JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 18, Iss. 5-6, May – June 2016, p. 440 - 444

Dispersive optical solitons in birefringent fibers with Schrödinger-Hirota equation

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Abstract. This paper obtains exact bright, dark and singular dispersive optical solitons in birefringent fibers in presence of several perturbation terms. The governing coupled Schrödinger-Hirota equation is integrated to extract these soliton solutions. The method of undetermined coefficients is applied to retrieve these soliton solutions. Constraint conditions naturally emerge for these solitons to exist.

(Received January 01, 2016; accepted June 09, 2016)

Keywords: Solitons, Dispersion, Birefringence, Integrability, Constraints

1. Introduction

Optical solitons in birefringent fibers is an important area of research in the field of nonlinear optics. The phenomena of birefringence occurs naturally in optical fibers. In a realistic situation, optical fibers are not perfectly symmetrical, diametrically. Therefore the two modes can propagate with different group velocity and phase. Thus a pulse injected into an optical fiber would split into two orthogonally polarized pulses and they will propagate with different group velocities. This leads to group velocity mismatch and hence differential group delay at the terminal end of the fiber. Therefore one encounters dispersed pulses and this phenomena is referred to as polarization mode dispersion. This is therefore a very unwanted feature that introduces several limitations to high bit-rate fiber optic communications, which eventually leads to erroneous data transmission across trans-oceanic and trans-continental distances.

This paper studies dispersive optical solitons in birefringent fibers in presence of spatio-temporal dispersion in addition to group-velocity dispersion (GVD), intermodal dispersion and higher order dispersion terms. The search is for exact bright, dark and singular 1-soliton solution to the model. The method of undetermined coefficients will be the integration scheme adopted in this paper. This will lead to the soliton solutions along with their respective constraint conditions, also known as integrability criteria, that will guarantee the existence of these solitons.

2. Governing equation

The dynamics of soliton propagation through optical fibers is governed by the nonlinear Schrödinger's equation (NLSE) [1-32]. However, in case of birefringence it is the vector coupled NLSE that is studied [3, 7, 25, 26]. For dispersive optical solitons, NLSE is transformed through Lie symmetry to Schrödinger-Hirota equation (SHE). This derivation has been discussed in several papers [1, 2, 4-6, 19, 20, 29]. Thus, in birefringent fibers, it is the vector-coupled SHE that represents the governing model, in this case. This paper will address the following dimensionless form of SHE [2, 11]:

$$\begin{aligned} &iq_t + i\alpha_1 q_x + a_1 q_{xx} + b_1 q_{xt} + (c_1|q|^2 + d_1|r|^2)q \\ &+ i\gamma_1 q_{xxx} + i(\sigma_1|q|^2 + \xi_1|r|^2)q_x = 0 \end{aligned} \tag{1}$$

$$\begin{aligned} &ir_t + i\alpha_2 q_x + a_2 r_{xx} + b_2 q_{xt} + (c_2|r|^2 + d_2|q|^2)r \\ &+ i\gamma_2 r_{xxx} + i(\sigma_2|r|^2 + \xi_2|q|^2)r_x = 0 \end{aligned} \tag{2}$$

Here, $q(x,t)$ and $r(x,t)$ represent the wave profile of the two pulses, α_j represents the inter-modal dispersion, a_j represents group velocity dispersion GVD, b_j gives the spatio-temporal dispersion, c_j is from self-phase modulation while d_j is due to the cross-phase modulation, then γ_j is the third order dispersion coefficient (3OD) while σ_j and ξ_j are from the nonlinear dispersions [7, 25, 26].

Here, $j = 1, 2$. Also, the first term in both of these equations represents the temporal evolution of the pulses in birefringent fibers. It needs to be noted that the inclusion of spatio-temporal dispersion was first proposed during 2012 to keep the model well-posed [13, 22].

