Dispersive optical solitons with Schrödinger–Hirota equation by a couple of integration schemes

ANWAR JA'AFAR MOHAMAD JAWAD[1], ANJAN BISWAS[2,3,4,5,6,*], YAKUP YILIDIRIM[7,8], ABDULAH A. ALGHAMDI[3]
[1]Al-Rafidain University College, Baghdad-10014, Iraq
[2]Department of Mathematics and Physics, Grambling State University, Grambling, LA–71245, USA
[3]Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah–21589, Saudi Arabia
[4]Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Hwy, Moscow–115409, Russian Federation
[5]Department of Applied Sciences, Cross–Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati–800201, Romania
[6]Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa–0204, Pretoria, South Africa
[7]Department of Computer Engineering, Biruni University, 34010 Istanbul, Turkey
[8]Department of Mathematics, Near East University, 99138 Nicosia, Cyprus

The perturbed nonlinear Schrödinger–Hirota equation with spatio–temporal dispersion that governs the propagation of dispersive pulses in optical fibers is investigated in this study using three methods that are the Csch method, Tanh–Coth method and modified simple equation method. The Kerr and power laws of nonlinearity are taken into account. Bright soliton, dark soliton, singular soliton, mixed bright–dark soliton and periodic solutions are retrieved. Many constraint conditions required for the existence of solutions emerge from the integration methods. Furthermore, we demonstrate the dynamical behaviors and physical significance of these solutions by using different parameter values.

(Received May 4, 2022; accepted April 5, 2023)

Keywords: Solitary wave solutions, Csch method, Tanh–Coth method, Modified simple equation method

1. Introduction

The nonlinear Schrödinger Equation (NLSE) is a nonlinear partial differential equation (NPDE) which occurs in many physical situations in fluid mechanics and hydrodynamics to describe the development of surface gravity water waves. It also has numerous applications in nonlinear optics, mathematical finance, fluid dynamics, plasma physics, biochemistry, nuclear physics, superconductivity describing solitary wave propagation in piezoelectric semiconductors, condensed matter, solid-state physics characterizing the propagation of a heat pulse in a solid, and so on [1–6]. Several techniques have been developed to extract soliton solutions to NPDEs, such as the following: the modified direct algebraic method [7], Jacobi elliptic function method [8], sub-equation method [9,10], F-expansion method [11], sine–Gordon expansion method [12,25], Sardar sub-equation method [13], (G′/G)-expansion method [14], Kudryashov method [15–17], new extended direct algebraic method [18,28], homogeneous balance method [19], modified simple equation [20,27], modified Kudryashov's method [25,26], exp-expansion method [25,26], and substantially more.

1.1. Governing model

The perturbed nonlinear Schrödinger–Hirota equation with spatio–temporal dispersion (STD) is considered in the form [21]:

$$iq_t + a_1\, q_{xx} + a_2\, q_{xt} + a_3\, |q|^2\, q$$
$$+ i(\, a_4\, q_{xxx} + a_5 |q|^2 q_x)$$
$$= i(b_1\, q_x + b_2\, (|q|^2\, q)_x + b_3\, (|q|^2)_x\, q), \qquad (1)$$

where a_1, a_2 and a_4 are respectively the coefficients of the group velocity dispersion (GVD), the STD, and the third order dispersion (3OD). The parameters a_3 and a_5 are respectively the coefficients of Kerr law nonlinearity and nonlinear dispersion. The coefficient b_1 , b_2 and b_3 are respectively the inter-modal dispersion (IMD), the self-steepening, and the nonlinear dispersion. The complex function $u = u(x,t)$ is the soliton profile, where x and t are spatial and temporal variables respectively. Lastly, the first term arises from the temporal evolution, where $i = \sqrt{-1}$.

We also investigate the optical solitons with the model having the power law nonlinearity that governs the propagation of dispersive pulses in optical fibers defined by [21]

$$iq_t + a_1\, q_{xx} + a_2\, q_{xt} + a_3\, |q|^{2n}\, q$$
$$+ i(\, a_4\, q_{xxx} + a_5 |q|^{2n} q_x)$$
$$= i(b_1\, q_x + b_2\, (|q|^{2n}\, q)_x + b_3\, (|q|^{2n})_x\, q), \quad (2)$$

where n is the general power law parameter.

