Characteristics of Laguerre–Gaussian beam scattering from a coated circular nihility cylinder

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To study the scattering characteristics of Laguerre–Gaussian (LG) beam from a coated circular nihility cylinder, a theoretical analysis is carried out in this work. A nihility cylinder is coated with metamaterials that may be double positive (DPS) or double negative (DNG), respectively. The electromagnetic field equations are reshaped with the help of LG potential and the usual classical scattering theory. The numerical solution for transverse magnetic (TM) and transverse electric (TE) polarizations are discussed. By implementing the boundary conditions at the interface, the undetermined scattering coefficients are calculated. On employing a special case, the numerical results for the present work were converted into already published literature. Bistatic echo widths have also been calculated and presented numerically. The impact of orbital angular momentum (OAM), i.e., OAM mode number (l) and beam frequency on the bistatic echo width is examined. The OAM index can also be used to control the scattering amplitude.

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1. Introduction

Metamaterials attract many researchers of the optical research community due to their unusual characteristics, i.e., they can control the electromagnetic properties for a varying and wide range of frequencies, a characteristic that does not exist in natural materials. The essential feature of metamaterials is their craved qualities owing to their specific structure and design [1]. The applications of metamaterials in electromagnetics are varied. For example, they can be applied as metamaterial absorbers, antennas and elements, waveguides, super lenses, invisibility cloaks, split–ring resonators, and many more [2-4].

Many researchers have been inspired by the idea proposed by [5] and carried out their investigations to study the characteristics of metamaterials [6-10]. The study of Photonic crystal fiber (PCF) based on metamaterial structure has transformed the communication industry [11, 12]. It intensifies not only the optical memory in modern computer systems but also used as a faster medium of transmission [13-15].

Some metamaterials reduce to nihility metamaterials by implementing the nihility condition on the constitutive parameters, i.e., by making the magnitude of both permittivity and permeability zero/null-valued [16]. The host nihility medium is regarded as electromagnetically nilpotent. However, it has an amazing property for optics and electromagnetics. The nihility material scheme was postulated by Lakhtakia [17] and found to be fascinating for many researchers.

As the permittivity and permeability of the abovesaid materials become zero, due to this electromagnetic wave/energy cannot propagate through it [18]. The study of the nihility medium for an electromagnetic wave is not limited to [17], but the scattering characteristics towards nihility material for different configurations such as, nihility sphere and cylinder [19, 20], chiral coated nihility cylinder [21], coated nihility cylinder settled in chiral metamaterials [22], and isotropic plasma coated nihility sphere [23] have been conducted to study the features of the nihility medium. On applying nihility conditions, Maxwell's equations become as,

$$\nabla \times \vec{E} = 0 \tag{1}$$

$$\nabla \times \vec{H} = 0 \tag{2}$$

The coated metamaterial transforms into nihility metamaterial by applying the nihility condition on constitutive parameters, i.e., $\varepsilon_r = 0$ and $\mu_r = 0$.

OAM [24] is associated with the LG beam and has received particular interest due to its fascinating characteristics. The LG beam has rotational symmetry along its axis of propagation and carries intrinsic rotational OAM, which is worthwhile from a practical viewpoint [25, 26]. The phase front for LG beams has a helical or twisted shape and it includes a term i.e., $e^{il\varphi}$. The beam orders are represented by the l values. By virtue of the OAM, the LG beam has the capability to rotate objects [27]. There is another radial parameter (p) for an LG beam that characterizes the node numbers. Kogelnik and Li defined the higher-order beam modes using cylindrical coordinates based on the Laguerre polynomial [28].

Allen et al. studied the OAM of light including transformation of LG beam and revolutionized the optical community [29]. Thakur and Berakdar studied the twisted light reflection and transmission at the phase-conjugating interfaces [30]. Zhao and Ou analyzed the LG beam scattering by a sphere [31]. Puyalto and Terriza investigated the role of the OAM in Mie scattering along with the excitation of dielectric spheres with LG modes [32]. The scattering of an LG vortex beam by a chiral sphere was done by Qu et al. [33]. Recently, Arfan et al. discussed the various factors of LG beam which can enhance the scattering rate using PEMC sphere as a metamaterial [34-37].

