

Determination of field profiles in buffer superconducting multilayer optical planar waveguides

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The effect on the propagation characteristics due to a low-index buffer layer added over a lossy superconductive leaky waveguide in a high-index substrate is related to a transition from the guided modes to the leaky one. For a guided waveguide structure, neglecting the high-index substrate, the TM mode field amplitude is more confined within the core layer and in superconducting thin film and more evanescent in the air claddings than the corresponding TE mode profile. Moreover, both the TE and TM mode profiles of the buffered waveguide leaks out more into the high-index substrate in comparison with those of the waveguide with a larger buffer thickness. This behaviour is confirmed by the fraction of the power carried by the modes along of the propagation direction.

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1. Introduction

The applications in photonics and optoelectronics of superconducting multilayer optical waveguides where the substrate (lossless) has refractive index lower than that of the lossy superconducting layers were recently analyzed [1-2]. In a superconductive traveling-wave photodetector [1], a superconducting multi-layer waveguide is simultaneously considered as an optical waveguide and a transmission line for high-frequency electric signal. The key phenomenon for the detection of light for frequencies above the gap frequency of superconductors is to annihilate Cooper pairs and create two times as many normal electrons as the broken Cooper pairs [1]. Thus, a new class of ultrasensitive, ultrafast and ultralow-noise detectors based on sensitive superconductive materials in an optical waveguide configuration can be realized.

The effects of a buffer material with a lower refractive index added between the waveguiding structure and the high-index silicon substrate where studied by using a finite element method leaky mode solver [3]. On the other hand, in a recent model of superconductive optical waveguides [1], the low-index interlayer is assumed sufficiently thick that it can serve as a virtual substrate and the high-index substrate is neglected. In this case only guided modes are obtained.

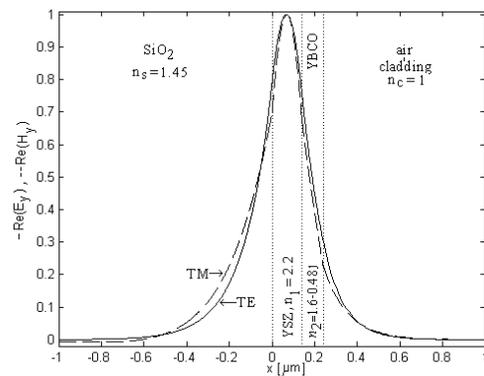


Fig. 1. The real part of the fundamental field profiles (E_y , H_x) for a waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 0.14\mu\text{m}$, $d_3 = 0.24\mu\text{m}$, $n_s = 1.45$, $n_1 = 2.2$, $n_2 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$) where the high-index substrate is neglected: TE (-) and TM (- -) guided modes. The field amplitude has been normalized to a maximum value of unity.

In this paper, we extended the study by considering the low-index interlayer as a buffer layer and including the high-index substrate. We applied a variational method [4] for exact location of each of the zeros of the dispersion equation, which correspond to the leaky modes of multilayer planar waveguides with layer consist of dielectric and superconducting materials, by using the known exact analytical eigenfunctions. Using this approach, the more physical leaky modes are obtained.

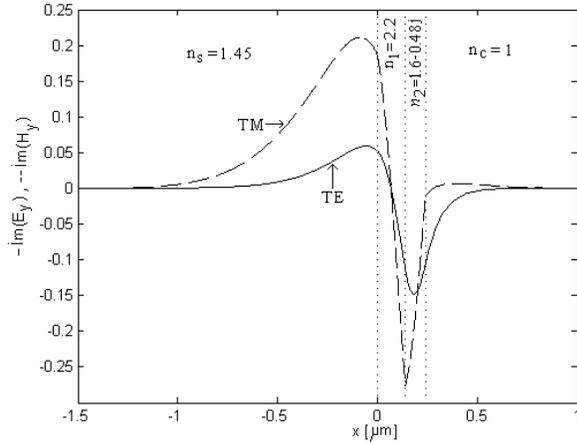


Fig. 2. The imaginary part of the fundamental field profiles (E_y , H_y) for a waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 0.14\mu\text{m}$, $d_3 = 0.24\mu\text{m}$, $n_s = 1.45$, $n_1 = 2.2$, $n_2 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$) where the high-index substrate is neglected: TE (-) and TM (-) guided modes.

