Cellular AlMg-SiC composites structure modeling by means of fractal analysis

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This paper aims to describe, based on fractal analysis, the configurations and geometrical characteristics of composite cellular structures (size, shape and distribution of the cells). The samples from AlMg10-SiC (with 5, 10 and 15% SiC) were obtained by means of gas injection method. We used fractal geometry modeling by means of fractal dimension types of composites obtained from performed experiments. We obtained the following fractal dimensions for the sample with 5% SiC the fractal dimension is 1.904, for 10% SiC – 1.939 and for 15% SiC –1.951. Fractal analysis results indicate that composite cells obtained from experiments have a statistically regular form. It seems that in the composite volume it is a tendency to obtain non-spherical open cells and bridges between adjacent cells. This fact is proved by microscopic analysis. We can conclude by correlating the determined fractal dimension with the pore size that the 15% SiC alloy has the smallest cell dimensions and the biggest relative porosity and, consequently, the highest complexity and mechanical properties.

(Received April 4, 2015; accepted October 25, 2015)

Keywords: Fractal analysis, Cellular structure, Composite, Porosity

1. Introduction

In this paper aims to describe, based on a quantitative analysis, configurations and geometrical characteristics of cellular composites structures (size, shape and distribution of cells) using fractal geometry modeling, the fractal dimension of the types of composites obtained from experiments performed.

First we present an overview of the concept of fractal analysis and an overview of the concept of fractal dimension of an object that can be natural or constructed experimentally from the fact that there is a difference between Euclidean geometry and fractal geometry.

Fractal theory is in fact a concept that comes to clarify the mathematical basis of the basic notions and concepts of the real world and difficult to address concepts such as orderly, chaotic, random, theory of developing a new vision of mathematical analysis and geometry [1,2].

Based on fractal analysis we can imagine and developed analogies of two or three-dimensional cellular materials (like composites tested during the researches). In this context we can apply some rules known as fractals development: Sierpinski carpet (figure 2), Von Koch curve (figure 3) and Menger sponge, with fractal dimension for Sierpinski carpet Df = 1.89, Von Koch curve Df = 1.26, Menger sponge Df = 2.72.

Fractals enables the generation of highly complex structures using only very simple equations. The fractal, defined as a geometric object, has in generally the following characteristics: a fine structure at arbitrarily small scales (it can reproduce the structures of representation until the atomic level); representation is too irregular to be described in the language of Euclidean geometry; is self-similar (at least approximately or stochastic development); it has the Hausdorff dimension greater than the topological dimension (although this requirement is not fulfilled for example for Hilbert curves); it has a simple and recursive mathematical definition [3].