However, to the best of the authors' knowledge, the scattering characteristics of an LG beam by a nihility cylinder has not been reviewed. The purpose of this study is to investigate the scattering features of a coated nihility cylinder illuminated by an LG beam, which is a key factor to stimulate applications in the optical field.

The layout of the manuscript is described in the following fashion. In Section 2, the LG beam expressions for electric fields (incident, scattered, and inside) are revised using the LG scalar potential along with the scattering characteristics of the plane wave from the nihility cylinder [38] within the domain of classical Mie theory. In Section 3, the influence of OAM mode index and beam frequency using nihility cylinder coated with metamaterials is analyzed for the proposed structure. The conclusions are given in Section 4.

2. Formulations

The scalar potential of an LG beam with finite OAM having amplitude $\widetilde{V_0}$ propagating along the z-axis in a cylindrical coordinate system can be given as [39-41],

$$V(r,t) = \widetilde{V_0}\, F_{pl}(r,z)\exp(il\varphi)\exp[i(kz-\omega t)] \quad (3)$$

The amplitude function of the LG beam can be defined as [41],

$$F_{pl}(r,z) = \frac{1}{2\sqrt{\pi}}\sqrt{\frac{(l+p)!}{p!}}\,(\mathrm{X})^{|l|}L_p^{|l|}(\mathrm{X})\exp\left(-\frac{\mathrm{X}}{2}\right) \quad (4)$$

The term $L_p^{|l|}(.)$ stands for the associated Laguerre-polynomial with p and l being the radial and azimuthal mode numbers, respectively. In Eq. 4, X=$\frac{r^2}{w^2(z)}$ and $w(z)$ denotes the beam waist. The electric field components of a beam using field equation in cylindrical coordinates $\vec{E}(r,t) = -\vec{\nabla}\varphi(r,t)$ can be expressed as [34],

$$E_r = -\frac{\partial V(r,t)}{\partial r} = -\frac{1}{F_{pl}}\frac{\partial F_{pl}}{\partial r}V(r,t) \quad (5)$$

$$E_\varphi = -\frac{\partial V(r,t)}{\partial \varphi} = -\frac{il}{r}V(r,t) \quad (6)$$

$$E_z = -\frac{\partial V(r,t)}{\partial z} = -\left(ik + \frac{1}{F_{pl}}\frac{\partial F_{pl}}{\partial z}\right)V(r,t) \quad (7)$$

The geometry of scattering problem is shown in Fig. 1. To simplify the problem, the whole space is sliced into three regions: 0, 1, and 2.

Region 0 is the free space region with $r > b$, the region $a < r < b$ is designated as 1, and the region with $r < a$ is termed as region 2. The coated metamaterial is considered homogeneous and isotropic. Here, the wave numbers are given as $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, $k_1 = \omega\sqrt{\mu_1\epsilon_1}$, and $k_2 = \omega\sqrt{\mu_2\epsilon_2}$ for the three regions, respectively.

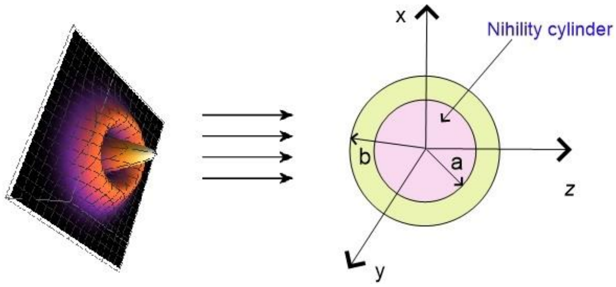

Fig. 1. The nihility coated cylinder illuminated by an LG beam (color online)