2. Buffer superconductor planar waveguide

In our analysis, the buffer leaky planar waveguide structure is obtained by placing a buffer material with a lower refractive index between the waveguiding structure and an yttria stabilized zirconia (YSZ) substrate. The scalar-wave equation for a buffered leaky slab waveguide is given by

$$\frac{d^2\psi(x)}{dx^2} + k^2 n^2(x)\psi(x) = \beta^2\psi(x), \quad (1)$$

where β is the propagation constant, k is the free space wave number, $n(x)$ is the refractive index profile

$$n(x) = \begin{cases} n_s, & \text{for } x < d_1 = 0, \\ n_i, & \text{for } d_i < x < d_{i+1}, i=1,2,3, \\ n_c, & \text{for } d_4 < x, \end{cases} \quad (2)$$

and n_s , n_1 , n_2 , n_3 , and n_c are the refractive index of the YSZ substrate, SiO_2 buffer, YSZ core, YBCO film and air cladding, respectively. The effective index β/k for the TE and TM modes can be found from the dispersion equation, which is obtained by applying the boundary conditions at the interfaces between different layers.

In the variational method, the exact value of β is obtained only when $\psi(x)$ coincides with the exact modal field. In the following we illustrate the application of the variational method to a five-layer slab waveguide that allow exact analytical solutions.

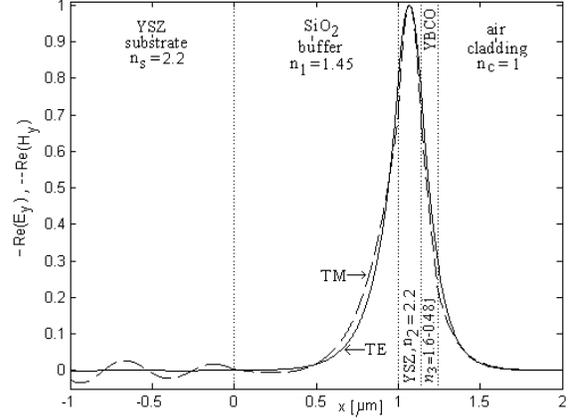


Fig. 3. The real part of the fundamental field profiles (E_y , H_y) of the buffered leaky waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 1\mu\text{m}$, $d_3 = 1.14\mu\text{m}$, $d_4 = 1.24\mu\text{m}$, $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$): TE (-) and TM (-) leaky modes. The field amplitude has been normalized to a maximum value of unity.

For TE (TM) modes, $\psi = E_y$ and $\frac{\partial\psi}{\partial x}$ ($\psi = H_y$ and $\frac{1}{n^2} \frac{\partial\psi}{\partial x}$) are continuous at each interface of the waveguide. The variational exact solution (Eq. (1) can be written as an eigenvalue equation) of the scalar wave Eq. (1) is found from the functional

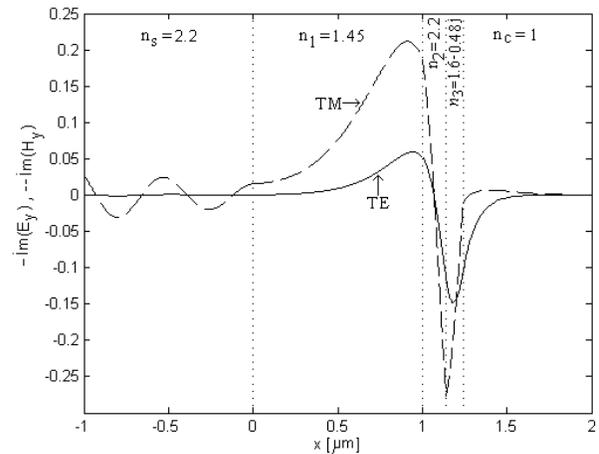


Fig.4. The imaginary part of the fundamental field profiles (E_y , H_y) of the buffered leaky waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 1\mu\text{m}$, $d_3 = 1.14\mu\text{m}$, $d_4 = 1.24\mu\text{m}$, $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$): TE (-) and TM (-) leaky modes.

