Dispersive optical solitons with DWDM technology using a couple of integration algorithms

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This paper employs modified Kudryashov's method as well as Riccati-Bernoulli's sub ODE method to address DWDM system to extract dispersive optical solitons. Dark and singular optical soliton solutions are established together with their existence criteria.

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1. Introduction

DWDM technology is essential for parallel transmission of loads of data across the globe. Therefore, it is imperative to advance this technology in its most optimal form. This DWDM system is modeled by vector coupled nonlinear evolution equation (NLEE). It typically stems from nonlinear Schrödinger's equation (NLSE). However, dispersive solitons stem from Schrödinger-Hirota's equation (SHE) that models the dynamics of dispersive optical solitons in a polarization-preserving fiber. This paper addresses DWDM system for dispersive optical solitons by the aid of a couple of integration techniques. DWDM system as well as other NLEEs have been extensively studied in the past by the aid of several integtaion tools that led to plentiful interesting solutions including solitons and shock waves. A few of them are the method of undetermined coefficients, extended trial equation method, G'/G-expansion scheme and several others [1-11]. This paper will employ a couple of such powerful tools to extract dispersive solitons in such a system.

1.1. Mathematical Model of DWDM Systems

The dimensionless form of the governing equation for DWDM system is given by [1]

$$\begin{aligned} & iq_t^{(l)} + i\alpha_l q_x^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + i\gamma_l q_{xxx}^{(l)} + \\ & \left\{ c_l \left| q^{(l)} \right|^2 + \sum_{n\neq l}^{N} d_{ln} \left| q^{(n)} \right|^2 \right\} q^{(l)} + \\ & i\left\{ \xi_l \left| q^{(l)} \right|^2 + \sum_{n\neq l}^{N} \eta_{ln} \left| q^{(n)} \right|^2 \right\} q_x^{(l)} = 0, \end{aligned} \tag{1}$$

where $l \le l \le N.$ The first term in (1) on left hand side is the linear temporal evolution term, while a_l represents the coefficient of group velocity dispersion (GVD) and the coefficient of b_l is the spatio-temporal dispersion. Then, the coefficient of γ_l is the third order dispersion. Also, c_l is the SPM while d_{ln} gives XPM. Finally, ξ_l and η_{ln} are from nonlinear dispersions.

This paper will integrate (1) to retrieve its soliton solutions by the application of Riccati-Bernoulli sub-ODE (ordinary differential equation) method and modified Kudryashov's algorithm.

2. Mathematical analysis

In order to solve (1) for solitons, the following phase-amplitude form of decomposition for the wave profile $q^{(l)}(x,t)$ is carried out.

$$q^{(l)}(x,t)=P_l(\tau)e^{i\Phi_l(x,t)} \quad (2)$$

where $P_l(\tau)$ represents the shape of the pulse and

$$\tau=K(x-vt), \quad (3)$$

and the phase component is defined as

$$\Phi_l(x,t)=-\kappa_l x+\omega_l t+\theta_l, \quad (4)$$

here $l\le l\le N.$ Here $P_l(x,t)$ represents the amplitude portion of the soliton and from the phase component, κ_l is the frequency of the soliton, ω_l is the wave number of the soliton and finally θ_l is the phase constant. Substituting (2) into (1) and decomposing into real and imaginary parts lead to

$$\begin{aligned}&\left\{\alpha_l\kappa_l+\omega_l\left(b_l\kappa_l-1\right)-a_l\kappa_l^2-\gamma_l\kappa_l^3\right\}P_l+\\&(c_l+\kappa_l\xi_l)P_l^3+(a_l+3\gamma_l\kappa_l-b_lv)K^2(P_l)''+\\&\sum_{l\neq n}^{N}\left\{\left(d_{ln}+\eta_{ln}\kappa_l\right)P_n^2P_l\right\}=0,\end{aligned} \quad (5)$$

and

$$\begin{aligned}&(b_l\kappa_l-1)Kv(P_l)'+\left(\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l^2\right)K(P_l)'\\&+\gamma_lK^3(P_l)'''+\xi_lKP_l^2(P_l)'+K\sum_{l\neq n}^{N}\eta_{ln}P_n^2(P_l)'=0.\end{aligned} \quad (6)$$

