Dispersive optical solitons with DWDM technology and four-wave mixing by modified simple equation method

AHMED H. ARNOUS[a], M. M. BABATIN[b], MIR ASMA[c], ANJAN BISWAS[b,d,*]
[a]Department of Physics and Engineering Mathematics, Higher Institute of Engineering, El-Shorouk, Cairo, Egypt
[b]Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh-13318, Saudi Arabia
[c]Institute of Mathematical Sciences, Faculty of Science, University of Malaya, 50603 Kuala Lumpur, Malaysia
[d]Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria-0008, South Africa

This paper studies dispersive optical solitons for DWDM systems in presence of four-wave mixing. The modified simple equation algorithm is implemented to retrieve soliton solutions to the model. These solitons appear with existence criteria that are refered to as constraints and they are also presented.

(Received May 31, 2017; accepted August 9, 2018)

Keywords: Solitons, Dispersion, DWDM system

1. Introduction

Performance enhancement as well as parallel transmission of optical solitons across trans-oceanic and trans-continental distances are the key features to technological marvel in telecom industry. These can be only achieved by the aid of DWDM system where parallel transmission of data is possible. This paper will address DWDM system for dispersive optical solitons that are governed by the Schrödinger-Hirota equation (SHE). Although DWDM systems have been studied in the past, this paper will consider it in presence of four-wave mixing (4WM) terms that will make the model much closer to reality. With the inclusion of 4WM, it is necessary to have phase-matching condition among the components to permit integrability. The integration algorithm that will be implemented is the modified simple equation scheme [1-25]. Both Kerr law and parabolic law of nonlinearity are studied in this paper. Dark and singular soliton solutions are obtained together with their existence criteria that are also presented.

2. Overview of modified simple equation method

Here is the nonlinear evolution equation:

$$P(u,u_t,u_x,u_{xx},u_{tt},u_{tx},...)=0. \tag{1}$$

In (1), P is a polynomial in $u(x,t)$ and its partial derivatives where the highest order derivatives and nonlinear terms are involved. The main steps of this scheme are as follows [2, 3]:

Step-1: The transformation

$$u(x,t)=u(\xi),\ \xi=x-ct, \tag{2}$$

where c is a constant to be determined, reduces Eq. (1) to the following ordinary differential equation:

$$Q(u,u',u'',...)=0, \tag{3}$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, while the notation is $'=\dfrac{d}{d\xi}$.

Step-2: Assume Eq. (3) permits the structural solution:

$$u(\xi)=\sum_{l=0}^{N}a_l\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{4}$$

where a_l are constants to be determined, such that $a_N\neq 0$, and $\psi(\xi)$ is an unknown function that is yet to be determined.

Step-3: We find the value of positive integer N in Eq. (4) by considering the homogeneous balance between the highest order derivatives with nonlinear terms in Eq. (3).

Step-4: We plug in (4) into (3) and compute all the necessary derivatives u', u'', Λ of the unknown function $u(\xi)$ and we account for the function $\psi(\xi)$. As a result of this substitution, we recover a polynomial in $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi), j=0,1,2,...$ and its derivatives, equate all coefficients of this polynomial to zero. This procedure yields a system of equations that needs to be solved to evaluate a_k and $\psi(\xi)$. Thus, finally, one recovers exact solutions of Eq. (1) .

3. Application to DWDM system with 4WM

This modified simple equation scheme will be applied to DWDM system that appears with 4WM. There are two laws of nonlinearity that are going to be considered. The study will now be split into the following two subsections based on the type of nonlinear medium.