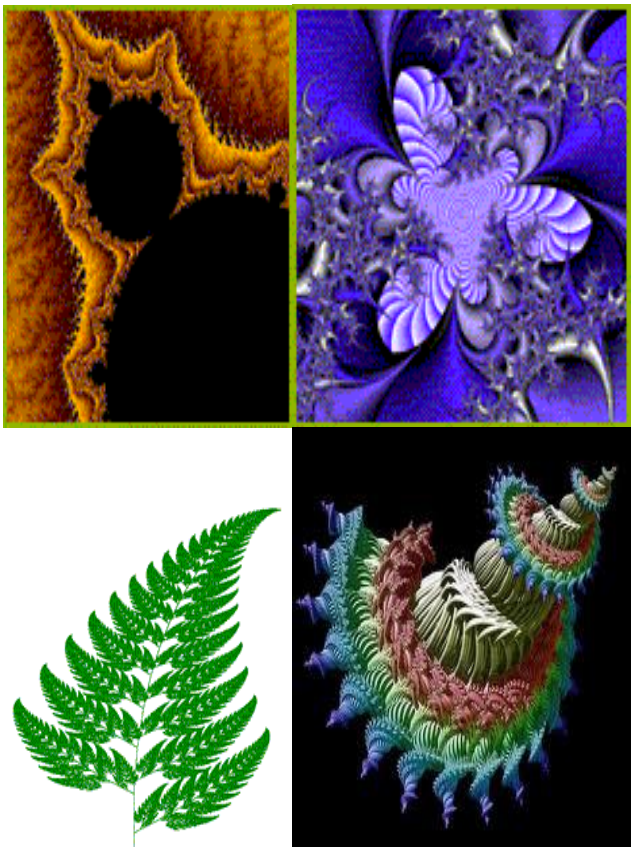

Fig. 1. Reproductions of some representations of fractal images

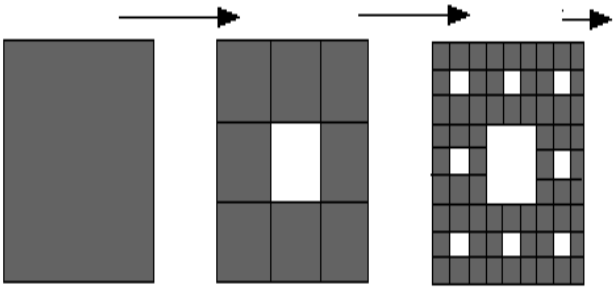

Fig. 2. Sierpinski carpet [4]

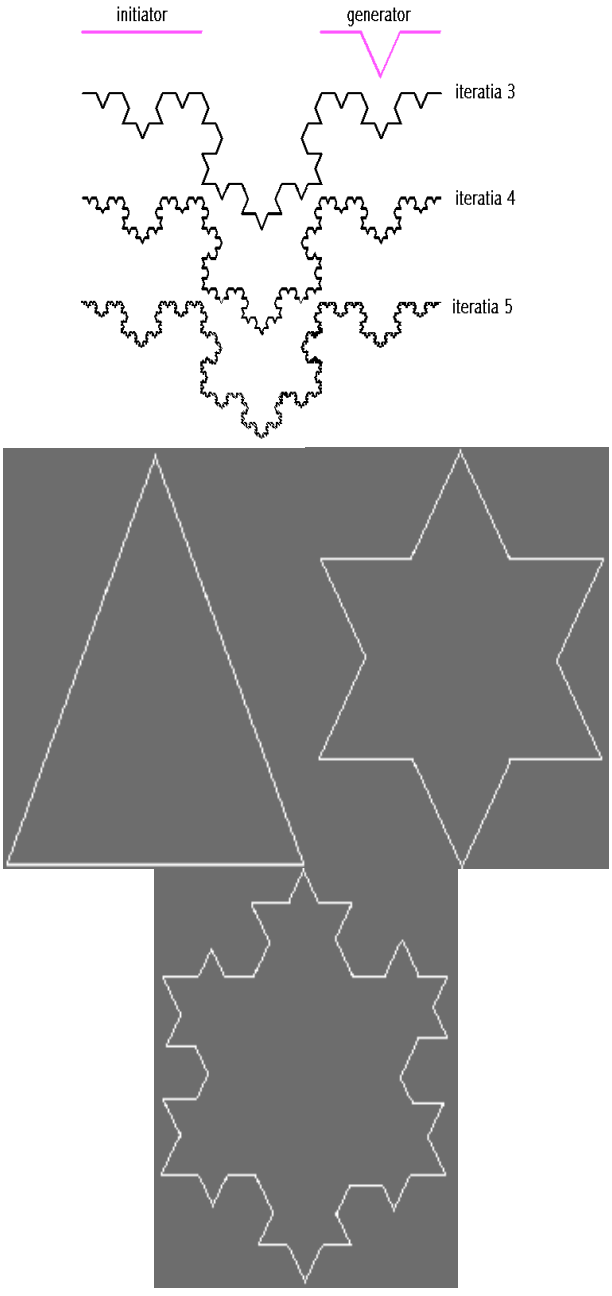

Fig. 3. Von Koch curve

Using a computer can easily construct a fractal curve associated with a data set to derive the following advantages: - fractals provides a method for storing images and data more compact than linear vectors; - if linear equations give errors in building inherent systems, unpredictable and chaotic (such as cells in which are structured the composite material) then realization and presentation is possible in a structural representation using fractals; - using fractal, irregularities in the cell structure of the composite material are essential parts of the pattern, instead of representing the difficulty of representation to be removed, such as in the case of conventional modelling based solely on the numerical analysis.