2.1. RICCATI – BERNOULI Sub – ODE method

The balancing effect leads to

$$P_l=P_n \quad (7)$$

Consequently, Eqs. (5), (6) modify to

$$\begin{aligned}&\left\{\alpha_l\kappa_l+\omega_l\left(b_l\kappa_l-1\right)-a_l\kappa_l^2-\gamma_l\kappa_l^3\right\}P_l+\\&(a_l+3\gamma_l\kappa_l-b_lv)K^2(P_l)''+\\&\left(c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)\right)P_l^3=0,\end{aligned} \quad (8)$$

and

$$\begin{aligned}&\left\{(b_l\kappa_l-1)v+\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l^2\right\}(P_l)'+\\&\gamma_lK^2(P_l)'''+\left\{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}\right\}P_l^2(P_l)'=0.\end{aligned} \quad (9)$$

Integrating Eq. (9) with respect to τ, gives

$$\begin{aligned}&\left\{(b_l\kappa_l-1)v+\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l^2\right\}P_l+\\&\gamma_lK^2(P_l)''+\frac{1}{3}\left\{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}\right\}P_l^3=0,\end{aligned} \quad (10)$$

where integration constant is taken to zero, without any loss of generality. In this section, the Riccati-Bernoulli sub-ODE method [8, 9, 11] will be introduced in details to obtain soliton solutions to Eq. (1). Suppose that the solution of Eqs. (8), (10) leads to Riccati-Bernoulli equation

$$(P_l)'=AP_l^{2-m}+BP_l+CP_l^m, \quad (11)$$

where $A,B,C,$ and m are constants to be determined later.

Substituting Eq. (11) into Eq. (8), we get

$$\begin{aligned}&\left\{\alpha_l\kappa_l+\omega_l\left(b_l\kappa_l-1\right)-a_l\kappa_l^2-\gamma_l\kappa_l^3\right\}P_l+\\&(a_l+3\gamma_l\kappa_l-b_lv)K^2\{AB(3-m)P_l^{2-m}+\\&A^2(2-m)P_l^{3-2m}+mC^2P_l^{2m-1}+BC(m+1)P_l^m+\\&(2AC+B^2)P_l\}+\left(c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)\right)P_l^3=0,\end{aligned} \quad (12)$$

Setting $m=0$, Eq. (12) is reduced to

$$\begin{aligned}&BC(a_l+3\gamma_l\kappa_l-b_lv)K^2+\\&\left\{\begin{matrix}\alpha_l\kappa_l+\omega_l\left(b_l\kappa_l-1\right)-a_l\kappa_l^2-\gamma_l\kappa_l^3+\\(a_l+3\gamma_l\kappa_l-b_lv)K^2(2AC+B^2)\end{matrix}\right\}P_l\\&+3AB(a_l+3\gamma_l\kappa_l-b_lv)K^2P_l^2+\\&\left(\begin{matrix}c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\\2A^2(a_l+3\gamma_l\kappa_l-b_lv)K^2\end{matrix}\right)P_l^3=0.\end{aligned} \quad (13)$$

Setting each coefficient of $P_l^j\,(j=0,1,2,3)$ to zero, we get

$$c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+ \\ 2A^2(a_l+3\gamma_l\kappa_l-b_l v)K^2=0, \tag{14}$$

$$3AB(a_l+3\gamma_l\kappa_l-b_l v)K^2=0, \tag{15}$$

$$\alpha_l\kappa_l+\omega_l\left(b_l\kappa_l-1\right)-a_l\kappa_l^2-\gamma_l\kappa_l^3+ \\ (a_l+3\gamma_l\kappa_l-b_l v)K^2(2AC+B^2)=0, \tag{16}$$

$$BC(a_l+3\gamma_l\kappa_l-b_l v)K^2=0. \tag{17}$$

Solving Eqs. (14) - (17), we get

$$v=\frac{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2 b_l K^2}, \tag{18}$$

$$\omega_l=\frac{a_l\kappa_l^2+\gamma_l\kappa_l^3-2AC(a_l+3\gamma_l\kappa_l-b_l v)K^2-\alpha_l\kappa_l}{b_l\kappa_l-1}, \tag{19}$$

$$A\neq 0, \tag{20}$$

$$C\neq 0, \tag{21}$$

$$B=0. \tag{22}$$

Substituting Eq. (18) into Eq. (19), we get

$$\omega_l=\frac{a_l\kappa_l^2+\gamma_l\kappa_l^3-2AC}{b_l\kappa_l-1} \\ \left\{a_l+3\gamma_l\kappa_l-\left(\frac{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2K^2}\right)\right\} \\ K^2-\alpha_l\kappa_l. \tag{23}$$