TM and TE polarizations of the incident field of LG beam are described as the following: (i) When the electric field of an incident LG beam becomes parallel to the incident plane, i.e., the plane containing the direction of propagation of the incident beam and the z-axis (x-z) plane, this is termed TM polarization; (ii) When the incident electric field becomes perpendicular to the incident plane, this is called TE polarization. For parallel/(TM) polarization, a coated nihility cylinder is illuminated by an incident LG beam that is traveling along the z-axis of the cylinder, then the field equations can be revised using [38] by substituting Eq. 3 in Eqs. 6–7, so the incident, scattered and total electric field components in regions 0, 1, and 2 can be expressed as

$$E_{0z}^{inc} = -\widetilde{V_0}\exp[i(l\varphi-\omega t)]\left(ik\,F_{pl} + \frac{\partial F_{pl}}{\partial z}\right)F_{pl}\sum_{n=-\infty}^{\infty} j^n J_n(k_0 r)e^{in\varphi} \quad (8)$$

$$E_{0z}^{sca} = -\widetilde{V_0}\exp[i(l\varphi-\omega t)]\left(ik\,F_{pl} + \frac{\partial F_{pl}}{\partial z}\right)F_{pl}\sum_{n=-\infty}^{\infty} j^n\, a_n\, H_n^{(1)}(k_0 r)\,e^{in\varphi} \quad (9)$$

There are two interfaces i.e., $r = a$ and $r = b$, in region 1.

$$E_{1z} = -\widetilde{V_0}\exp[i(l\varphi-\omega t)]\left(ik\,F_{pl} + \frac{\partial F_{pl}}{\partial z}\right)F_{pl}\sum_{n=-\infty}^{\infty} j^n\left[b_n\,H_n^{(2)}(k_1 r) + c_n\,H_n^{(1)}(k_1 r)\right]e^{in\varphi} \quad (10)$$

$$E_{2z} = -\widetilde{V}_0 \exp[i(l\varphi - \omega t)] \left(ik\, F_{pl} + \frac{\partial F_{pl}}{\partial z}\right) F_{pl} \sum_{n=-\infty}^{\infty} j^n d_n J_n(k_2 r) e^{in\varphi} \quad (11)$$

In these expressions $J_n(.)$, $H_n^{(1)}(.)$, and $H_n^{(2)}(.)$ are the Bessel functions of the 1st and Hankel functions of the 1st and 2nd kinds, respectively.

The above expressions comprising the unknown scattering coefficients, i.e., a_n, b_n, c_n, and d_n, respectively. To find out these scattering coefficients, the boundary conditions [38] are implemented at interface $r = b$ and $r = a$, respectively.

$$E_{0z}^{inc} + E_{0z}^{sca} = E_{1z} \qquad r = b \qquad 0 \le \varphi \le 2\pi \quad (12)$$

$$H_{0z}^{inc} + H_{0z}^{sca} = H_{1z} \qquad r = b \qquad 0 \le \varphi \le 2\pi \quad (13)$$

$$E_{1z} = E_{2z} \qquad r = a \qquad 0 \le \varphi \le 2\pi \quad (14)$$

$$H_{1\varphi} = H_{2\varphi} \qquad r = a \qquad 0 \le \varphi \le 2\pi \quad (15)$$

For TE polarization, all electromagnetic field equations can be revised using the duality method. Therefore, only their numerical results are given. After finding out these unknown coefficients, the scattered field can be obtained.

3. Results and discussion

Here the numerical results of the analytical calculations performed above are presented. To check the accuracy of our work, we compared the results for the LG_{pl} beam with $p = 0$, $l = 0$ and the plane wave for a nihility coated cylinder. We observed that our results were in accordance with the published literature.

For the LG_{00} beam, we obtained the same results as given in [38] for both polarizations (TM and TE), so our present scattering problem was converted into electromagnetic scattering by a nihility cylinder coated with metamaterials as shown in Figs. 2 (a & b).

After successful comparison with the published literature by applying special condition for the LG beam, further results of the normalized bistatic radar cross section (RCS) regarding the coated nihility cylinder with metamaterials are presented. As our interest is to the study of the effects of the OAM mode number (l) for RCS, so we set $p = 0$. We fixed the nihility coated circular cylinder and coating metamaterial (DPS/DNG), which was replaced one by one to see its scattering response.