2. Application of fractal analysis to the study of cellular material AlMg10-SiC

The analyzed materials have been obtained by mixing the alloy melt AlMg10 with SiC powder 120 μm size, at a temperature of 710ºC and with C_4H_{10} injection at 1.2 atm pressure. The foam formed on the melting surface has been removed using a specially designed device, adapted to the respective oven. After cooling in the air, the samples (figure 4) were adapted for the scanning electron microscope study.

The analysis method is based on the properties of fractals, fragmentation to infinity (an infinity of components); self-similarity - defined in the theory of fractals as a property of an object to have every detail of the entire or similar to it at any scale; the shift-invariance is the property of a fractal object, to retrieve details from itself, by overlaying them over a different area of the fractal, after translating in a certain direction; fractional dimension or "fractal dimension" or " self-similarity size" [5].

Fig. 4. Macrography of the three types of cellular composite studied for achieving modeling and fractal analysis

Using these properties we can imagine a model which aims to an evaluation, based on the theory of fractals, of the spacing form (design or General geometry) of a cellular type composite, of the size and distribution of cells within the composite volume and of the theoretical distribution of the composite volumetric properties.

By applying fractal geometry, we obtain a direct method for the study of different cellular type forms that are found within these composites, as well as a general estimate of material homogeneity, based on a numerical analysis [6].

It was investigated the effect of the added magnesium amount to the AlMg10-SiC alloys on its structure. For alloys and composite materials containing regular microstructures, a prediction of mechanical properties can be made by a quantitative measurement of features such as grain size, particle size, and spacing. In our case, where irregular microstructure is involved, the numerical characterization of the structure is difficult, but the application of fractal geometry offers a method by which even this late structure can be described in a numerical manner [7, 8].

Scanning electron microscopy (SEM) is a technique normally used for 2D-analysis of surface features. By fractal dimension analysis of the SEM images, it is possible to get quantitative topographical measurements. In this work, three different surfaces (representing AlMg-SiC alloys with different SiC content) were analyzed. Clear surface topographical changes can be qualitatively observed. In order to quantify such changes, two steps were followed:

(i) a black-white digitalization from each image was used in order to reproduce topographical features on the analyzed surface;

(ii) the fractal dimension (*D*) was determined using Fractalyse software (www.fractalyse.org). It is based on the counting of black spots (pixels) contained in a window of counting [9].

The measuring process is split in two parts:

1. the counting method in the first part the analysis goes step by step following an iteration principle and consists in counting the number of black pixels contained in a counting window. From one step to the next, the size of the counting window is enlarged. Thus, a series of points is obtained, that can be represented on a Cartesian graph; the Y-axis corresponds to the number of counted elements (*N*) and the X-axis corresponds to the size of the counting window (ε);

2. the estimation module consists in fitting the above graph with another one, the estimated curve, usually dictated by a power law (parabolic or hyperbolic), $N = \varepsilon^D$, where *D* represents the fractal dimension [10].

In figures 5, 6, 7 are shown the structures of materials on which fractal dimensions were calculated. We obtain: for the sample number 1 (AlMg10-SiC with 5%SiC, figure 5) fractal dimension is 1.904, for the sample number 2 (AlMg10-SiC with 10%SiC, figure 6) fractal dimension is 1.939 and for the sample number 3 (AlMg10-SiC with 15%SiC, figure 7) fractal dimension is 1.951.

It is noted that the fractal dimensions of these three samples are close to the fractal dimension of the 2D percolation cluster, 1.8958 (fig. 8).