3.1. Kerr law nonlinearity

For Kerr law nonlinearity, DWDM model with 4WM reads [4]

$$iq_t^{(l)}+a_l q_{xx}^{(l)}+b_l q_{xt}^{(l)}+\left\{c_l\left|q^{(l)}\right|^2+\sum_{n\neq l}^{N}\alpha_{ln}\left|q^{(n)}\right|^2\right\}q^{(l)}+\sum_{n\neq l}^{N}\beta_{ln}q^{(l)*}\left|q^{(n)}\right|^2=0. \tag{5}$$

Here, $1\le l\le N$. The first term in (5) on left hand side is the linear evolution term, while a_l represents the coefficient of GVD; b_l represents the STD. Then, c_l is the coefficient of self-phase modulation (SPM) while α_{ln} are the coefficients of cross-phase modulation (XPM), while β_{ln} accounts for 4WM. The independent variables are x and t that represents the spatial and temporal variables respectively. The dependent variable is $q^{(l)}(x,t)$ that represents soliton profile in every single channel for $1\le l\le N$.

In order to solve (5) for solitons, the following solution structure is taken into consideration:

$$q^{(l)}(x,t)=P_l(\xi)e^{i\Phi_l(x,t)}, \tag{6}$$

where the wave variable ξ is given by

$$\xi=k(x-vt). \tag{7}$$

Here, $P_l(\xi)$ represents the amplitude component of the soliton solutions and v is the speed of the soliton, while the phase component $\Phi_l(x,t)$ is defined as

$$\Phi_l(x,t)=-\kappa x+\omega t+\theta, \tag{8}$$

where $1\le l\le N$. Here $P_l(x,t)$ represents the amplitude portion of the soliton and from the phase component, κ is the frequency of the soliton, ω is the wave number of the soliton and finally θ is the phase constant. Substituting (6) into (5) and decomposing into real and imaginary parts lead to

$$k^2\left(a_l-vb_l\right)P_l''+\left(\kappa\omega b_l-\kappa^2a_l-\omega\right)P_l+c_lP_l^3+P_l\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})P_n^2=0, \tag{9}$$

and

$$(1-b_l\kappa)P_l'+(b_l\omega-2a_l\kappa)P_l'=0. \tag{10}$$

The imaginary part equation leads to the speed of the soliton that is given by

$$v=\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}. \tag{11}$$

Using the balancing principle leads to

$$P_n=P_l. \tag{12}$$

Consequently, Eqs. (9) reduces to

$$k^2\left(a_l-vb_l\right)P_l''+\left(\kappa\omega b_l-\kappa^2a_l-\omega\right)P_l+\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}P_l^3=0, \tag{13}$$

Balancing P_l'' with P_l^3 in Eqs. (13), then we get $N=1$. Consequently we reach

$$P_l(\xi)=a_0+a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right),\quad a_1\neq 0. \tag{14}$$

Substituting Eq. (14) in Eq. (13) and then setting the coefficients of $\psi^{-j}(\xi),\ j=0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving a_0, a_1, k, κ, α_{ln}, β_{ln} b_l, v and ω as follows:

ψ^{-3} coeff.:

$$a_1\psi'^3\left[\begin{array}{l}a_1^2\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}+\\ 2k^2\left(a_l-vb_l\right)\end{array}\right]=0, \tag{15}$$

ψ^{-2} coeff.:

$$3a_1\psi'\begin{bmatrix} a_0a_1\psi'\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}+ \\ k^2\psi''\left(vb_l-a_l\right) \end{bmatrix}=0, \quad (16)$$

ψ^{-1} coeff.:

$$a_1\begin{bmatrix} \psi'\begin{Bmatrix} 3a_0^2\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right)- \\ a_l\kappa^2+\kappa\omega b_l-\omega \end{Bmatrix}+ \\ k^2\psi'''\left(a_l-vb_l\right) \end{bmatrix}=0, \quad (17)$$

ψ^{0} coeff.:

$$a_0\begin{bmatrix} a_0^2\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}- \\ \kappa^2a_l+\omega\left(\kappa b_l-1\right) \end{bmatrix}=0. \quad (18)$$

Solving this system, we obtain

$$a_0=\sqrt{\frac{\kappa^2a_l-\kappa\omega b_l+\omega}{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})}},$$
$$a_1=\mp\sqrt{-\frac{2k^2\left(a_l-vb_l\right)}{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})}}, \quad (19)$$

and

$$\psi''=\pm\sqrt{-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}}\psi', \quad (20)$$

$$\psi'''=-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}\psi'. \quad (21)$$