To start off with the integration of the model (1) and (2), the following hypothesis for the soliton solution structure is selected:

$$q(x,t) = P_1(x,t)\exp[i\phi_1(x,t)] \tag{3}$$

and

$$r(x,t) = P_2(x,t)\exp[i\phi_2(x,t)] \tag{4}$$

where

$$\phi_j(x,t) = -\kappa_j x + \omega_j t + \theta_j \tag{5}$$

for $j = 1, 2$. Here in (3) and (4), $P_j(x,t)$ represents the solitary wave profile and $\phi_j(x,t)$ is the phase component of the solitons. Also from (5), κ_j is the soliton frequency for the two components, ω_j is the wave number of the two components and finally θ_j represents the centers of phase. Substituting (3), (4) and (5) into (1) and (2) leads to

$$\begin{aligned} &P_j\{\omega_j - \kappa_j(\alpha_j + \omega_j b_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\} \\ &-b_j\frac{\partial^2 P_j}{\partial x\partial t} - (a_j + 3\gamma_j\kappa_j)\frac{\partial^2 P_j}{\partial x^2} \\ &-(c_j + \sigma_j\kappa_j)P_j^3 - (d_j + \kappa_j\xi_j)P_j P_{\bar{j}}^2 = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{\partial P_j}{\partial t}(1 - b_j\kappa_j) - \frac{\partial P_j}{\partial x}(2a_j\kappa_j - \alpha_j - b_j\omega_j + 3\gamma_j\kappa_j^2) \\ &+ P_j^2\frac{\partial P_j}{\partial x}\sigma_j + P_{\bar{j}}^2\frac{\partial P_j}{\partial x}\xi_j + \gamma_j\frac{\partial^3 P_j}{\partial x^3} = 0 \end{aligned} \tag{7}$$

for real and imaginary part, respectively. Here, the notation $\bar{j} = 3 - j$ was introduced. This pair of relations (6) and (7) will now be analyzed for four different types of solitons that will now be discussed in details in subsequent section.

3. Soliton solutions

This section will obtain solutions to the model that was introduced in the previous section, given by (1) and (2). Bright, dark and singular soliton solutions will be obtained along with their respective constraint conditions for its existence. The methodology that will be adopted is the principle of undetermined coefficients. The study will now be split into the following four subsections.

3.1 Bright solitons

For bright soliton solution, starting hypothesis is given by [1-3, 7]

$$P_j(x,t) = A_j\,\mathrm{sech}^{p_j}\tau \tag{8}$$

where

$$\tau = B(x - vt) \tag{9}$$

In (8) and (9), A_j represents the amplitude of the solitons for the two components and B is the width of the solitons and finally v is the speed of the solitons for the two components. Substituting this hypothesis into (6) and (7) reduces them to

$$\begin{aligned} &\{\omega_j - \kappa_j(\alpha_j + \omega_j b_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\} \\ &- p_j B^2(a_j + 3\gamma_j\kappa_j - vb_j) \\ &+ p_j(p_j+1)B^2(a_j + 3\gamma_j\kappa_j - vb_j) \\ &- A_j(c_j + \sigma_j\kappa_j)\,\mathrm{sech}^{2p_j}\tau \\ &- A_{\bar{j}}^2(d_j + \kappa_j\xi_j)\,\mathrm{sech}^{2p_{\bar{j}}}\tau = 0 \end{aligned} \tag{10}$$

and

$$\begin{aligned} &v_j(1 - b_j\kappa_j) - \alpha_j + 2a_j\kappa_j - b_j\omega_j + 3\gamma_j\kappa_j^2 - \gamma_j p_j^2 B^2 \\ &+ \gamma_j B^2(p_j+1)(p_j+2)\,\mathrm{sech}^2\tau \\ &- \sigma_j A_j^2\mathrm{sech}^{2p_j}\tau - \xi_j A_{\bar{j}}^2\,\mathrm{sech}^{2p_{\bar{j}}}\tau = 0 \end{aligned} \tag{11}$$