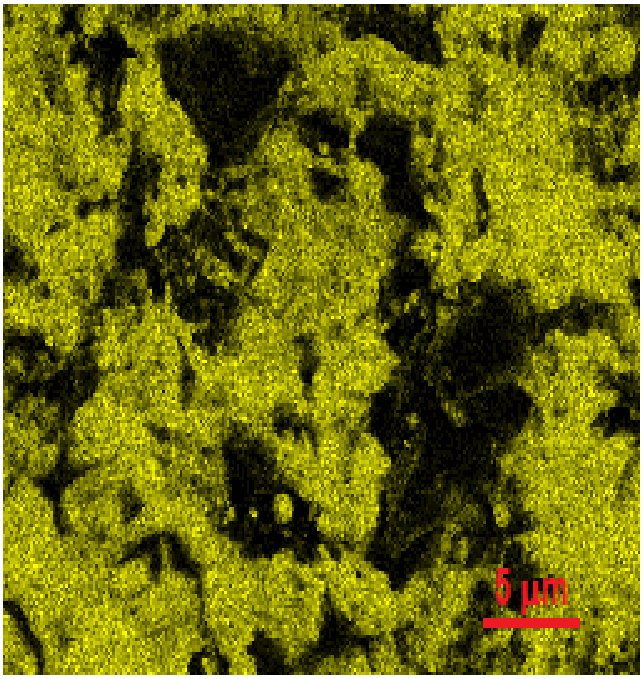

Fig. 5. Composite 1 – AlMg10-SiC, 5%SiC

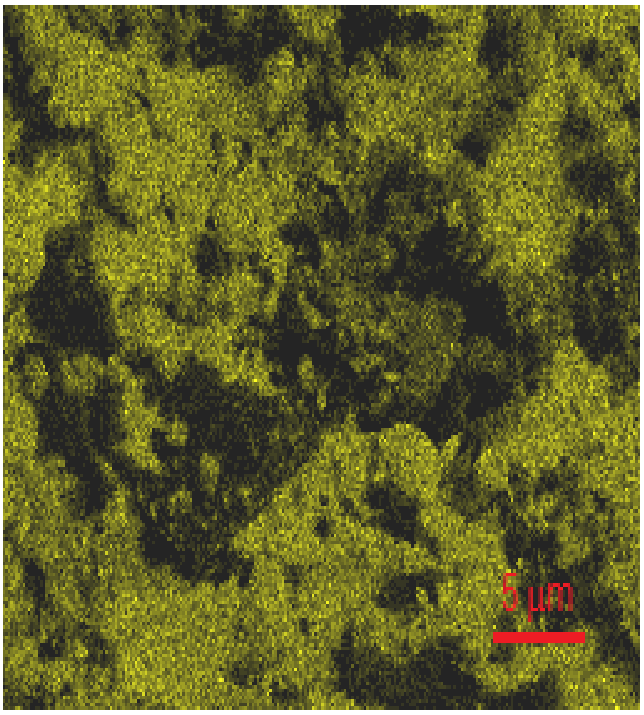

Fig. 6. Composite 2 – AlMg10-SiC, 10% SiC

Another method that we performed for calculating fractal dimension is Sandbox Method. The analysis was performed on the same type of composites (AlMg10-SiC with-5%, 10% and 15% SiC).

The simulation of morphology/geometry of an object that can be reproduced by an identical topology, it will be done on AlMg10-SiC composite cell, based implicitly on a computer self-generated equation that represents a fractal number and then by plotting it based on fractal analysis. Therefore, the procedure that we use for fractal analysis is Sandbox Method.