From Eqs. (20) and (21), we can deduce that

$$\psi'(\xi)=\pm\sqrt{-\frac{k^2\left(a_l-vb_l\right)}{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}}\, c_1e^{\pm\sqrt{-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}}\xi}, \quad (22)$$

$$\psi(\xi)=-\frac{k^2\left(a_l-vb_l\right)}{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}\, c_1e^{\pm\sqrt{-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}}\xi}+c_2, \quad (23)$$

where c_1 and c_2 are constants of integration. Substituting Eq. (22) and Eq. (23) into Eq. (14), we obtain following the following exact solution to Eq. (5).

$$q^{(l)}(x,t)=\left\{\frac{\sqrt{\frac{\kappa^2a_l-\kappa\omega b_l+\omega}{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})}}-\frac{k^2\left(a_l-vb_l\right)}{\sqrt{\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right)\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}}\, c_1e^{\pm\sqrt{-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}}\xi}}{-\frac{k^2\left(a_l-vb_l\right)}{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}\, c_1e^{\pm\sqrt{-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}}\xi}+c_2}\right\}\times e^{i(-\kappa x+\omega t+\theta)}. \quad (24)$$

If we set

$$c_1=-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}e^{\pm\sqrt{-\frac{2\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)}{k^2\left(a_l-vb_l\right)}}\xi_0}, \quad c_2=\pm 1,$$

we obtain:

$$q^{(l)}(x,t)=\pm\sqrt{\frac{\kappa^2a_l-\kappa\omega b_l+\omega}{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})}}\tanh\left[\sqrt{-\frac{\kappa^2a_l-\kappa\omega b_l+\omega}{2k^2\left(a_l-vb_l\right)}}(k(x-vt)+\xi_0)\right]\times e^{i(-\kappa x+\omega t+\theta)}, \quad (25)$$

or

$$q^{(l)}(x,t)=\pm\sqrt{\frac{\kappa^2a_l-\kappa\omega b_l+\omega}{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})}}\coth\left[\sqrt{-\frac{\kappa^2a_l-\kappa\omega b_l+\omega}{2k^2\left(a_l-vb_l\right)}}(k(x-vt)+\xi_0)\right] \quad (26)$$

$$\times e^{i(-\kappa x+\omega t+\theta)},$$

where v is given by (11). Solutions (30) and (31) are valid when

$$\kappa^2 a_l - \kappa\omega b_l + \omega > 0, \quad c_l + \sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln}) > 0, \quad (27)$$

$$a_l - v b_l < 0. \quad (28)$$

3.2. Parabolic law nonlinearity

This law is alternatively known as the cubic-quintic nonlinearity. For parabolic law nonlinearity, DWDM, with 4WM, is modeled as [14]:

$$\begin{aligned}
&iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left\{ c_l \left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N}\alpha_{ln}\left|q^{(n)}\right|^2 \right\} q^{(l)} + \\
&\left\{ d_l \left|q^{(l)}\right|^4 + \sum_{n\neq l}^{N}\left|q^{(n)}\right|^2 \left(\beta_{ln}\left|q^{(n)}\right|^2 + \gamma_{ln}\left|q^{(l)}\right|^2\right) \right\} q^{(l)} + \\
&\sum_{n\neq l}^{N}\delta_{ln} q^{(l)*}\left|q^{(n)}\right|^2 + \sum_{n\neq l}^{N}\lambda_{ln}\left|q^{(l)}\right|^2 q^{(l)*}\left(q^{(n)}\right)^2 + \\
&\sum_{n\neq l}^{N}\nu_{ln} q^{(l)*}\left|q^{(n)}\right|^2\left(q^{(n)}\right)^2 + \sum_{n\neq l}^{N}\sigma_{ln} q^{(l)*}\left(q^{(n)}\right)^3 = 0.
\end{aligned} \quad (29)$$

for $1 \le l \le N$. In (29), SPM terms are the coefficients of c_l and d_l , while XPM coefficients are α_{ln}, β_{ln} and γ_{ln} . Also, the terms with δ_{ln}, λ_{ln}, ν_{ln} and σ_{ln} are accounted for 4WM in parabolic law medium.