$$J_{11} = \int_{-\infty}^{d_1} \left[-\frac{f_s^2}{n_s^2 \xi^2} + (kn_s^{1-\xi} f_s)^2 \right] dx + \sum_{i=1}^3 \int_{d_i}^{d_{i+1}} \left[-\frac{f_i^2}{n_i^2 \xi^2} + (kn_i^{1-\xi} f_i)^2 \right] dx + \int_{d_4}^{\infty} \left[-\frac{f_c^2}{n_c^2 \xi^2} + (kn_c^{1-\xi} f_c)^2 \right] dx, \quad (3)$$

subject to the constraint that

$$I_{11} = \int_{-\infty}^{d_1} \frac{f_s^2}{n_s^2 \xi^2} dx + \sum_{i=1}^3 \int_{d_i}^{d_{i+1}} \frac{f_i^2}{n_i^2 \xi^2} dx + \int_{d_4}^{\infty} \frac{f_c^2}{n_c^2 \xi^2} dx, \quad (4)$$

$$\beta^2 = \frac{J_{11}}{I_{11}} \quad (5)$$

where ξ reads as 0 for TE polarized waves and 1 for TM polarized waves and the exact functions f_s , f_i and f_c are given by

$$\begin{aligned} f_s(x) &= A_s \exp(\gamma_s x), & x < d_1, \\ f_i(x) &= A_i \cos[\alpha_i(x - d_i)] + B_i \sin[\alpha_i(x - d_i)], & d_i < x < d_{i+1}, i=1,2,3, \\ f_c(x) &= A_c \exp(-\gamma_c(x - d_4)), & x > d_4, \end{aligned} \quad (6)$$

and

$$\alpha_s = \pm \sqrt{\beta^2 - (n_s k)^2}, \alpha_i = \sqrt{(n_i k)^2 - \beta^2}, \alpha_c = \sqrt{\beta^2 - (n_c k)^2}, i=1,2,3, \quad (7)$$

$$A_1 = A_s = 1, B_1 = \frac{n_1^{2\xi} \alpha_s}{n_s^{2\xi} \alpha_1},$$

$$A_i = A_{i-1} \cos[\alpha_{i-1}(d_i - d_{i-1})] + B_{i-1} \sin[\alpha_{i-1}(d_i - d_{i-1})], i=2,3,$$

$$B_i = \frac{n_i^{2\xi} \alpha_{i-1}}{n_{i-1}^{2\xi} \alpha_i} (B_{i-1} \cos[\alpha_{i-1}(d_i - d_{i-1})] - A_{i-1} \sin[\alpha_{i-1}(d_i - d_{i-1})]), i=2,3,$$

$$A_c = A_3 \cos[\alpha_3(d_4 - d_3)] + B_3 \sin[\alpha_3(d_4 - d_3)], \quad (8)$$

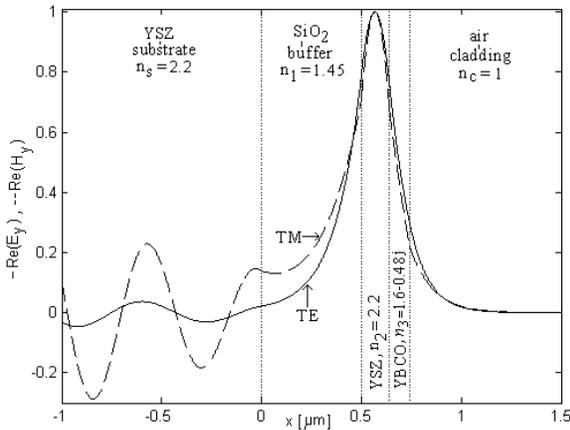


Fig.5. The real part of the fundamental field profiles (E_y , H_y) of the buffered leaky waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 0.5\mu\text{m}$, $d_3 = 0.64\mu\text{m}$, $d_4 = 0.74\mu\text{m}$, $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$): TE (-) and TM (- -) leaky modes. The field amplitude has been normalized to a maximum value of unity.

The minus (plus) sign for α_s corresponds to the leaky (guiding) modes in substrate. The integrals in equations (3 - 4) with the chosen exact functions are evaluated analytically to reduce the amount of numerical computation. The solutions of the dispersion equation (5)

$$\frac{J_{11}}{I_{11}} - \beta^2 = 0 \quad (9)$$

give the propagation constants β and the effective index β/k of the waveguide. The novelty of our dispersion relation that is determined by a variational principle lies in the simultaneous presence of the propagation constant and the known exact field profile.