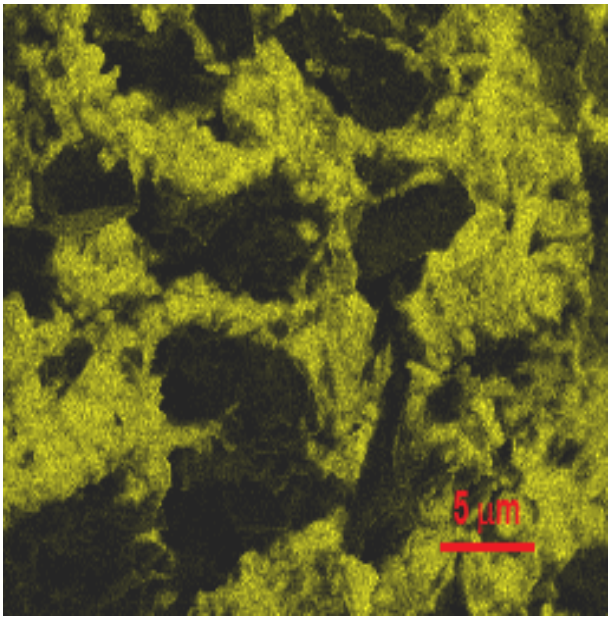

Fig. 7. Composite 3 – AlMg10-SiC, 15% SiC

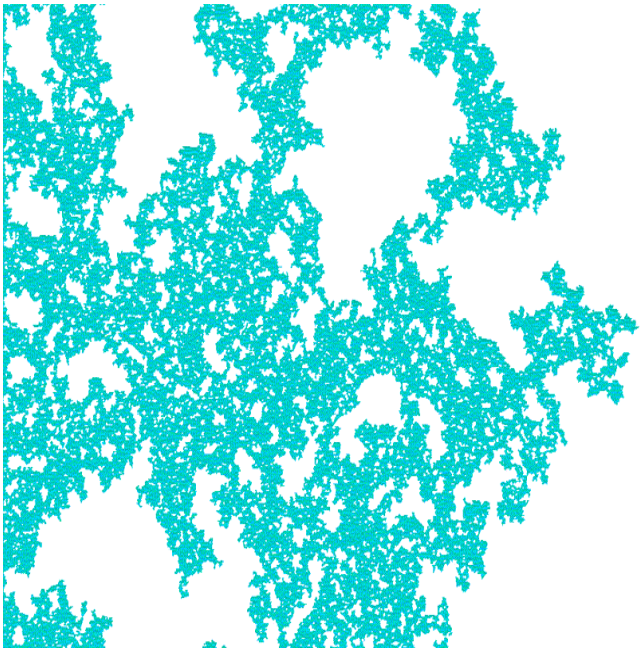

Fig. 8. Fractal dimension of the 2D percolation cluster

To initiate the algorithm we need for the beginning a representative image of each type of bi-dimensional composite, image that can be adopted by a conventional computer program, which works based on successive iterations calculations, in my case MATLAB (figure 12). For this, we adopted an equation for the determination of the fractal dimension Df in a 2D representation of the considered object. For example, a fractal dimension Df is given by a relationship such as [11]:

$$\mathbf{D_f} = \ln N(L)/\ln L = \ln 10/\ln 4 = 1{,}66 \qquad (1)$$

In this equation the number of pixels that covers a minimum area of representation of dimension L will be a value N(L); by logarithming and reporting this quantity we will obtain a dimension of a fractal type (see equation 1).

Therefore, we select three images for structural representation of the three types of obtained composites (figure 5, 6, 7).

To get the best image is necessary to determine a minimum level pixels resolution (or a critical dimension to the pixel level in order not to cause image distortion) that we note p_{xmin}, value that can be picked up and played by the computer without modifying the information, i.e. without producing changes/distortion on the image. To do this we must translate from a monochromatic image to a grayscale one, and then, for simplicity, we will use only a black and white picture (Fig. 9, 10 and 11).

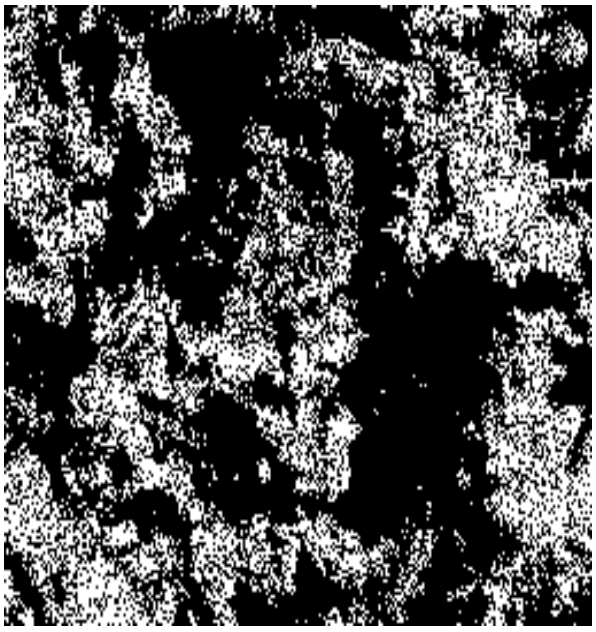

Fig. 9. Electron microscopy image in black and white of AlMg10-SiC alloy with 5% SiC. Twigs binary microstructure