In this case, substituting (6) into (29), leads to the same imaginary part as given by (10). Again, the speed will be the same as (11). The real part equation however is

$$\begin{aligned}
&k^2\left(a_l - v b_l\right)P_l'' + \left(\kappa^2 a_l - \kappa\omega b_l + \omega\right)P_l + c_l P_l^3 + d_l P_l^5 + \\
&\sum_{n\neq l}^{N}\left\{\begin{array}{l}(\alpha_{ln}+\delta_{ln})P_n^2 P_l + (\beta_{ln}+\nu_{ln})P_n^4 P_l + \\ (\gamma_{ln}+\lambda_{ln})P_n^2 P_l^3 \\ +\sigma_{ln}P_n^3 P_l^2\end{array}\right\} = 0.
\end{aligned} \quad (30)$$

Using the balancing principle leads to

$$P_n = P_l. \quad (31)$$

Consequently, Eqs. (30) reduces to

$$\begin{aligned}
&k^2\left(a_l - v b_l\right)P_l'' + \left(\kappa\omega b_l - \kappa^2 a_l - \omega\right)P_l + \\
&\left\{c_l + \sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}P_l^3 + \\
&\left\{d_l + \sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right\}P_l^5 = 0.
\end{aligned} \quad (32)$$

Set

$$P_l = U_l^{\frac{1}{2}}. \quad (33)$$

so that (32) transform to

$$\begin{aligned}
&k^2\left(a_l - v b_l\right)\left\{2U_l U_l'' - U_l'^2\right\} + \\
&4\left(\kappa\omega b_l - \kappa^2 a_l - \omega\right)U_l^2 + \\
&4\left\{c_l + \sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}U_l^3 + \\
&4\left\{d_l + \sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right\}U_l^4 = 0.
\end{aligned} \quad (34)$$

Balancing $U_l U_l''$ with U_l^4 in Eqs. (34), then we get $N = 1$. Consequently we reach

$$U_l(\xi) = a_0 + a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right), \quad a_1 \neq 0. \quad (35)$$

Substituting Eq. (35) in Eq. (34) and then setting the coefficients of $\psi^{-j}(\xi),\ j = 0,1,2,3,4$ to zero, then we obtain a set of algebraic equations involving a_0, a_1, k, κ, α_{ln}, ν_{ln}, δ_{ln}, γ_{ln}, β_{ln} b_l, v and ω as follows:

ψ^{-4} coeff.:

$$a_1^2\left(\psi'\right)^4\left[\begin{array}{l}3k^2\left(a_l - v b_l\right) + 4a_1^2 \\ \left\{\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln}) + d_l\right\}\end{array}\right] = 0, \quad (36)$$

ψ^{-3} coeff.:

$$\begin{aligned}
&4a_1\psi'^2(a_1^2\psi'(4a_0\left\{\begin{array}{l}\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln}) \\ +d_l\end{array}\right\} \\
&+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln}) + c_l) + a_1 k^2\psi''\left(v b_l - a_l\right) + \\
&a_0 k^2\psi'\left(a_l - v b_l\right)) = 0,
\end{aligned} \quad (37)$$

ψ^{-2} coeff.:

$$\begin{aligned}
&a_1(a_1(4\psi'^2(3a_0\left\{c_l + \sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\} + \\
&6a_0^2\left\{\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln}) + d_l\right\} - \\
&\kappa^2 a_l + \kappa\omega b_l - \omega) + k^2\left(\psi''\right)^2\left(v b_l - a_l\right) + \\
&2k^2\psi'''\psi'\left(a_l - v b_l\right)) - 6a_0 k^2\psi'\psi''\left(a_l - v b_l\right)) = 0,
\end{aligned} \quad (38)$$