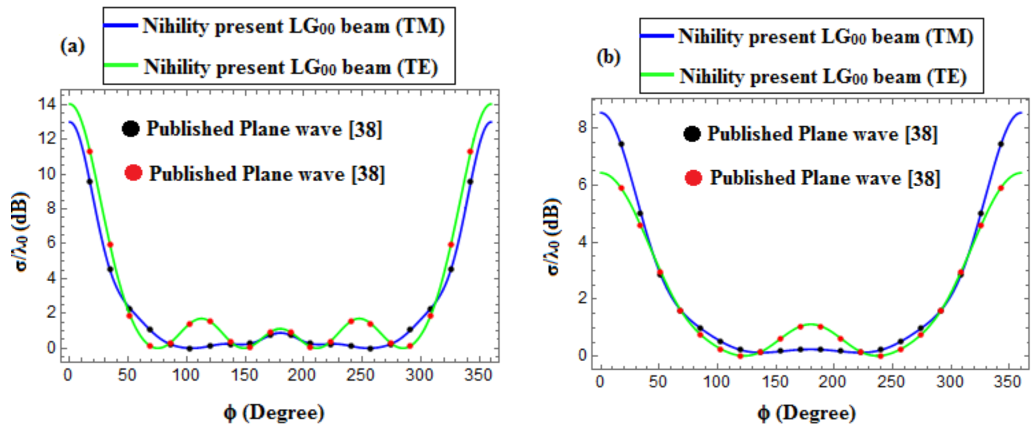

Fig. 2 Graphical comparison between TM and TE scattering field coefficients of the nihility cylinder coated with metamaterials (f = 1 GHz, a = 5cm, b = 10cm, $\varepsilon_{r1} = \pm 9.8$, and $\mu_{r1} = \pm 1$) (a) DPS (b) DNG (color online)

Fig. 3 (a & b) depict the influence of the OAM mode on the normalized bistatic echo width of the coated nihility cylinder for TM polarization for the DPS and DNG coating metamaterials. It is obvious from Figs. 3 and 4 that with increasing the OAM index, the RCS/bistatic echo width changes. It increases for the co-polarized component and decreases for the cross-polarized component, respectively. The contribution of scattering intensity toward the co-polarized field is greater as compared to the cross-polarized field. The formation of lobes is more prominent for the cross-polarized field component.

The scattering response toward the DPS and DNG coatings for the co-polarized component is the same; however, this becomes totally distinct in terms of the cross-polarized scattering component regarding DPS/DNG coating. The distance between the scattering peaks becomes shorter for DNG coating as compared to DPS coating for the cross-polarized scattering field coefficient.

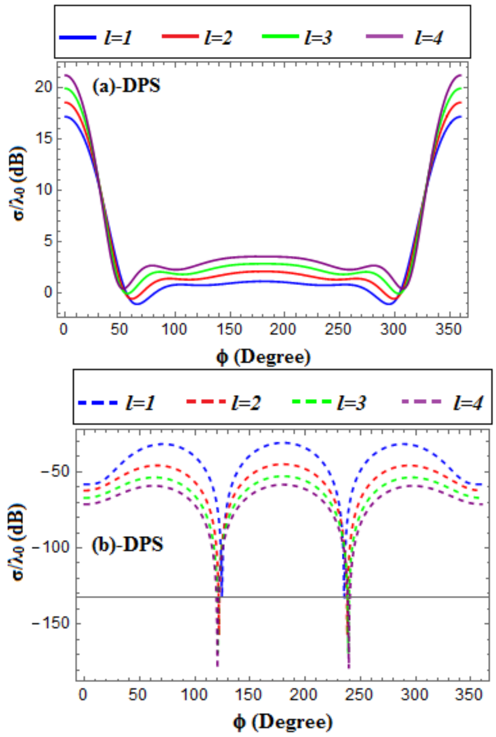

Fig. 3. The influence of the OAM mode index (l) on the normalized bistatic echo width under TM polarization for DPS-coated metamaterial: (a) co-polarized field component and (b) cross-polarized field component (color online)