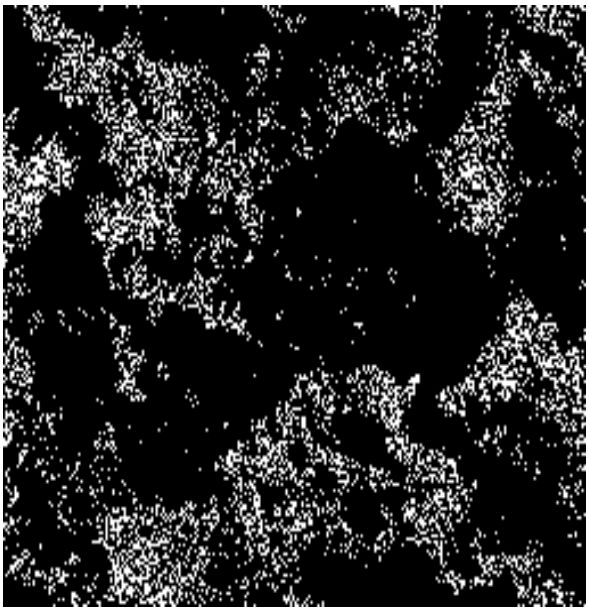

Fig. 10. Electron microscopy image in black and white of AlMg10-SiC alloy with 10% SiC

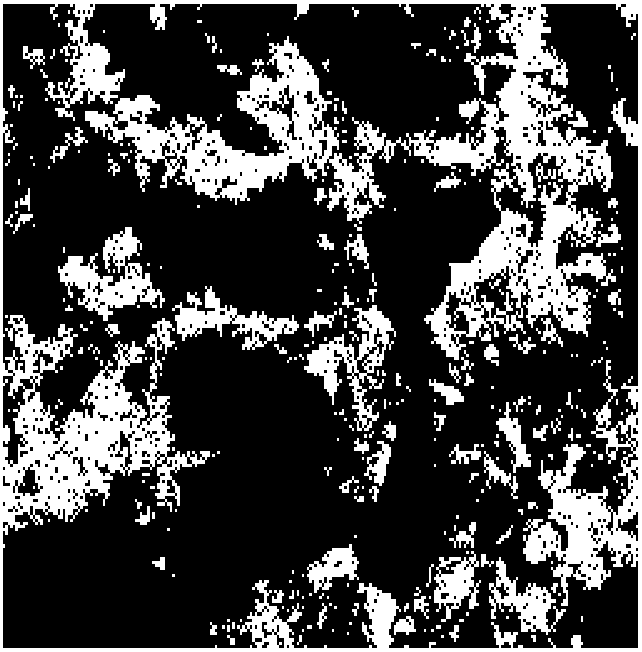

Fig. 11. Electron microscopy image in black and white of AlMg10-SiC alloy with 15% SiC

The taken experimental measurements used for the calculation of the occupied area of the broken images, lead to some unavoidable errors of estimation and, that is why the value obtained for the fractal dimension of image analysis differs slightly from the theoretical amount that was determined for the fractal definition.

Statistical values can be estimated more accurately and represented in such fractal dimension calculation, the obtained results the obtained being closer to the theoretical value of a fractal.

It is thus seen that in the case of the composite sample no. 1, the cell shape close to the uniformisation trend appears in a small number of cases, respectively on the positions 8 and 10. Best fractal dimension is given by the position 10 where the fractal dimension Df = 1.3110, close to the theoretical fractal dimension D = 1.66.

Table 1. Values of β and Df for composite sample No. 1 (AlMg10-SiC with 5% SiC)

Analyzed cell	Fractal dimension $\mathbf{D_f}$	Cell uniformity degree β
1	1,1131	0,6770
2	1,1580	0,8101
3	1,1229	0,8055
4	1,1464	0,3341
5	1,5262	0,0321
6	1,2841	0,3888
7	1,3110	0,5931
8	1,1530	0,9891
9	1,2330	0,7865
10	1,3110	0,8350