Application of balancing principle to (10) and (11) gives

$$p_j = 1 \tag{12}$$

for $j = 1, 2$. Next, from real part equation (10), setting the coefficients of linearly independent functions to zero leads to

$$\omega_j = \frac{B^2(a_j - vb_j) + \kappa_j(\alpha_j + 3\gamma_j B^2) - a_j\kappa_j^2 - \gamma_j\kappa_j^3}{1 - \kappa_j b_j} \tag{13}$$

and

$$\begin{aligned} &2B^2(a_j + 3\gamma_j\kappa_j - vb_j) \\ &= (c_j + \sigma_j\kappa_j)A_j^2 + (d_j + \kappa_j\xi_j)A_{\bar{j}}^2 \end{aligned} \tag{14}$$

which is the relation between the amplitudes and the widths of the solitons for the two components. From the imaginary part, similarly, the principle of undetermined coefficients leads to

$$v = \frac{\alpha_j - 2a_j\kappa_j + \gamma_j B^2 + b_j\omega_j - 3\gamma_j\kappa_j^2}{1 - \kappa_j b_j} \tag{15}$$

$$\omega_j = \frac{v(1 - \kappa_j b_j) - \alpha_j + 2a_j b_j - \gamma_j B^2 + 3\gamma_j\kappa_j^2}{b_j} \tag{16}$$

and

$$6\gamma_j B^2 = \sigma_j A_j^2 + \xi_j A_{\bar{j}}^2 \tag{17}$$

Thus (17), with $j = 1, 2$ implies

$$A_j = B\sqrt{\frac{6(\gamma_j\sigma_{\bar{j}} - \gamma_{\bar{j}}\xi_j)}{\sigma_j\sigma_{\bar{j}} - \xi_j\xi_{\bar{j}}}} \tag{18}$$

which is the relation between the bright solitons' amplitudes and their width. The constraint relation therefore is

$$(\gamma_j\sigma_{\bar{j}} - \gamma_{\bar{j}}\xi_j)(\sigma_j\sigma_{\bar{j}} - \xi_j\xi_{\bar{j}}) > 0 \tag{19}$$

Finally (15), for $j = 1, 2$ gives the width of the solitons as

$$B = \left[\frac{\begin{pmatrix} v(\kappa_{\bar{j}}b_{\bar{j}} + 2(a_j\kappa_j - a_{\bar{j}}\kappa_{\bar{j}}) \\ +3(\gamma_j\kappa_j^2 - \gamma_{\bar{j}}\kappa_{\bar{j}}^2) + (\alpha_{\bar{j}} - \alpha_j) \end{pmatrix}}{\gamma_j - \gamma_{\bar{j}}}\right]^{1/2} \tag{20}$$

Therefore the width of the bright soliton for the two components is completely determined from the given parameters. Now, this provokes the constraint

$$(\gamma_j - \gamma_{\bar{j}})\begin{pmatrix} v(\kappa_{\bar{j}}b_{\bar{j}} + 2(a_j\kappa_j - a_{\bar{j}}\kappa_{\bar{j}}) \\ +3(\gamma_j\kappa_j^2 - \gamma_{\bar{j}}\kappa_{\bar{j}}^2) + (\alpha_{\bar{j}} - \alpha_j) \end{pmatrix} > 0 \tag{21}$$

for these bright solitons to exist. Thus, dispersive bright solitons in birefringent fibers are:

$$q(x,t) = A_1 \operatorname{sech}[B(x - vt)]e^{i(-\kappa_1 x + \omega_1 t + \theta_1)} \tag{22}$$

$$r(x,t) = A_2 \operatorname{sech}[B(x - vt)]e^{i(-\kappa_2 x + \omega_2 t + \theta_2)} \tag{23}$$

with the deinition of parameters and constraints listed above.

