving this equation.

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1. Introduction

During the past decades a tremendous international research development both in this area of medicine and in biomedical engineering has been observed. A particular emphasis has been put on the thermography use in breast pathology diagnosis. The abnormal thermical differences that appear between certain regions of the breast could be created by a modified metabolism or vascularisation. The thermical differences are reflected on the breast surface that could serve to the breast cancer detection. The thermical asymmetries that appear and the time-evolution of the thermal field are also important factors for the diagnose of these tumours.

The main advantages of the thermographic method for breast tumour detection are: it allows an early diagnose of the breast cancer, in that stage when the mammography or echography can not easily detect the changes of the tissue; the method is totally non-invasive; the costs are very low; bras with temperature sensors or thermosensitive sheets can be manufactured and sold allowing the women to do a self thermographic testing.

An extended number of papers (for ex. [1], [2], [3]) where focused on the relation between tumour presence and temperature distribution on breast surface, mathematical and numerical models for thermal field being developed. The problem of tumour dimensions and position is very complex and, as a consequence, not enough analysed. Eddie Y-K Ng, S.C Fok and col. [4] studied the computerized detection of breast cancer with artificial intelligence and thermograms, presenting the concurrent use of thermography and artificial neural networks (ANN) for the early diagnosis of breast cancer. It has been reported that breast thermography itself could detect breast cancer up to 10 years earlier than the conventional methods such as mammography, in particular in the younger patient. However, the accuracy of thermography is dependent on many factors such as the symmetry of the breasts' temperature and temperature stability, the physiological state and the microclimate of the investigation room. This paper examines the use of ANN to complement the infrared heat radiating from the surface of the body with other physiological data. Four backpropagation neural networks were developed and trained using the results from the Singapore General Hospital patients' physiological data and thermographs. Owing to the inaccuracies found in thermography and the low population size gathered for this project, the networks developed could only accurately diagnose about 61.54% of the breast cancer cases. Nevertheless, the basic neural network framework has been established and it has great potential for future development of an intelligent breast cancer diagnosis system. This would be especially useful to the teenagers and young adults who are unsuitable for mammography at a young age. An intelligent breast thermography-neural network will be able to give an accurate diagnosis of breast cancer and can make a positive impact on breast disease detection.

2. Qualitative aspects

As mentioned before, one of the main functions of the blood flow within a biological system is the capacity of heating or cooling a tissue, with respect to the local temperature. The difference of temperature between blood and tissue can be considered a proof for this function of releasing or absorbing the heat.

Based on this presumption, Pennes [1948] proposed one of the most known models of thermal transfer within alive systems: **« Pennes' thermal equation »**. He suggests that the effect of the blood flow can be modelled as a heat source term added to the traditional equation of thermal conduction:

$$-div\ \lambda grad T - (c_b w_b)(T_v - T) = p_t \qquad (1)$$

where $(c_b w_b)(T_v - T)$ represents the contribution of the blood flow for tissue heating, T_v is blood temperature, considered as known (it can be taken as $37^0 C$). The values of the coefficient $(c_b w_b)$ are described in Table 1, that contains also values for thermal conductivity λ and metabolic heat production p for different breast tissues.

Table 1. Thermophysical properties used in a 3D model of breast thermography [5]

Layers	$(c_b w_b)$ W m^{-3} °C^{-1}	λ W m^{-1} °C^{-1}	p W m^{-3}
Areola	800	0.21	400
Subcutaneous	800	0.21	400
Lower-inner Gland	2400	0.48	700
Upper-inner Gland	2400	0.48	700
Lower-outer Gland	2400	0.48	700
Upper-outer Gland	2400	0.48	700
Muscle Core	2400	0.48	700
Tumour (32 mm)	48×10^3	0.48	5.5×10^3

The boundary condition is:

$$-\lambda \frac{\partial T}{\partial n} = \alpha_c (T - T_e) + r(T^4 - T_e^4) \qquad (2)$$

where α_c and r are coefficients that represents the thermal convection and radiation and T_e is the environmental temperature outside the domain Ω.

Equation (1) is Helmholz type, with real positive coefficient, with favourable properties for numerical solving. We can obtain the difference of thermal field between the normal and malignant tissue if subtracting equations (1) written for both tissues:

$$-div\ \lambda grad T + \gamma T = p - \Delta\gamma(T_0 - T_v) \qquad (3)$$

where:

- T is the difference of temperature;
- T_0 is the temperature of the healthy tissue obtained in Pennes' equation;
- $\gamma = (c_b w_b)$ for malignant tissue;
- $\gamma_0 = (c_b w_b)_0$ for healthy tissue;
- $\Delta\gamma = \gamma - \gamma_0$;
- p is the difference between the metabolic heat production of the tumour and healthy tissue, respectively.