The total power carried by the TE (TM) mode is related to the electric (magnetic) field through the relation:

$$P = \frac{1}{2\omega\mu_0^{1-\xi}\epsilon_0^\xi} \int_{-\infty}^{\infty} \text{Re} \left[\beta \frac{|\psi(x)|^2}{n^2 \xi^2(x)} \right] dx, \quad (10)$$

where ξ reads as 0 for TE ($\psi = E_y$) polarized waves and 1 for TM ($\psi = H_y$) polarized waves, μ_0 is the magnetic permeability of free space, ϵ_0 is the permittivity in a vacuum and ω is the angular frequency. The effect of the buffer thickness on the confinement of the light in different parts of the waveguide structure is related to the fractional power P_i/P , where P_i is the power carried by a mode in a specific part of the waveguide.

3. Numerical results and conclusions

We have calculated the exact value of the effective index β/k for a waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 0.14\mu\text{m}$, $d_3 = 0.24\mu\text{m}$, $n_s = 1.45$, $n_1 = 2.2$, $n_2 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$), where the high-index substrate is neglected (Table 1). The real and imaginary parts of the TE and TM guided mode profiles of this structure are shown at the same scale in Fig. 1 and in Fig.2, respectively. The field amplitude has been normalized to a maximum value of unity (modal fields are divided for the maximum value of the function f_2 in (6) for the YSZ core). For this guided waveguide structure the real and imaginary parts of the TM mode amplitude are more confined within the core layer and in superconducting thin film and more evanescent in the air claddings in comparison with the TE mode profile.

Table 1. Comparison of the effective index β/k for two waveguides which differ by the buffer thicknesses: ($d_1 = 0\mu\text{m}$, $d_2 = 0.5\mu\text{m}$, $d_3 = 0.64\mu\text{m}$, $d_4 = 0.74\mu\text{m}$) and ($d_1 = 0\mu\text{m}$, $d_2 = 1\mu\text{m}$, $d_3 = 1.14\mu\text{m}$, $d_4 = 1.24\mu\text{m}$) with the same values of $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$. TE_{0n} and TM_{0n} are the guided modes for a very large buffer thickness when the high-index substrate is neglected.

Mode	$d_2 = 0.5\mu\text{m}$, β/k
TE ₀	1.7643573628 - 0.0649473572j
TM ₀	1.5547398356 - 0.1120650116j
Mode	$d_2 = 1\mu\text{m}$, β/k
TE ₀	1.7642468876 - 0.0648113648j
TM ₀	1.5499978452 - 0.1135094283j
Mode	β/k
TE _{0n}	1.7642467870 - 0.0648113597j
TM _{0n}	1.5500447183 - 0.1135462563j

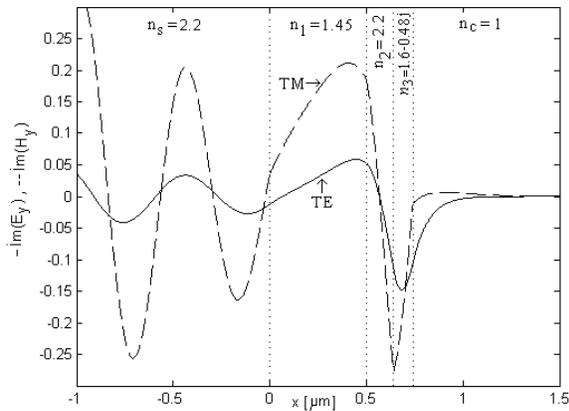


Fig. 6. The imaginary part of the fundamental field profiles (E_y , H_y) of the buffered leaky waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 0.5\mu\text{m}$, $d_3 = 0.64\mu\text{m}$, $d_4 = 0.74\mu\text{m}$, $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$): TE (-) and TM (-) leaky modes.

Comparing the results with those in [1], we report a typographical error in [1] regarding the values of the effective index β/k for the fundamental TE₀ mode of a waveguide in which the high-index substrate is neglected and the thickness of the superconducting film is fixed to 100nm. Thus, the correct values are $\beta/k = 1.83 - 0.05j$ for a thickness of the YSZ layer $d_{\text{YSZ}} = 170\text{nm}$ (not 400nm) and $\beta/k = 2.17 - 0.001j$ for $d_{\text{YSZ}} = 1000\text{nm}$ (not 170nm).