3.2 Dark solitons

For dark soliton solutions, the starting hypothesis is [1-3, 7]

$$P_j(x,t) = A_j \tanh^{p_j}\tau \tag{24}$$

for $j = 1, 2$ and the definition of τ being the same as in (9). In this case A_j , for $j = 1, 2$ and B are free parameters. From this hypothesis (6) and (7) respectively reduce to:

$$\begin{aligned}
&p_j(p_j - 1)B^2(a_j + 3\gamma_j\kappa_j - vb_j) \\
&-\{\omega_j - \kappa_j(\alpha_j + \omega_j b_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3 \\
&+2p_j^2 B^2(a_j + 3\gamma_j\kappa_j - vb_j)\}\tanh^2\tau \\
&+p_j(p_j + 1)B^2(a_j + 3\gamma_j\kappa_j - vb_j)\tanh^4\tau \\
&+A_j^2(c_j + \sigma_j\kappa_j)\tanh^{2p_j+2}\tau + (d_j + \xi_j\kappa_j)\tanh^{2p_{\bar{j}}+2}\tau = 0
\end{aligned} \tag{25}$$

$$\begin{aligned}
&\gamma_j B^2(p_j - 1)(p_j - 2) \\
&-\{v(1 - b_j\kappa_j) - \alpha_j + 2a_j\kappa_j - b_j\omega_j + 3\gamma_j\kappa_j^2 \\
&+(3p_j^2 - 3p_j + 2)\gamma_j B^2\}\tanh^2\tau \\
&+\{v(1 - b_j\kappa_j) - \alpha_j + 2a_j\kappa_j - b_j\omega_j + 3\gamma_j\kappa_j^2 \\
&+(3p_j^2 + 3p_j + 2)\gamma_j B^2\}\tanh^4\tau \\
&-(p_j + 1)(p_j + 2)\gamma_j B^2\tanh^6\tau \\
&+(\sigma_j A_j^2\tanh^{2p_j}\tau + \xi_{\bar{j}}A_{\bar{j}}^2\tanh^{2p_{\bar{j}}}\tau(\tanh^2\tau - \tanh^4\tau) = 0
\end{aligned} \tag{26}$$

From balancing principle, the same value of p_j as in (12) is recovered. Next, from the undetermined coefficients of real part equation (25), the following relations are obtained:

$$\omega_j = \frac{\kappa_j\alpha_j - 2a_jB^2 + 2b_j\gamma_jB^2 - 6\gamma_j\kappa_jB^2 - \gamma_j\kappa_j^3 - a_j\kappa_j^2}{1 - \kappa_j b_j} \tag{27}$$

and

$$\begin{aligned}
&2B^2(a_j + 3\gamma_j\kappa_j - vb_j) \\
&+(c_j + \sigma_j\kappa_j)A_j^2 + (d_j + \kappa_j\xi_j)A_{\bar{j}}^2 = 0
\end{aligned} \tag{28}$$

The imaginary part equation (26) gives

$$v = \frac{\alpha_j - 2a_j\kappa_j - 2\gamma_jB^2 + b_j\omega_j - 3\gamma_j\kappa_j^2}{1 - \kappa_j b_j} \tag{29}$$

$$v = \frac{\begin{pmatrix} \alpha_j - 2a_j\kappa_j - 8\gamma_jB^2 \\ +b_j\omega_j - 3\gamma_j\kappa_j^2 - \alpha_jA_j^2 - \xi_jA_{\bar{j}}^2 \end{pmatrix}}{1 - \kappa_j b_j} \tag{30}$$

and

$$6\gamma_jB^2 + \alpha_jA_j^2 + \xi_jA_{\bar{j}}^2 = 0 \tag{31}$$

From (29), one gets

$$B = A_j\sqrt{\frac{\xi_j\sigma_{\bar{j}} - \sigma_j\sigma_{\bar{j}}}{6(\gamma_j\sigma_{\bar{j}} - \gamma_{\bar{j}}\xi_j)}} \tag{32}$$

This introduces the integrability criteria for dark solitons as

$$(\xi_j\sigma_{\bar{j}} - \sigma_j\sigma_{\bar{j}})(\gamma_j\sigma_{\bar{j}} - \gamma_{\bar{j}}\xi_j) > 0 \tag{33}$$