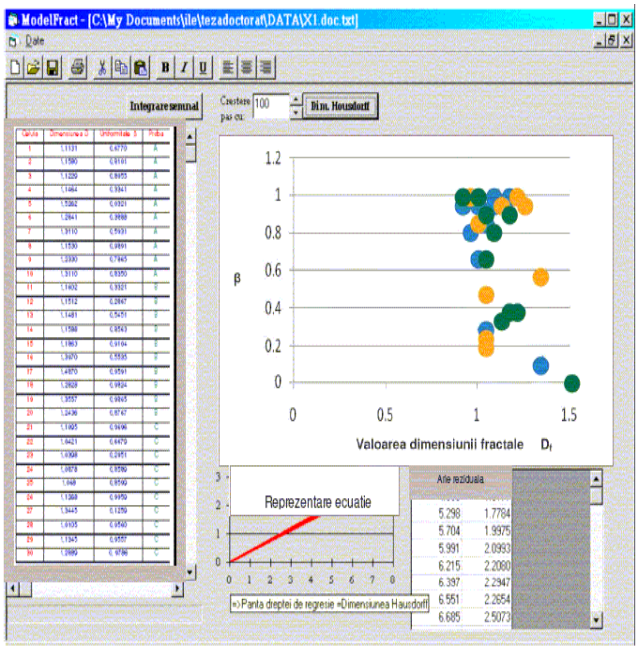

Fig. 12. Correlation between the β degree of uniformity and fractal dimension Df for the three types of obtained samples

Table 2. Values of β and Df for composite sample No. 2 (AlMg10-SiC with 10% SiC)

Analyzed cell	Fractal dimension $\mathbf{D_f}$	Cell uniformity degree β
1	1,1602	0,3321
2	1,1512	0,2867
3	1,1481	0,5451
4	1,1588	0,8563
5	1,1863	0,9104
6	0,9104	0,5535
7	1,4870	0,9591
8	1,2828	0,9824
9	1,3557	0,9865
10	1,2436	0,8767

Table 3. Values of β and Df for composite sample No. 3 (AlMg10-SiC with-15% SiC)

Analyzed cell	Fractal dimension $\mathbf{D_f}$	Cell uniformity degree β
1	1,1095	0,9696
2	1,0421	0,6679
3	1,0398	0,2951
4	1,0878	0,8589
5	1,0680	0,8599
6	1,1268	0,9959
7	1,3445	0,1259
8	1,0105	0,9560
9	1,1345	0,9557
10	1,3889	0, 9786

In the case of the composite sample no. 2, the cell shape close to the uniformisation trend appears in four cases, respectively on the positions 5, 7, 8 and 9. Best fractal dimension is given by the position 9 where the fractal dimension Df = 1.3557.

For the composite sample no. 3, the cell shape close to the uniformisation trend appears in the biggest number of cases, respectively on the positions 1, 6, 8, 9 and 10. Best fractal dimension is given by the position 10 where the fractal dimension Df = 1.3889, the closest to the theoretical fractal dimension D = 1.66.

3. Conclusions

We can conclude that in 15% SiC alloy, the structure have the following properties:

- the smallest porosity (pores) and, consequently, highest complexity (this conclusion was drawn by correlating the determined fractal dimension with the pore size -observable from picture);
- highest real surface, corresponding to highest fractal dimension , corrosion processes will be stronger due to higher available area of the alloy in contact with the corrosive media;
- rougher surfaces.

Taking into account the evolution of fractal dimension, and of the corresponding alloy properties related to SiC content in the alloy, we can affirm that 15% SiC represents a critical point of alloy composition.

Fractal analysis result indicates that composite cells obtained from experiments were of statistically regular form, with β > 0.5 and with the approach of fractal dimension to the value D = 1.

Based on numerical analysis and plotting the values obtained, the resulting distribution map shows a uniform trend in the whole volume of the composite structure and the fractal dimension over 0.5 value indicates the tendency of obtaining non-spherical open cells and bridges between adjacent cells that make up the composite volume.

From the graphical representation in fig. 8 we can observe that as the value of β decreases the fractal D increases the uniformity of the composite cell geometry; there are minimum values for D and β for which the Df critical value is between 1.0165 for the composite no. 3 and 1.113 for the composite no. 1.

The obtained analysis results by means of fractal analysis, show that there are critical values for Df fractal dimension and β degree of uniformity (D = 1.3445 and β = 0.1259), under which if they decreased, some properties of composites, such as hardness and mechanical strength will increase the number of cells will become smaller and it will appear the danger of intercellular cracking.

For $1 < D_f < 2$ the composites have less regular cell shape, the deviation from ideal geometry being considerable. In these cases the composites are less suitable for industrial applications, composites with irregular cell structure having less predictable properties and therefore less controllable properties (that is why such composites were not used in the study).

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