Also, we have calculated the exact value of the effective index β/k for a waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 1\mu\text{m}$, $d_3 = 1.14\mu\text{m}$, $d_4 = 1.24\mu\text{m}$, $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$) with a larger buffer thickness (Table 1). The real and imaginary parts of the TE and TM leaky mode profiles of this structure are shown at the same scale in Fig. 3 and in Fig. 4,

respectively. The TM mode is more leaky out into the high-index substrate in comparison with the TE mode.

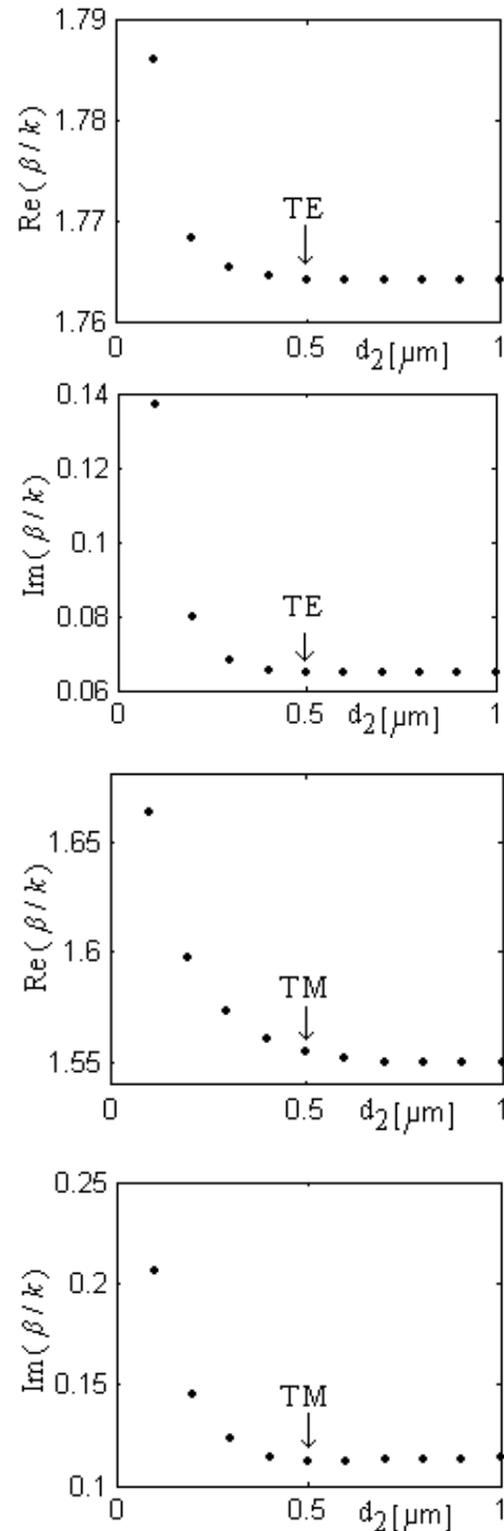


Fig. 7. The real and imaginary parts of the effective index versus the buffer thickness d_2 for TE and TM modes.