This relation (23) introduce the constraint:

$$b_l\kappa_l\neq 1. \tag{24}$$

Substituting Eq. (11) into Eq. (9), we get

$$\left\{(b_l\kappa_l-1)v+\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l^2\right\}P_l+ \\ \gamma_l K^2\{AB(3-m)P_l^{2-m}+A^2(2-m)P_l^{3-2m}+ \\ mC^2P_l^{2m-1}+BC(m+1)P_l^m+ \\ (2AC+B^2)P_l\}+\frac{1}{3}\left\{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}\right\}P_l^3=0, \tag{25}$$

Setting $m=0$, Eq. (25) is reduced to

$$BC\gamma_l K^2+\left\{\begin{matrix}(b_l\kappa_l-1)v+\alpha_l+b_l\omega_l+2a_l\kappa_l \\ -3\gamma_l\kappa_l^2+\gamma_l K^2(2AC+B^2)\end{matrix}\right\}P_l \\ +3AB\gamma_l K^2P_l^2+\left(\frac{1}{3}\left\{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}\right\}+2A^2\gamma_l K^2\right)P_l^3=0, \tag{26}$$

Setting each coefficient of $P_l^j\,(j=0,1,2,3)$ to zero, we get

$$\frac{1}{3}\left\{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}\right\}+2A^2\gamma_l K^2=0, \tag{27}$$

$$3AB\gamma_l K^2=0, \tag{28}$$

$$(b_l\kappa_l-1)v+\alpha_l+b_l\omega_l+2a_l\kappa_l- \\ 3\gamma_l\kappa_l^2+\gamma_l K^2(2AC+B^2)=0, \tag{29}$$

$$BC\gamma_l K^2=0. \tag{30}$$

Solving Eqs. (27) - (30), we get

$$\omega_l=\frac{3\gamma_l\kappa_l^2-(b_l\kappa_l-1)v-\alpha_l-2a_l\kappa_l-2AC\gamma_l K^2}{b_l}, \tag{31}$$

$$\gamma_l=-\frac{1}{6A^2K^2}\left\{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}\right\}, \tag{32}$$

$$A\neq 0, \tag{33}$$

$$C\neq 0, \tag{34}$$

$$B=0. \tag{35}$$

Substituting Eq. (18) into Eq. (31), we get

$$\omega_l = \frac{3\gamma_l\kappa_l^2 - (b_l\kappa_l - 1)}{b_l}$$

$$\left\{ \frac{c_l + \kappa_l\xi_l + \sum_{l\neq n}^{N}\left(d_{ln} + \eta_{ln}\kappa_l\right) + 2A^2(a_l + 3\gamma_l\kappa_l)K^2}{2A^2 b_l K^2} \right\} \quad (36)$$

$$-\alpha_l - 2a_l\kappa_l - 2AC\gamma_l K^2.$$

Equating the wave number from (19) and (31) gives the constraint as

$$2A^2K^2\begin{Bmatrix} 3\gamma_l\kappa_l^2 - (b_l\kappa_l - 1)v - \\ \alpha_l - 2a_l\kappa_l - 2AC\gamma_l K^2 \end{Bmatrix} = a_l\kappa_l^2 + \gamma_l\kappa_l^3 - 2AC\left\{a_l + 3\gamma_l\kappa_l - b_l v\right\}K^2 - \alpha_l\kappa_l. \quad (37)$$

When $m \neq 1, A \neq 0,$ and $B^2 - 4AC < 0,$ the solutions of Eq. (11) are [11]

$$P_l(\tau) = \left\{ -\frac{B}{2A} + \frac{\sqrt{4AC - B^2}}{2A} tan\left[\frac{(1-m)\sqrt{4AC-B^2}}{2}\left(\tau + \tau_0\right)\right]\right\}^{\frac{1}{1-m}}, \quad (38)$$

and

$$P_l(\tau) = \left\{ -\frac{B}{2A} - \frac{\sqrt{4AC - B^2}}{2A} cot\left[\frac{(1-m)\sqrt{4AC-B^2}}{2}\left(\tau + \tau_0\right)\right]\right\}^{\frac{1}{1-m}}. \quad (39)$$