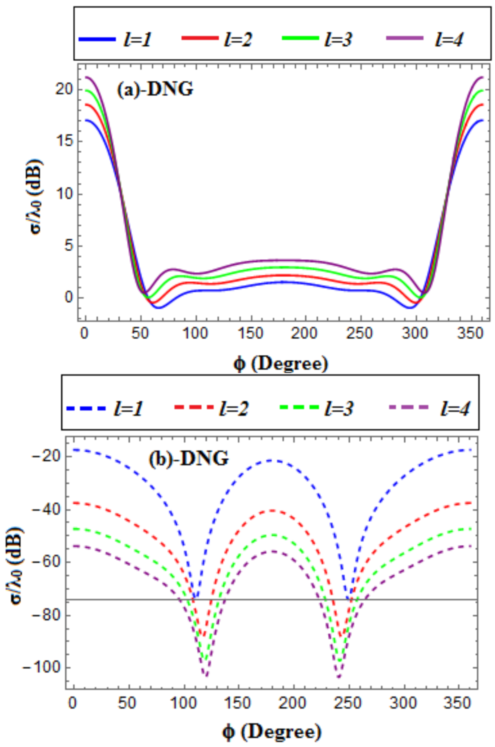

Fig. 4. The influence of the OAM mode index (l) on the normalized bistatic echo width under TM polarization for DNG-coated metamaterial: (a) co-polarized field component and (b) cross-polarized field component (color online)

For TE polarization, changing the l parameter, the RCS clearly changes as can be seen in Figs. 5-6. In Fig. 5(a), the size of the main lobe changes for the scattering angle $\varphi^0 = 80^0 - 280^0$. Basically, the internal field modes are involved for the OAM, and changing this number changes the scattering behavior toward the LG beam.

Fig. 5(b) shows that the RCS decreases for the cross-polarized scattering coefficient as the l increases. Now, the nihility cylinder is coated by DNG metamaterial, and its scattering features are seen in Fig. 6. Fig. 5(b) shows that the bistatic echo width of the scattering coefficient versus scattering angle for $70^0 - 290^0$ decreases, but in Fig. 6(b), it shows an increasing trend due to the specific nature of OAM mode involvement.

The effect of changing frequency is depicted in Fig.7. On varying the incident LG beam frequency in equal number for the unit magnitude of OAM, a visible change in scattering amplitude can be seen. Moreover, the scattering of LG beam with metamaterials i.e., PEMC (sphere/cylinder) will also be beneficial for researchers of optical community [42, 43].

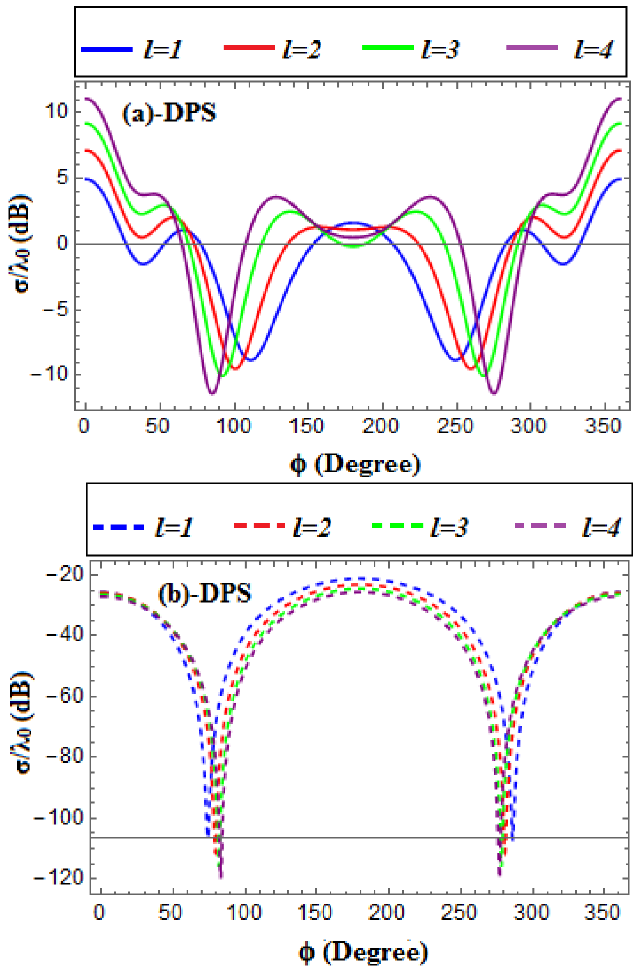

Fig. 5. The influence of the OAM mode index (l) on the normalized bistatic echo width under TE polarization for DPS-coated metamaterial: (a) co-polarized field component and (b) cross-polarized field component (color online)