ψ^{-1} coeff.:

$$2a_0a_1(\psi'(6a_0\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}- 4\left(\kappa^2a_l-\kappa\omega b_l+\omega\right)+ 8a_0^2\left\{\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})+d_l\right\})+ k^2\psi'''\left(a_l-vb_l\right))=0, \tag{39}$$

ψ^{0} coeff.:

$$4a_0^2\begin{bmatrix} a_0\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}+ \\ a_0^2\left\{\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})+d_l\right\}- \\ \kappa^2a_l+\omega\left(\kappa b_l-1\right) \\ \end{bmatrix}=0, \tag{40}$$

Solving this system, we obtain

$$a_0=-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)}{4\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)},$$

$$a_1=\frac{1}{2}\sqrt{-\frac{3k^2\left(a_l-vb_l\right)}{d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})}},$$

$$\omega=\frac{16\kappa^2a_l\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)+3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{16\left(\kappa b_l-1\right)\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)} \tag{41}$$

and

$$\psi''=\sqrt{-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}}\,\psi', \tag{42}$$

$$\psi'''=-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}\,\psi', \tag{43}$$

From Eqs. (42) and (43), we can deduce that

$$\psi'(\xi)=\sqrt{-\frac{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}}\; c_1e^{\sqrt{-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}}\,\xi}, \tag{44}$$

$$\psi(\xi)=-\frac{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}\; c_1e^{\sqrt{-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}}\,\xi}+c_2, \tag{45}$$

where c_1 and c_2 are constants of integration. Substituting Eq. (44) and Eq. (45) into Eq. (35), we obtain following the following exact solution to Eq. (29).

$$q^{(l)}(x,t)=\left(-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)}{4\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)}-\frac{\dfrac{k^2(a_l-vb_l)}{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})}\,c_1e^{\sqrt{-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}}\,\xi}}{-\dfrac{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}\,c_1e^{\sqrt{-\frac{3\left(c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\nu_{ln}+\gamma_{ln}+\lambda_{ln}+\sigma_{ln})\right)\left(a_l-vb_l\right)}}\,\xi}+c_2}\right)\times e^{i(-\kappa x+\omega t+\theta)}. \tag{46}$$

If we set

$$c_1 = -\frac{3\left(c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right)^2}{4k^2\left(d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right)(a_l - vb_l)}$$

$$c_1 e^{\sqrt{-\frac{3\left(c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right)^2}{4k^2\left(d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right)(a_l - vb_l)}}\xi_0}, \quad c_2 = \pm 1,$$

we obtain:

$$q^{(l)}(x,t) = \frac{3\left(c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right)}{4\left(d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right)} \times \left\{1 \pm \tanh\left[\sqrt{\frac{3\left(c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right)^2}{16k^2\left(d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right)(a_l - vb_l)}}\,(k(x - vt) + \xi_0)\right]\right\} \times e^{i(-\kappa x + \omega t + \theta)}, \tag{47}$$

or

$$q^{(l)}(x,t) = \frac{3\left(c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right)}{4\left(d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right)} \times \left\{1 \pm \coth\left[\sqrt{\frac{3\left(c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right)^2}{16k^2\left(d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right)(a_l - vb_l)}}\,(k(x - vt) + \xi_0)\right]\right\} \times e^{i(-\kappa x + \omega t + \theta)}, \tag{48}$$

Solutions (47) and (48) are valid when

$$\left\{d_l + \sum_{n\neq l}^{N}(\beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\right\}(a_l - vb_l) < 0. \tag{49}$$

4. Conclusions

This paper obtained dark and singular soliton solutions to DWDM system that was studied in presence of 4WM. The modified simple equation approach was adopted to obtain the soliton solutions that appeared under restrictive conditions which, in this paper, are referred to as constraint conditions. The results are novel and meaningful although an inherent drawback of this integration algorithm is that it fails to retrieve the much needed bright soliton solution. Several other integration schemes are nevertheless available to secure bright soliton solutions. A few of them are extended trial equation method, Bernoulli's algorithm, Kudryashov's technique and several others. The results of the application of these schemes will be reported elsewhere.