When $m \neq 1, A \neq 0,$ and $B^2 - 4AC > 0,$ the solutions of Eq. (11) are

$$P_l(\tau) = \left\{ -\frac{B}{2A} - \frac{\sqrt{B^2 - 4AC}}{2A} tanh\left[\frac{(1-m)\sqrt{B^2-4AC}}{2}\left(\tau + \tau_0\right)\right]\right\}^{\frac{1}{1-m}}, \quad (40)$$

and

$$P_l(\tau) = \left\{ -\frac{B}{2A} - \frac{\sqrt{B^2 - 4AC}}{2A} coth\left[\frac{(1-m)\sqrt{B^2-4AC}}{2}\left(\tau + \tau_0\right)\right]\right\}^{\frac{1}{1-m}}. \quad (41)$$

Case-A: When $AC > 0,$ we get exact solutions of Eq. (1),

$$q_1^{(l)}(x,t) = \sqrt{\frac{C}{A}} tan\left[\sqrt{AC}\left(K\left(x - \left\{\frac{c_l + \kappa_l\xi_l + \sum_{l\neq n}^{N}\left(d_{ln} + \eta_{ln}\kappa_l\right) + 2A^2(a_l + 3\gamma_l\kappa_l)K^2}{2A^2 b_l K^2}\right\}t\right) + \tau_0\right)\right] \times e^{i\left\{-\kappa_l x + \omega_l t + \theta_l\right\}} \quad (42)$$

and

$$q_2^{(l)}(x,t) = -\sqrt{\frac{C}{A}} cot\left[\sqrt{AC}\left(K\left(x - \left\{\frac{c_l + \kappa_l\xi_l + \sum_{l\neq n}^{N}\left(d_{ln} + \eta_{ln}\kappa_l\right) + 2A^2(a_l + 3\gamma_l\kappa_l)K^2}{2A^2 b_l K^2}\right\}t\right) + \tau_0\right)\right] \times e^{i\left\{-\kappa_l x + \omega_l t + \theta_l\right\}}, \quad (43)$$

where ω_l is given by Eq. (23) or Eq. (36). Three constraint conditions for the existence of these analytical solutions to exist are given by Eqs. (24), (32) and (37).

Case-B: When $AC < 0,$ we get exact solutions of Eq. (1),

$$q_3^{(l)}(x,t) = -\sqrt{-\frac{C}{A}} tanh\left[\sqrt{-AC}\left(K\left(x - \left\{\frac{c_l + \kappa_l\xi_l + \sum_{l\neq n}^{N}\left(d_{ln} + \eta_{ln}\kappa_l\right) + 2A^2(a_l + 3\gamma_l\kappa_l)K^2}{2A^2 b_l K^2}\right\}t\right) + \tau_0\right)\right] \times e^{i\left\{-\kappa_l x + \omega_l t + \theta_l\right\}}, \quad (44)$$

and

$$q_4^{(l)}(x,t) = -\sqrt{-\frac{C}{A}} coth \left[\sqrt{-AC}\left(K\left(x - \left\{ \frac{c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} (d_{ln} + \eta_{ln}\kappa_l) + 2A^2(a_l + 3\gamma_l \kappa_l)K^2}{2A^2 b_l K^2} \right\} t \right) + \tau_0 \right) \right] \times e^{i\{-\kappa_l x + \omega_l t + \theta_l\}}, \quad (45)$$

which are dark and singular soliton solutions respectively. Here, ω_l is given by Eq. (23) or Eq. (36). Three constraint conditions for the existence of these analytical solutions to exist are given by Eqs. (24), (32) and (37).

Fig. 1 shows the profile of a dark 1-soliton solution for the parametrers

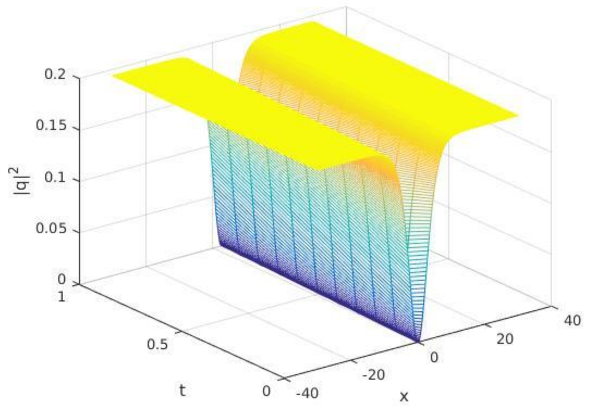

Fig. 1.