The boundary condition for differential thermal field is:

$$-\lambda \frac{\partial T}{\partial n} = \alpha T \qquad (4)$$

where, this time, α is an equivalent convection coefficient:

$$\alpha = \alpha_c + 4rT_0^3$$

3. The rapid solution of the direct problem

Because we have to solve a huge number of direct problems in order to determine the dimensions and the position of the tumour, the fastness of the solution is essential for obtaining a very large number of calculated values for the temperature on the breast surface. Using the comparison between these values and the measured ones, the position and the most probable shape of the tumour can be obtained. Unlike the reconstruction methods for the devices defects, the extremely variable geometry of the breast does not allow the achievement of a data base that can be improved in the future.

A. Using the submatrix associated to the malignant area

Equation (3) can be written as follows:

$$-div\ \lambda grad T + \gamma_0 T + \Delta\gamma T = p - \Delta\gamma(T_0 - T_v) \qquad (5)$$

The numerical expression corresponding to equation (5), under matrix form, is:

$$WT + GT = P \qquad (6)$$

where:

- the matrix T contains the coefficients a_k;
- the matrix W has the elements:

$$w_{jk} = \int_{S_F} \alpha\varphi_k\varphi_j dA + \int_\Omega \lambda grad\varphi_k grad\varphi_j dv + \int_\Omega \gamma_0\varphi_k\varphi_j dv \qquad (7)$$

and does not depend on the tumour presence;

- the matrix G has non-zero elements associated only to the nodes of the tumour surroundings Ω_c:

$$g_{jk} = \int_{\Omega_c} \Delta\gamma\varphi_k\varphi_j dv \qquad (8)$$

- the matrix P has non-zero elements associated only to the nodes of the tumour:

$$p_j = \int_{\Omega_c} (p - \Delta\gamma(T_0 - T_v))\varphi_j dv \qquad (9)$$

Multiplying the relation (6) by W^{-1}, we obtain:

$$\begin{pmatrix} T_C \\ T_S \end{pmatrix} + \begin{pmatrix} \left(W^{-1}\right)_{cc} & \left(W^{-1}\right)_{cs} \\ \left(W^{-1}\right)_{sc} & \left(W^{-1}\right)_{ss} \end{pmatrix} \begin{pmatrix} G_{cc} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} T_C \\ T_S \end{pmatrix} = \begin{pmatrix} \left(W^{-1}\right)_{cc} & \left(W^{-1}\right)_{cs} \\ \left(W^{-1}\right)_{sc} & \left(W^{-1}\right)_{ss} \end{pmatrix} \begin{pmatrix} P_C \\ 0 \end{pmatrix} \tag{10}$$

where the indices *c* and *s* refer to the malignant and healthy domain, respectively. Results:

$$\left(I + \left(W^{-1}\right)_{cc} G_{cc}\right) T_C = \left(W^{-1}\right)_{cc} P_C \tag{11}$$

$$T_S = -\left(W^{-1}\right)_{sc} G_{cc} T_C + \left(W^{-1}\right)_{sc} P_C \tag{12}$$

As the tumour domain is small enough comparing to the breast volume, especially in the case of an early detection, the dimensions of the matrices $\left(W^{-1}\right)_{cc}$, G_{cc} and $\left(I + \left(W^{-1}\right)_{cc} G_{cc}\right)$ are small. Another reduction of the computational effort can be done retaining only the submatrix corresponding to the boundary temperatures.

$$T_F = -\left(W^{-1}\right)_{Fc} G_{cc} T_C + \left(W^{-1}\right)_{Fc} P_C \tag{13}$$

From relations (11) and (13) we obtain:

$$T_F = -\left(W^{-1}\right)_{Fc} G_{cc} \left(I + \left(W^{-1}\right)_{cc} G_{cc}\right)^{-1} \left(W^{-1}\right)_{cc} P_C + \left(W^{-1}\right)_{Fc} P_C \tag{14}$$

When searching the tumour, only the reduced dimensions matrix $G_{cc}\left(I + \left(W^{-1}\right)_{cc} G_{cc}\right)^{-1}$ is recalculated, where $\left(W^{-1}\right)_{cc}$ is not recalculated and it is tested for different values of the vector P_C.

B. Using the iterative method

The numerical expression of the Fourier equation has a positive defined and diagonal dominant matrix. Solving the system using a Gauss Seidel iterative procedure is highly efficient. Indeed, the second term of the left part of Pennes' equation (1) forces the temperature field to get close to blood temperature (37^0). The initial value in the iterative procedure can be the blood temperature or, even better, the final temperature of the previous search. The great advantage of the iterative method with respect to the first one consists in the smaller memorizing effort. The matrix of the system is extremely rare and only the non-zero elements are retained.

4. A representative example

We consider the simple breast shape showed in Fig. 1 and the tumour area detailed in Fig. 2. We use a triangular mesh with nodal elements of first order. The healthy region of the breast is considered to be homogenous, with $\lambda = 0.4\frac{W}{m\cdot^\circ C}$, $\gamma = 1500\frac{W}{m^3\cdot^\circ C}$ and $p = 700\frac{W}{m^3}$. For the tumour region we choose $\gamma = 4000\frac{W}{m^3\cdot^\circ C}$ and $p = 6000\frac{W}{m^3}$. The equivalent thermal conduction coefficient that contains the effects of convection, radiation and evaporation was set as $\alpha = 13.5\frac{W}{m^2\cdot^\circ C}$.