Three methods (Csch, Tanh–Coth and modified simple equation) are employed to extract optical soliton solutions.

2. Travelling wave solution

The solutions of Eq. (1) or Eq. (2) are supposed as

$$q(x,t) = u(\xi) e^{i\,\theta(x,t)}, \quad (3)$$

where $\xi = x - \gamma\, t$ and $\theta(x,t) = -kx + \omega t + \theta_0$. $u(\xi)$ and $\theta(x,t)$ are the amplitude and phase components of the soliton wave, respectively. γ is the soliton speed, k is the soliton frequency, ω is the soliton wave number and θ_0 is the phase constant.

Eq. (1) can be also decomposed into real and imaginary parts. The real part of Eq. (1) is:

$$[a_3 + a_5 k - (3b_2 + 2b_3)\, k]\; u^3$$
$$+[a_1 - a_2\, \gamma + 3\, a_4 k]\, u''$$
$$+[a_2\, \omega k - a_4\, k^3 - \omega - b_1\, k - a_1\, k^2]\, u = 0, \quad (4)$$

while the complex part of Eq. (1) is:

$$a_4 u''' + \begin{bmatrix} a_2\, (\gamma k + \omega) - b_1 \\ -\gamma - 2a_1\, k - 3a_4\, k^2 \end{bmatrix} u'$$
$$+[a_5 - (3b_2 + 2b_3)] u^2\, u' = 0. \quad (5)$$

Integrating Eq. (5) gives:

$$a_4\, u'' + \begin{bmatrix} a_2\, (\gamma k + \omega) - 3a_4\, k^2 \\ -b_1 - \gamma - 2a_1 k \end{bmatrix} u$$
$$+\tfrac{1}{3}[a_5 - (3b_2 + 2b_3)]\, u^3 = 0. \quad (6)$$

Now comparing the coefficient of u'', u^3, and u in Eqs. (4) and (6) results the following conditions:

$$a_1 = a_2 \gamma + a_4 - 3\, a_4 k, \quad (7)$$

$$a_3 = \frac{[1-3\,k][a_5 - (3b_2 + 2b_3)]}{3}, \quad (8)$$

and the soliton's speed as:

$$\gamma = \frac{(1-k)b_1 + 2k[1-2k+k^2]a_4 + [a_2\,(k-1) - 1]\omega}{[a_2 k^2 - a_2 k - 1]}. \quad (9)$$

Substituting Eqs. (7)–(9) in Eq. (4) gives:

$$\alpha_1\, u'' + \alpha_2\, u^3 + \alpha_3\, u = 0, \quad (10)$$

where

$$\alpha_1 = 3\, a_4, \quad (11)$$

$$\alpha_2 = [a_5 - (3b_2 + 2b_3)], \quad (12)$$

$$\alpha_3 = \begin{bmatrix} a_2\, \omega k - \omega - b_1\, k \\ +2\, a_4\, k^3 - a_2 \gamma\, k^2 - 3\, a_4\, k^2 \end{bmatrix}. \quad (13)$$

3. Methodology

In this section, we will apply three different methods to solve Eq. (10). These methods are the Csch method, extended Tanh–Coth method and modified simple equation method.

3.1. Csch function method

The solutions of many are expressed in the form [22]:

$$u(\xi) = A\text{csch}^{\tau}(\mu\xi), \quad (14)$$

and

$$u'(\xi) = -A\tau\mu\text{csch}^{\tau}(\mu\xi)\coth(\mu\xi), \quad (15)$$

$$u''(\xi) = A\tau\mu^2 \begin{pmatrix} (\tau+1)\text{csch}^{\tau+2}(\mu\xi) \\ +\tau\, \text{csch}^{\tau}(\mu\xi) \end{pmatrix}, \quad (16)$$

where A, μ and τ are parameters to be determined. μ and λ are the wave number and wave speed of the soliton, respectively.