$l = 1$, $\alpha_1 = 0.0445$, $\alpha_2 = 0.7549$, $a_1 = 0.2428$, $a_2 = 0.4424$, $b_1 = 0.6878$, $b_2 = 0.3592$, $\gamma_1 = -0.5154$, $\gamma_2 = -0.2779$, $c_1 = 0.7363$, $c_2 = 0.3947$, $d_{11} = 0.6834$, $d_{12} = 0.4423$, $d_{21} = 0.7040$, $d_{22} = 0.0196$, $\xi = 0.3309$, $\xi = 0.4243$, $\eta_{11} = 0.2703$, $\eta_{12} = 0.8217$, $\eta_{21} = 0.1971$, $\eta_{22} = 0.4299$, $K = 0.8085$, $\kappa_1 = 0.8878$, $\kappa_2 = 0.3912$, $v = 2.6520$, $A = -0.7551$, $C = 0.1464$

2.1.1. BÄCKLUND Transformation of RICATTI-BERNOULLI equation

When $g_{n-1}(\tau)$ and $g_n(\tau)(g_n(\tau) = g_n(g_{n-1}(\tau)))$ are the solutions of Eq. (11), we get

$$\frac{dg_n(\tau)}{d\tau} = \frac{dg_n(\tau)}{dg_{n-1}(\tau)} \cdot \frac{dg_{n-1}(\tau)}{d\tau} = \frac{dg_n(\tau)}{dg_{n-1}(\tau)}\left(Ag_{n-1}^{2-m} + Bg_{n-1} + Cg_{n-1}^{m} \right), \quad (46)$$

namely

$$\frac{dg_n(\tau)}{Ag_n^{2-m} + Bg_n + Cg_n^m} = \frac{dg_{n-1}(\tau)}{Ag_{n-1}^{2-m} + Bg_{n-1} + Cg_{n-1}^m}. \quad (47)$$

Integrating above equation once with respect to τ and simplifying it, we get

$$g_n(\tau) = \left\{ \frac{-CK_1 + AK_2 (g_{n-1}(\tau))^{1-m}}{BK_1 + AK_2 + AK_1 (g_{n-1}(\tau))^{1-m}} \right\}^{\frac{1}{1-m}}, \quad (48)$$

where K_1 and K_2 are arbitrary constants. Equation (48) is a Bäcklund transformation of Eq. (11). If we get a solution of Eq. (48), we can search for new infinite sequence of solutions of Eq. (11) by using Eq. (48). Then an infinite sequence of solutions of Eq. (1) is obtained.

Applying Eq. (48) to $q_j^{(l)}(x,t) (j = 1,2,3,4)$, we can get an infinite sequence of solutions of Eq. (1). By applying Eq. (48) to $q_j^{(l)}(x,t)$ for $j = 5,6,7,8$ once, when $AC < 0$, we get new solutions of Eq. (1),

$$q_5^{(l)}(x,t) = \left\{ \frac{-CK_1 - K_2\sqrt{-AC} tanh}{AK_2 - K_1\sqrt{-AC} tanh} \right\} \frac{\left[\sqrt{-AC}\left(K\left(x - \left\{ \frac{c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} (d_{ln} + \eta_{ln}\kappa_l) + 2A^2(a_l + 3\gamma_l \kappa_l)K^2}{2A^2 b_l K^2} \right\} t \right) \right) \right]}{\left[\sqrt{-AC}\left(K\left(x - \left\{ \frac{c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} (d_{ln} + \eta_{ln}\kappa_l) + 2A^2(a_l + 3\gamma_l \kappa_l)K^2}{2A^2 b_l K^2} \right\} t \right) \right) \right]} \times e^{i\{-\kappa_l x + \omega_l t + \theta_l\}}, \quad (49)$$

and

$$q_6^{(l)}(x,t)=\left\{\frac{-CK_1-K_2\sqrt{-AC}coth}{AK_2-K_1\sqrt{-AC}coth}\right\}$$
$$\left[\frac{\sqrt{-AC}\left(K\left(x-\left\{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\frac{2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2b_lK^2}\right\}t\right)\right)}{\sqrt{-AC}\left(K\left(x-\left\{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\frac{2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2b_lK^2}\right\}t\right)\right)}\right]$$
$$\times e^{i\{-\kappa_lx+\omega_lt+\theta_l\}}, \tag{50}$$

where ω_l is given by Eq. (23) or Eq. (36). Three constraint conditions for the existence of these analytical solutions to exist are given by Eqs. (24), (32) and (37).