Similarly, as in bright solitons, the free parameter B is given in terms of soliton prameters as

$$B = \left[\frac{\begin{pmatrix} v(\kappa_{\bar{j}}b_{\bar{j}} - \kappa_jb_j) + 2(a_j\kappa_j - a_{\bar{j}}\kappa_{\bar{j}}) \\ +3(\gamma_j\kappa_j^2 - \gamma_{\bar{j}}\kappa_{\bar{j}}^2) + (\alpha_{\bar{j}} - \alpha_j) \end{pmatrix}}{2(\gamma_j - \gamma_{\bar{j}})}\right]^{1/2} \tag{34}$$

with the same restriction (21). Finally, dispersive dark solitons in birefringent fibers are:

$$q(x,t) = A_1 \tanh[B(x-vt)]e^{i(-\kappa_1 x+\omega_1 t+\theta_1)} \quad (35)$$

$$r(x,t) = A_2 \tanh[B(x-vt)]e^{i(-\kappa_2 x+\omega_2 t+\theta_2)} \quad (36)$$

with the deinition of parameters and constraints listed above.

3.3 Singular solitons (Type-1)

For singular solitons, the starting hypothesis is given by [1,2]

$$P_j(x,t) = A_j \operatorname{csch}^{p_j}\tau \quad (37)$$

with A_j and B again being free parameters. This choice reduces (6) and (7) to

$$\begin{aligned}&\{\omega_j - \kappa_j(\alpha_j + \omega_j b_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\}\\&- p_j B^2(a_j + 3\gamma_j\kappa_j - vb_j)\\&- p_j(p_j+1)B^2(a_j + 3\gamma_j\kappa_j - vb_j)\\&- A_j(c_j + \sigma_j\kappa_j)\operatorname{csch}^{2p_j}\tau\\&- A_{\bar{j}}^2(d_j + \kappa_j\xi_j)\operatorname{csch}^{2p_{\bar{j}}}\tau = 0\end{aligned} \quad (38)$$

and

$$\begin{aligned}&v_j(1-b_j\kappa_j) - \alpha_j + 2a_j\kappa_j - b_j\omega_j + 3\gamma_j\kappa_j^2 - \gamma_j p_j^2 B^2\\&- \gamma_j B^2(p_j+1)(p_j+2)\operatorname{csch}^2\tau\\&- \sigma_j A_j^2 \operatorname{csch}^{2p_j}\tau - \xi_j A_{\bar{j}}^2 \operatorname{csch}^{2p_{\bar{j}}}\tau = 0\end{aligned} \quad (39)$$

respectively. Application of balancing principle to (38) and (39) gives (12). Next, from real part equation (38), setting the coefficients of linearly independent functions to zero leads to

$$\omega_j = \frac{B^2(a_j - vb_j) + \kappa_j(\alpha_j + 3\gamma_j B^2) - a_j\kappa_j^2 + \gamma_j\kappa_j^3}{1-\kappa_j b_j} \quad (40)$$

and

$$\begin{aligned}&2B^2(a_j + 3\gamma_j\kappa_j - vb_j)\\&+ (c_j + \sigma_j\kappa_j)A_j^2 + (d_j + \kappa_j\xi_j)A_{\bar{j}}^2 = 0\end{aligned} \quad (41)$$

which is the relation between the amplitudes and the widths of the solitons for the two components. From the imaginary part, similarly, the principle of undetermined coefficients leads to (15), (16) and (17). Now, (17), with $j=1,2$ implies

$$A_j = B\sqrt{-\frac{6(\gamma_j\sigma_{\bar{j}} - \gamma_{\bar{j}}\xi_j)}{\sigma_j\sigma_{\bar{j}} - \xi_j\xi_{\bar{j}}}} \quad (42)$$

which is the relation between the bright solitons' amplitudes and their width. Also relations (20) and (21) are applicable in this case. The constraint condition for this relation to hold is given by

$$(\gamma_j\sigma_{\bar{j}} - \gamma_{\bar{j}}\xi_j)(\sigma_j\sigma_{\bar{j}} - \xi_j\xi_{\bar{j}}) < 0 \quad (43)$$

Therefore singular solitons, of first kind, for birefringent fibers are:

$$q(x,t) = A_1 \operatorname{csch}[B(x-vt)]e^{i(-\kappa_1 x+\omega_1 t+\theta_1)} \quad (44)$$

$$r(x,t) = A_2 \operatorname{csch}[B(x-vt)]e^{i(-\kappa_2 x+\omega_2 t+\theta_2)} \quad (45)$$

with the deinition of parameters and constraints are given.