Substituting Eqs. (14)–(16) into the reduced equation (10), we get

$$\alpha_1 \tau\mu^2 \left((\tau+1)\text{csch}^{\tau+2}(\mu\xi) + \tau\text{csch}^{\tau}(\mu\xi) \right)$$
$$+\alpha_2\, A^2\text{csch}^{3\tau}(\mu\xi) + \alpha_3\text{csch}^{\tau}(\mu\xi) = 0. \quad (17)$$

Balance the terms of the csch functions in Eq. (17), i.e. $3\tau = \tau + 2$, then

$$\tau = 1. \quad (18)$$

Collecting all terms in Eq. (17) with the same power in $\text{csch}^k(\mu\xi)$ for k =1 and 3 and setting their coefficients to zero, we get a system of algebraic equations among the unknown's A, and μ, to the subsequent system:

$$2\,\alpha_1\, \mu^2 + \alpha_2\, A^2 = 0,$$
$$\alpha_1\, \mu^2 + \alpha_3 = 0. \quad (19)$$

Solving the system of equations in (19), we get:

$$A = \mp\sqrt{\frac{2\,\alpha_3}{\alpha_2}}, \quad \mu = \mp\sqrt{-\frac{\alpha_3}{\alpha_1}}. \quad (20)$$

The singular soliton solution to the model is therefore given as:

$$q(x,t) = \mp\sqrt{\frac{2\,\alpha_3}{\alpha_2}}\operatorname{csch}\left(\mp\sqrt{-\frac{\alpha_3}{\alpha_1}}\;(x-\gamma\,t)\right)$$

$$\times\, e^{i(-kx+\omega\,t+\theta_0)}, \tag{21}$$

where

$$\alpha_1\,\alpha_3 < 0.$$

3.2. Tanh–Coth method

To introduce the ansatz, the new independent variable is considered as [23]

$$Y = \tanh(\xi), \tag{22}$$

and

$$\frac{du}{d\xi} = (1-Y^2)\frac{du}{dY}, \tag{23}$$

$$\frac{d^2u}{d\xi^2} = -2Y(1-Y^2)\frac{du}{dY} + (1-Y^2)^2\frac{d^2u}{dY^2}. \tag{24}$$

The solution is expressed in the form

$$u(\xi) = \sum_{i=0}^{p} c_i\, Y^i + \sum_{i=1}^{p} d_i\, Y^{(-i)}, \tag{25}$$

where the parameter p can be found by balancing the highest–order linear term with the nonlinear terms in the reducing equation.

For the reducing equation (10), we balance u^3 with u'' to obtain $3p = p + 2$, then

$$p = 1. \tag{26}$$

The Tanh–Coth method admits the use of the finite expansion for

$$u = (c_0 + c_1 Y + d_1\, Y^{-1}), \tag{27}$$

$$\frac{du}{dY} = (c_1 - d_1\, Y^{-2}), \tag{28}$$

$$\frac{d^2u}{dY^2} = (2d_1\, Y^{-3}). \tag{29}$$

Substituting u, u^3 and u'' into Eq. (10) yields

$$2\,\alpha_1\,(-c_1 Y + c_1 Y^3 + d_1\, Y^{-3} - d_1\, Y^{-1})$$

$$+\alpha_2\begin{pmatrix} c_0{}^3 + 6c_0c_1d_1 + 3c_0{}^2c_1Y + 3c_1{}^2d_1Y \\ +3\,c_0c_1{}^2Y^2 + c_1{}^3Y^3 + 3c_0{}^2d_1\,Y^{-1} \\ +3\,c_1d_1{}^2\,Y^{-1} + 3c_0d_1{}^2\,Y^{-2} + d_1{}^3\,Y^{-3}\end{pmatrix}$$

$$+\alpha_3\,(c_0 + c_1Y + d_1\,Y^{-1}) = 0, \tag{30}$$

where c_0, c_1 and d_1 are to be determined.

Equating expressions at Y^i, $(i = -3, -1, 0, 1, 3)$ to zero, we have the following system of algebraic equations:

$$Y^{-3}: 2\,\alpha_1 + \alpha_2\,d_1{}^2 = 0,$$

$$Y^{-1}: 2\,\alpha_1 - \alpha_2\,(3c_0{}^2 + 3c_1d_1) - \alpha_3 = 0,$$

$$Y^{0}: \alpha_2\,(c_0{}^2 + 6d_1c_1) + \alpha_3 = 0,$$

$$Y^{1}: 2\alpha_1 - \alpha_2\,(3\,c_0{}^2 + 3d_1c_1) - \alpha_3 = 0,$$

$$Y^{3}: 2\,\alpha_1 + \alpha_2\,c_1{}^2 = 0. \tag{31}$$

Solving the system of equations (31), we get:

$$c_1 = d_1 = \mp\sqrt{-\frac{2\,\alpha_1}{\alpha_2}},\; c_0 = \mp\sqrt{\frac{8\,\alpha_1 - \alpha_3}{3\alpha_2}},$$

$$\omega = \frac{42\,a_4 + b_1\,k - 2\,a_4\,k^3 + a_2\gamma\,k^2 + 3\,a_4\,k^2}{(a_2k-1)}. \tag{32}$$