When $AC>0$, we get new solutions of Eq. (1),

$$q_7^{(l)}(x,t)=\left\{\frac{-CK_1+K_2\sqrt{AC}tan}{AK_2+K_1\sqrt{AC}tan}\right\}$$
$$\left[\frac{\sqrt{AC}\left(K\left(x-\left\{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\frac{2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2b_lK^2}\right\}t\right)\right)}{\sqrt{AC}\left(K\left(x-\left\{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\frac{2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2b_lK^2}\right\}t\right)\right)}\right]$$
$$\times e^{i\{-\kappa_lx+\omega_lt+\theta_l\}}, \tag{51}$$

and

$$q_8^{(l)}(x,t)=\left\{\frac{-CK_1-K_2\sqrt{AC}cot}{AK_2-K_1\sqrt{AC}cot}\right\}$$
$$\left[\frac{\sqrt{AC}\left(K\left(x-\left\{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\frac{2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2b_lK^2}\right\}t\right)\right)}{\sqrt{AC}\left(K\left(x-\left\{c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)+\frac{2A^2(a_l+3\gamma_l\kappa_l)K^2}{2A^2b_lK^2}\right\}t\right)\right)}\right]$$
$$\times e^{i\{-\kappa_lx+\omega_lt+\theta_l\}}, \tag{52}$$

where ω_l is given by Eq. (23) or Eq. (36). Three constraint conditions for the existence of these analytical solutions to exist are given by Eqs. (24), (32) and (37).

Fig. 2 shows the profile of a dark 1-soliton solution for the parametrers

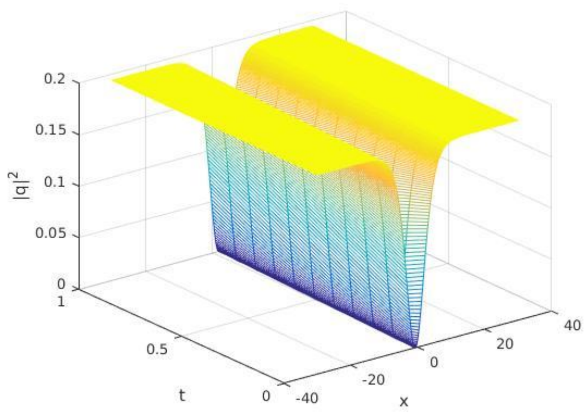

Fig. 2.

$l=1,\ \alpha_1=0.0445,\ \alpha_2=0.7549,\ a_1=0.2428,$
$a_2=0.4424,\ b_1=0.6878,\ b_2=0.3592,\ \gamma_1=-0.5154,$
$\gamma_2=-0.2779,\ c_1=0.7363,\ c_2=0.3947,\ d_{11}=0.6834,$
$d_{12}=0.4423,\ d_{21}=0.7040,\ d_{22}=0.0196,\ \xi=0.3309,$
$\xi=0.4243,\ \eta_{11}=0.2703,\ \eta_{12}=0.8217,\ \eta_{21}=0.1971,$
$\eta_{22}=0.4299,\ K=0.8085,\ \kappa_1=0.8878,\ \kappa_2=0.3912,$
$v=2.6520,\ A=-0.7551,\ C=0.1464,\ K_1=0.1999,$
$K_2=0.4070.$

2.2. Modified KUDRYASHOV'S method

In this section, a new and effective version of Kudryashov method is used to produce new exact traveling wave solutions of the Eq. (1) in non-linear optics. According to the modified Kudryashov method [3, 4], Eq. (8) has the solution in the form

$$P_l(\tau) = c_0^{(l)} + c_1^{(l)} Q(\tau), \tag{53}$$

where $c_0^{(l)}$ and $c_1^{(l)}$ are unknown constants and

$$Q(\tau) = \frac{1}{1 + dA^{\tau}}, \tag{54}$$

satisfies an ODE as

$$Q'(\tau) = Q(\tau)\big(Q(\tau) - 1\big) lnA, \tag{55}$$

where K and A are nonzero constants with $A > 0$ and $A \neq 1$.