3.4 Singular solitons (Type-2)

For dark soliton solutions, the starting hypothesis is

$$P_j(x,t) = A_j \coth^{p_j}\tau \quad (46)$$

where the definition of τ being the same as in (9). In this case A_j, for $j=1,2$ and B are free parameters. From this hypothesis (6) and (7) respectively reduce to:

$$\begin{aligned}&p_j(p_j-1)B^2(a_j + 3\gamma_j\kappa_j - vb_j)B^2\\&-\{\omega_j - \kappa_j(\alpha_j + \omega_j b_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\\&+ 2p_j^2 B^2(a_j + 3\gamma_j\kappa_j - vb_j)\}\coth^2\tau\\&+ p_j(p_j+1)B^2(a_j + 3\gamma_j\kappa_j - vb_j)\coth^4\tau\\&+ A_j^2(c_j + \sigma_j\kappa_j)\tanh^{2p_j+2}\tau + (d_j + \xi_j\kappa_j)\coth^{2p_{\bar{j}}+2}\tau = 0\end{aligned} \quad (47)$$

$$\begin{aligned}&\gamma_j B^2(p_j-1)(p_j-2)\\&-\{v(1-b_j\kappa_j) - \alpha_j + 2a_j\kappa_j - b_j\omega_j + 3\gamma_j\kappa_j^2\\&+ (3p_j^2 - 3p_j + 2)\gamma_j B^2\}\coth^2\tau\\&+\{v(1-b_j\kappa_j) - \alpha_j + 2a_j\kappa_j - b_j\omega_j + 3\gamma_j\kappa_j^2\\&+ (3p_j^2 + 3p_j + 2)\gamma_j B^2\}\coth^4\tau\\&- (p_j+1)(p_j+2)\gamma_j B^2\coth^6\tau\\&+ (\sigma_j A_j^2\coth^{2p_j}\tau + \xi_{\bar{j}} A_{\bar{j}}^2 \coth^{2p_{\bar{j}}}\tau)\\&\times(\coth^2\tau - \coth^4\tau) = 0\end{aligned} \quad (48)$$

The same logical argument as in the case of dark solitons follow for this kind of singular solitons. Therefore all parameter definitions and their respective constraints stay the same as in Section 4.2. Therefore singular solitons, of second kind, for birefringent fibers are:

$$q(x,t) = A_1 \coth[B(x-vt)]e^{i(-\kappa_1 x+\omega_1 t+\theta_1)} \quad (49)$$

$$r(x,t) = A_2 \coth[B(x-vt)]e^{i(-\kappa_2 x+\omega_2 t+\theta_2)} \quad (50)$$

4. Conclusion

This paper studied dispersive optical solitons in birefringent fibers with vector-coupled SHE. The method of undetermined coefficients retrieved bright, dark and singular soliton solutions to the model. The constraint conditions ensure their existence. The results of this paper carry a lot of prospect into future. Additional integration schemes will be applied to such model. These include Lie symmetry analysis, G'/G-expansion scheme, Kudryashov's method, Riccati equation schem, exp-function method and others. Later, these results will be extended to the case of DWDM systems and also to optical fibers as well as magneto-optic waveguides. Additionally, conservation laws for bright solitons will be obtained and these results will be dissemenated in other journals.

Acknowledgement

The research work of first (IB) and second (NM) authors is supported by NSF-CREST Grant Number: 0630388 and is sincerely appreciated. This research for fifth (AB) and sixth (MB) authors is funded by Qatar National Research Fund (QNRF) under the grant number NPRP 6-021-1-005 and this support is thankfully acknowledged. The authors also declare that there is no conflict of interest.

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