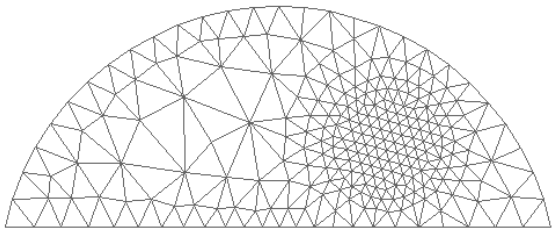

Fig. 1. Breast and the investigated region

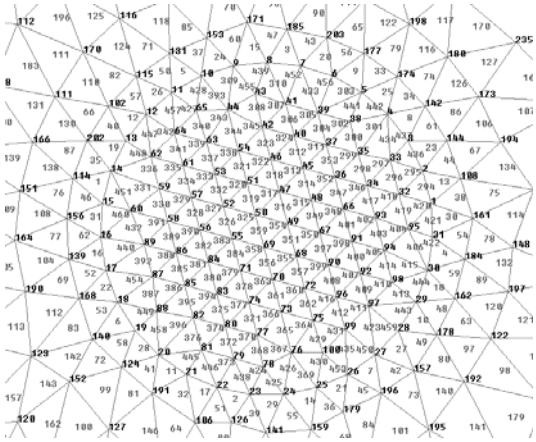

Fig. 2. Detail of the investigated area (bounded by a circle)

We chose a tumour placed in the region of triangles around the node 47. The isotherms are plotted in Fig.3. The differences between the healthy and malignant breast tissue is represented in Fig.4. We randomly modify the temperatures field on the breast surface with a certain error that represents the measurements error of the infrared camera. Then we search for tumours that contain the

triangles around the nodes placed in the suspicious region; for these nodes we determine the thermal fields on the breast surface and we compare them with that one obtained for the node 47. For a maximum measurement error of 0.03°C we localise the tumour, by using an iterative method.

Another example consists in searching for a small tumour of a triangle dimension from the explored region, localised in triangle no. 353. The graphics for the temperature difference is represented in Fig.5. In order to localize the tumour a measurement error of 0.01 °C is needed.

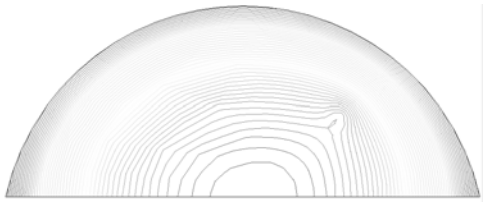

Fig.3. Breast temperature scale (the presence of the tumour is obvious)

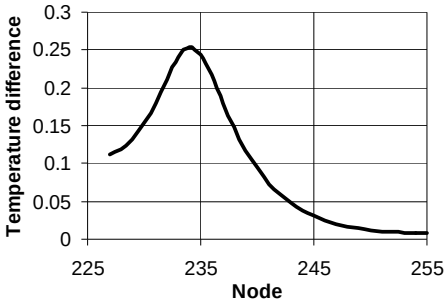

Fig.4. Difference of the temperature (breast contour is counter-clockwise covered)

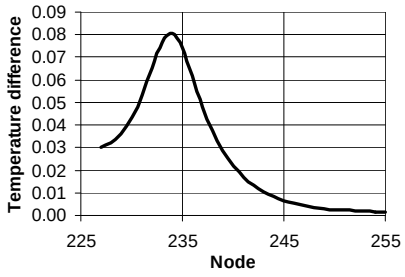

Fig.5. Difference of the temperature for a small tumour (breast contour is counter-clockwise covered)

5. Conclusions

The proposed procedure is a non-invasive one, and allows the detection of breast tumour location and dimensions. It can be repeated later on in order to confirm an early diagnose. If the thermal investigation gives the same results the patient can be directed to more precise, but invasive, investigations (mammography, MRN, etc.).

The huge computational effort comes from the enormous number of direct problems. Within these problems, different positions, shapes and dimensions of the tumour are proposed. Two procedures destined to computational effort reduction were proposed, the iterative one being the most advantageous. For solving the thermal problem with an error of:

$$\varepsilon = \left[\frac{\frac{1}{N}\sum_{k=1}^{N}\left(T^k - T^{k-1}\right)^2}{\frac{1}{N}\sum_{k=1}^{N}\left(T^k\right)^2} \right]^{\frac{1}{2}} < 10^{-7} \qquad (15)$$

17-50 iterations are necessary. The tumour search needs 1.4 *sec*. on a 2.128 GHz processor notebook. For the representative example a two-dimensional model was chosen, but the procedure can be easily applied for a 3D structure.

The computational effort can be reduced, decreasing the parameters that describe the inspected area. For this purpose, "zooming" techniques can be used [6] or different categories of subdomains can be inspected.

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*Corresponding author: hantila@elth.pub.ro,