By substituting Eq. (53) into Eq. (8) and equating the coefficients of the same powers of $Q(\tau)$, we will find a non-linear algebraic system as the following form:

$$\begin{aligned} &2(a_l + 3\gamma_l\kappa_l - b_l v) K^2 (lnA)^2 c_1^{(l)} + \\ &\left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) (c_1^{(l)})^3 = 0, \end{aligned} \tag{56}$$

$$\begin{aligned} &-3(a_l + 3\gamma_l\kappa_l - b_l v) K^2 (lnA)^2 c_1^{(l)} + \\ &3\left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) c_0^{(l)} (c_1^{(l)})^2 = 0, \end{aligned} \tag{57}$$

$$\begin{aligned} &(a_l + 3\gamma_l\kappa_l - b_l v) K^2 (lnA)^2 c_1^{(l)} + \\ &\left\{ \alpha_l\kappa_l + \omega_l \left(b_l\kappa_l - 1 \right) - a_l\kappa_l^2 - \gamma_l\kappa_l^3 \right\} c_1^{(l)} \\ &+3\left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) (c_0^{(l)})^2 c_1^{(l)} = 0, \end{aligned} \tag{58}$$

$$\begin{aligned} &\left\{ \alpha_l\kappa_l + \omega_l \left(b_l\kappa_l - 1 \right) - a_l\kappa_l^2 - \gamma_l\kappa_l^3 \right\} c_0^{(l)} + \\ &\left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) (c_0^{(l)})^3 = 0. \end{aligned} \tag{59}$$

Now, upon solving the resulting system, we find

$$c_0^{(l)} = -\frac{1}{2} c_1^{(l)}, \tag{60}$$

$$v = \frac{\begin{aligned} &2K^2 (lnA)^2 a + 6K^2 (lnA)^2 \gamma_l \kappa_l + \\ &\left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) (c_1^{(l)})^2 \end{aligned}}{2K^2 (lnA)^2 b_l}, \tag{61}$$

$$\omega_l = \frac{\begin{aligned} &4\alpha_l\kappa_l - 4a_l\kappa_l^2 - 4\gamma_l\kappa_l^3 + \\ &\left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) (c_1^{(l)})^2 \end{aligned}}{4(1 - b_l\kappa_l)}. \tag{62}$$

This relation (62) introduce the constraint:

$$b_l \kappa_l \neq 1. \tag{63}$$

By substituting Eq. (53) into Eq. (10) and equating the coefficients of the same powers of $Q(\tau)$, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$c_0^{(l)} = \pm \frac{KlnA}{2} \sqrt{\frac{-6\gamma_l}{\xi_l + \sum_{l \neq n}^{N} \eta_{ln}}}, \tag{64}$$

$$c_1^{(l)} = \mp KlnA \sqrt{\frac{-6\gamma_l}{\xi_l + \sum_{l \neq n}^{N} \eta_{ln}}}, \tag{65}$$

$$\omega_l = \frac{\gamma_l K^2 (lnA)^2 + 2v(1 - b_l\kappa_l) - 2\alpha_l - 4a_l\kappa_l + 6\gamma_l\kappa_l^2}{2b_l^2}. \tag{66}$$

Additionally, Eqs. (64), (65) poses the restriction that is given by

$$\gamma_l \left(\xi_l + \sum_{l \neq n}^{N} \eta_{ln} \right) < 0. \tag{67}$$

Equating the wave number from (62) and (66) gives the constraint as

$$\begin{aligned} &2(1 - b_l\kappa_l) \begin{Bmatrix} \gamma_l K^2 (lnA)^2 + 2v(1 - b_l\kappa_l) - \\ 2\alpha_l - 4a_l\kappa_l + 6\gamma_l\kappa_l^2 \end{Bmatrix} \\ &= b_l^2 \begin{Bmatrix} 4\alpha_l\kappa_l - 4a_l\kappa_l^2 - 4\gamma_l\kappa_l^3 + \\ \left(c_l + \kappa_l \xi_l + \sum_{l \neq n}^{N} \left(d_{ln} + \eta_{ln}\kappa_l \right) \right) (c_1^{(l)})^2 \end{Bmatrix}. \end{aligned} \tag{68}$$