References

[1] A. H. Arnous, M. Mirzazadeh, Q. Zhou, M. F. Mahmood, A. Biswas, M. Belic, Optoelectron. Adv. Mat. **9**(9–10), 1214 (2013).
[2] A. H. Arnous, M. Mirzazadeh, S. Moshokoa, S. Medhekar, Q. Zhou, M. F. Mahmood, A. Biswas, M. Belic, Journal of Computational and Theoretical Nanoscience **12**, (12), 5940 (2015).
[3] A. H. Arnous, M. Mirzazadeh, Q. Zhou, S. P. Moshokoa, A. Biswas, M. Belic. Optik **127**(23), 11450 (2016).
[4] A. H. Arnous, M. Z. Ullah, S. P. Moshokoa, Q. Zhou, H. Triki, M. Mirzazadeh, A. Biswas, Optik **130**, 996 (2017).
[5] A. H. Arnous, M. Z. Ullah, M. Asma, S. P. Moshokoa, Q. Zhou, M. Mirzazadeh, A. Biswas, M. Belic, Optik **136**, 445 (2017).
[6] A. H. Arnous, M. Zaka Ullah, S. P. Moshokoa, Q. Zhou, H. Triki. M. Mirzazadeh, A. Biswas, Nonlinear Dynamics DOI 10.1007/s11071-017-3351-2 (2017).
[7] F. I. Chicharro, B. Ortega, J. Mora, Optics Communications **370**, 239 (2016).
[8] A. Dideban, H. Habibiyan, H. Ghafoorifard, Physica E**87**, 77 (2017).
[9] J. Du, Z. Teng, N. Shen, Optics Communications **358**, 180 (2016).
[10] J. Du, Y. Xu, F. Han, W. Yang, To appear in Optik.
[11] M. Ekici, M. Mirzazadeh, A. Sonmezoglu, Qin Zhou, H. Triki, M. Z. Ullah, S. P. Moshokoa, A. Biswas, Optik **131**, 964 (2017).
[12] M. M. El-Borai, H. M. El-Owaidy, H. M. Ahmed, A. H. Arnous, S. Moshokoa, A. Biswas, M. Belic, Optik **130**, 324 (2017).
[13] M. Mirzazadeh, A. H. Arnous, M. F. Mahmood, E. Zerrad, A. Biswas, Nonlinear Dynamics **81**, 277 (2015).
[14] M. Savescu, A. A. Alshareby, E. M. Hilal, A. H. Bhrawy, Q. Zhou, A. Biswas, Optoelectron. Adv. Mat. **9**, 14 (2015).
[15] Q. Zhou, M. Ekici, A. Sonmezoglu, M. Mirzazadeh, M. Eslami, Journal of Modern Optics **63**, 2131 (2016).
[16] E. C. Aslan, M. Inc, D. Baleanu, Superlattices and Microstructures **109**, 784 (2017).
[17] M. Inc, A. I. Aliyu, A. Yusuf, Optik **142**, 509 (2017).
[18] M. M. Al Quarashi, E. Ates, M. Inc, Optik **142**, 343 (2017).

[19] B. Kilic, M. Inc, Optik **138**, 65 (2017).
[20] M. M. Al Quarashi, D. Baleanu, M. Inc, Optik **140**, 114 (2017).
[21] E. C. Aslan, F. Tchier, M. Inc, Superlattices and Microstructures **105**, 183 (2017).
[22] M. Inc, A. I. Aliyu, A. Yusuf, D. Baleanu, Superlattices and Microstructures **112**, 628 (2017).
[23] F. Tchier, E. C. Aslan, M. Inc, Nonlinear Dynamics **85**, 2577 (2016).
[24] M. Inc, E. Ates, F. Tchier, Nonlinear Dynamics **85**, 1319 (2016).
[25] E. C. Aslan, M. Inc, Waves in Random and Complex Media **27**, 594 (2017).

*Corresponding author: biswas.anjan@gmail.com