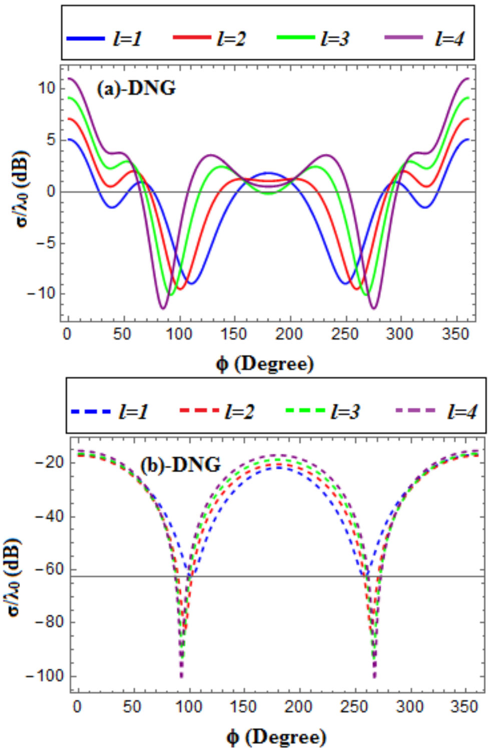

Fig. 6. The influence of the OAM mode index (l) on the normalized bistatic echo width under TE polarization for DNG-coated metamaterial: (a) co-polarized field component and (b) cross-polarized field component (color online)

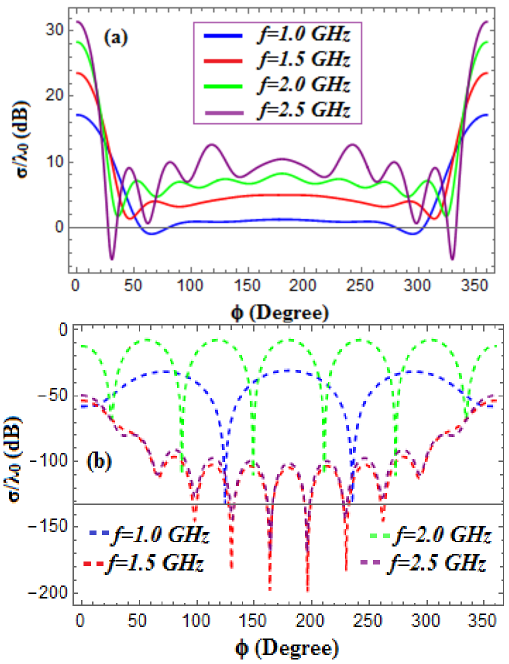

Fig. 7. The influence of beam frequency on the normalized bistatic echo width for a = 5 cm, b = 10 cm: (a) co-polarized field component and (b) cross-polarized field component (color online)

4. Conclusions

In this manuscript, LG beam scattering from a coated circular nihility cylinder by using extended classical wave theory is analyzed. It is observed that the scattering cross section strongly depends on OAM mode number. Moreover, it has been seen that normalized bistatic echo widths of an LG_{pl} beam with beam parameters i.e., $p = 0, l = 0$, and plane wave toward a nihility cylinder coated with metamaterial (DPS/DNG) are the same. The normalized bistatic echo width for a cross-polarized scattering component is very distinct as compared to the co-polarized component regarding DPS and DNG coatings. However, this behavior for DPS coating toward the co-polarized field component is more similar. For certain regions, versus the scattering angle, the overlapping becomes more prominent. The scattering amplitude for TM and TE polarization changes on changing the OAM mode index. The RCS for LG beams depends on the following factors i.e., OAM index, beam frequency, and coated metamaterial. Thus, the proposed structure is well suitable for different scattering problems and its applications in optical research field using different coatings, i.e., plasma, plasmonic, PEMC metamaterials, etc.

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