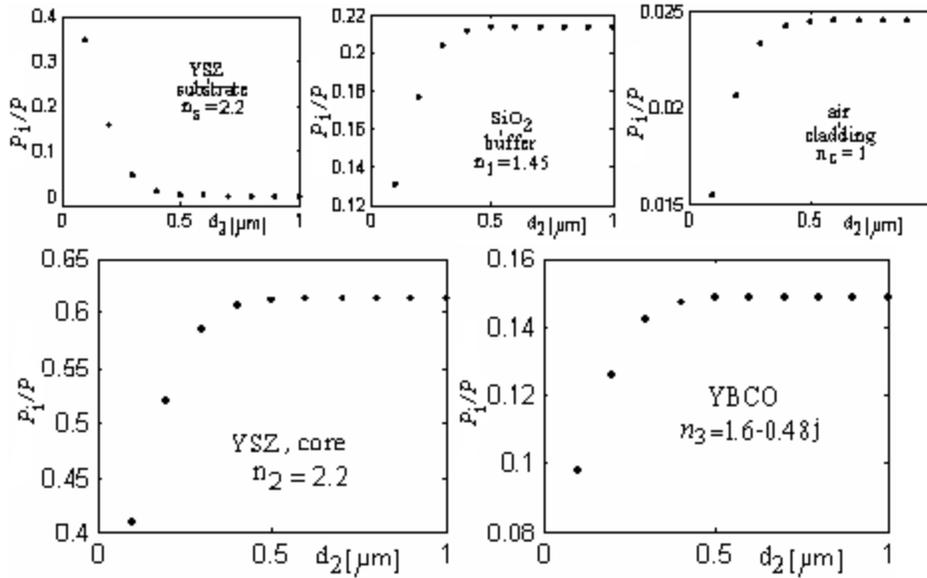


Fig. 8. The fraction of the power P_1/P , versus the buffer thickness d_2 , carried by the fundamental mode TE in different parts of the waveguide structure.

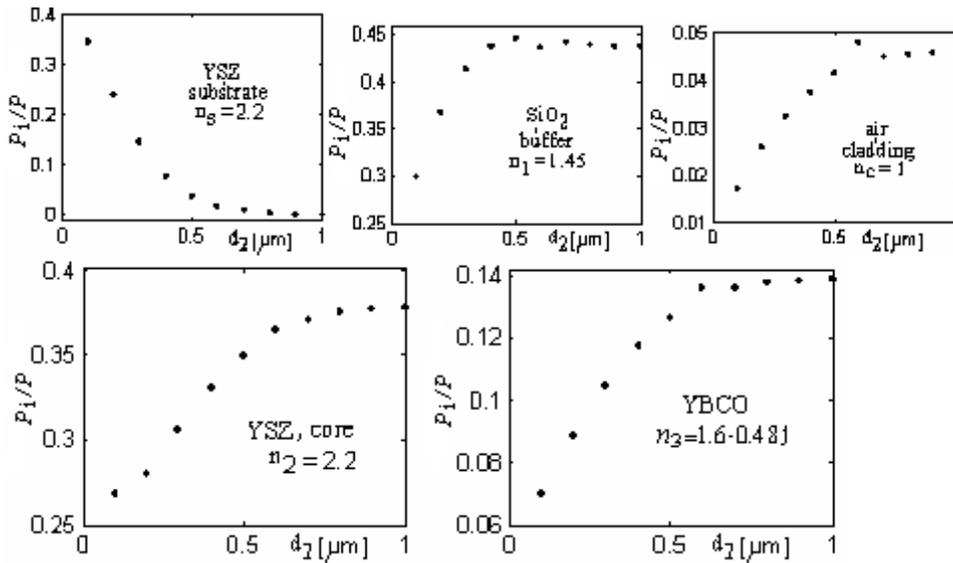


Fig. 9. The fraction of the power P_1/P , versus the buffer thickness d_2 , carried by the fundamental mode TM in different parts of the waveguide structure.

In another example we have calculated the exact value of the effective index β/k for a waveguide ($d_1 = 0\mu\text{m}$, $d_2 = 0.5\mu\text{m}$, $d_3 = 0.64\mu\text{m}$, $d_4 = 0.74\mu\text{m}$, $n_s = 2.2$, $n_1 = 1.45$, $n_2 = 2.2$, $n_3 = 1.6 - 0.48j$, $n_c = 1$, $\lambda = 0.85\mu\text{m}$) with a smaller buffer thickness (Table 1).

The real and imaginary parts of the TE and TM leaky mode profiles of this structure are shown at the same scale in Fig.5 and in Fig.6, respectively. The TE and TM modes

for this waveguide are more leaky out into the high-index substrate in comparison with the waveguide with a larger buffer thickness (compare with Figs 3-4).

Fig.7 shows the effect of thickening the buffer layer on the real and imaginary parts of the effective index. If the thickness of the buffer layer is increased, the real and imaginary parts of the effective index are decreased. In [3], for another buffered leaky planar waveguide structure,

the real parts of the effective index will increase asymptotically with the buffer thickness.

Figs. 8 - 9 show the fraction of the power, versus the buffer thickness, carried by the TE and TM modes (per unit length in y direction) along the z axis, for different parts of the waveguide structure. With a increase in the buffer layer thickness, the fraction of the power in YSZ substrate is diminished and the power is increased in the core layer.

Our analyses are important for engineering design and for understanding the physical processes of multilayer waveguides with layers consisting of dielectric and superconducting materials.

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