This yields

$$P_l(x,t)=\pm\frac{K\ln A}{2}\sqrt{\frac{-6\gamma_l}{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}}}\left(\frac{1-dA^{K\left\{x-\frac{2K^2(\ln A)^2a+6K^2(\ln A)^2\gamma_l\kappa_l+\left(c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)\right)(c_1^{(l)})^2}{2K^2(\ln A)^2b_l}t\right\}}}{1+dA^{K\left\{x-\frac{2K^2(\ln A)^2a+6K^2(\ln A)^2\gamma_l\kappa_l+\left(c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)\right)(c_1^{(l)})^2}{2K^2(\ln A)^2b_l}t\right\}}}\right). \quad (69)$$

From Eqs. (2) and (69), we get new exact solution of Eq. (1),

$$q^{(l)}(x,t)=\pm\frac{K\ln A}{2}\sqrt{\frac{-6\gamma_l}{\xi_l+\sum_{l\neq n}^{N}\eta_{ln}}}\left(\frac{1-dA^{K\left\{x-\frac{2K^2(\ln A)^2a+6K^2(\ln A)^2\gamma_l\kappa_l+\left(c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)\right)(c_1^{(l)})^2}{2K^2(\ln A)^2b_l}t\right\}}}{1+dA^{K\left\{x-\frac{2K^2(\ln A)^2a+6K^2(\ln A)^2\gamma_l\kappa_l+\left(c_l+\kappa_l\xi_l+\sum_{l\neq n}^{N}\left(d_{ln}+\eta_{ln}\kappa_l\right)\right)(c_1^{(l)})^2}{2K^2(\ln A)^2b_l}t\right\}}}\right)\times e^{i\{-\kappa_l x+\omega_l t+\theta_l\}}, \quad (70)$$

where ω_l is given by Eq. (62) or Eq. (66). Three constraint conditions for the existence of these analytical solutions to exist are given by Eqs. (63), (67) and (68).

3. Conclusions

This paper retrieved dispersive dark and singular optical soliton solutions to DWDM system that stems from SHE. Two integration schemes were employed. They are Riccati-Bernoulli sub-ODE approach and the modified Kudryashov's method. The constraint conditions for the existence of such solitons are also given. These two powerful techniques yielded soliton solution to a class of important NLEEs and thus the scheme stands on a strong footing for future research activities. Later, this scheme will be applied to other models that will also retrieve soliton solutions in optical fibers, PCF, metamaterials, couplers and other forms of optical devices. The results are however awaited at the present time.

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References

[1] M. Ekici, A. Sonmezoglu, Q. Zhou, A. Biswas, M. Z. Ullah, M. Asma, S. P. Moshokoa, M. Belic, Optik **141**, 157 (2017).
[2] M. Ekici, A. Sonmezoglu, Q. Zhou, S. P. Moshokoa, M. Z. Ullah, A. Biswas, M. Belic, Superlattices and Microstructures **107**, 254 (2017).
[3] K. Hosseini, P. Mayeli, R. Ansari, Optik **130**, 737 (2016).
[4] A. Korkmaz, arXiv:1611.07086v2 [nlin.SI] 3 Dec 2016.
[5] M. Mirzazadeh, M. Eslami, M. Savescu, A. H. Bhrawy, A. A. Alshaery, E. M. Hilal, A. Biswas, Journal of Nonlinear Optical Physics and Materials **24**(1), 1550006 (2015).
[6] M. Mirzazadeh, M. Eslami, Q. Zhou, A. Biswas, M. Belic, J. Optoelectron. Adv. M. **17**(3–4), 499 (2015).
[7] M. Savescu, A. A. Alshaery, E. M. Hilal, A. H. Bhrawy, Q. Zhou, A. Biswas, Optoelectron. Adv. Mat. **9**(1-2), 14 (2015).
[8] M. M. A. Qurashi, A. Yusuf, A. I. Aliyu, M. Inc. Superlattices and Microstructures **105**, 183 (2017).
[9] F. Tchier, A. Yusuf, A. I. Aliyu, M. Inc. Superlattices and Microstructures **107**, 320 (2017).
[10] M. Z. Ullah, A. Biswas, S. P. Moshokoa, Q. Zhou, M. Mirzazadeh, M. Belic. Optik **132**, 210 (2017).
[11] X. F. Yang, Z. C. Deng, Y. Wei, Advances in Difference Equations **2015**, 117. (2015).

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