The dark–singular soliton solution to the model is therefore presented as below

$$q(x,t) = \begin{bmatrix} \mp\sqrt{\dfrac{8\,\alpha_1-\alpha_3}{3\alpha_2}} \mp\sqrt{-\dfrac{2\,\alpha_1}{\alpha_2}} \\ \times\{\tanh\,(x-\gamma t) + \coth(x-\gamma t)\}\end{bmatrix}$$

$$\times\; e^{i(-kx+\omega t+\theta_0)}, \tag{33}$$

where

$$\alpha_1\,\alpha_2 < 0.$$

The periodic soliton solution to the model is also structured as

$$q(x,t) = \begin{bmatrix} \mp\sqrt{\dfrac{8\,\alpha_1-\alpha_3}{3\alpha_2}} \mp\sqrt{-\dfrac{2\,\alpha_1}{\alpha_2}} \\ \times\{\tan\,(x-\gamma t) + \cot(x-\gamma t)\}\end{bmatrix}$$

$$\times\; e^{i(-kx+\omega t+\theta_0)}, \tag{34}$$

where

$$\alpha_1\,\alpha_2 > 0.$$

3.3. The modified simple equation method

We look for the solutions of Eq. (10) in the form [24]:

$$u = A_0 + A_1 \frac{\psi'}{\psi}, \tag{35}$$

$$u' = A_1 \left(\frac{\psi''}{\psi} - \left(\frac{\psi'}{\psi} \right)^2 \right), \tag{36}$$

$$u'' = A_1 \left(\frac{\psi'''}{\psi} - 3 \frac{\psi'' \psi'}{\psi^2} + 2 \left(\frac{\psi'}{\psi} \right)^3 \right). \tag{37}$$

Then Eq. (10) can be written as

$$\alpha_1 A_1 \left(\frac{\psi'''}{\psi} - 3 \frac{\psi'' \psi'}{\psi^2} + 2 \left(\frac{\psi'}{\psi} \right)^3 \right)$$
$$+\alpha_2 \left[\begin{array}{c} A_0{}^3 + 3 A_0{}^2 A_1 \frac{\psi'}{\psi} \\ +3A_0 A_1{}^2 \left(\frac{\psi'}{\psi} \right)^2 + A_1{}^3 \left(\frac{\psi'}{\psi} \right)^3 \end{array} \right]$$
$$+\alpha_3 \left(A_0 + A_1 \frac{\psi'}{\psi} \right) = 0. \tag{38}$$

Equating expressions at $\psi^0, \psi^{-1}, \psi^{-2}$, and ψ^{-3} to zero, we have the following system of equations:

$$\alpha_2 A_0{}^2 + \alpha_3 = 0,$$
$$\alpha_1 \psi''' + \left(3 A_0{}^2 \alpha_2 + \alpha_3 \right) \psi' = 0,$$
$$\alpha_1 \psi'' - \alpha_2 A_0 A_1 \psi' = 0,$$
$$2 \alpha_1 + \alpha_2 A_1{}^2 = 0. \tag{39}$$

Solving the system of equations in (39), we get:

$$A_0 = \mp \sqrt{-\frac{\alpha_3}{\alpha_2}}, A_1 = \mp \sqrt{-\frac{2 \alpha_1}{\alpha_2}}, \tag{40}$$

$$\psi''' + \frac{2 \alpha_3}{\alpha_1} \psi' = 0, \tag{41}$$

$$\psi'' - \sqrt{\frac{2 \alpha_3}{\alpha_1}} \psi' = 0. \tag{42}$$

From Eqs. (41) and (42), we can deduce that

$$\psi' = c_1 \sqrt{\frac{\alpha_1}{2 \alpha_3}} e^{\sqrt{\frac{2 \alpha_3}{\alpha_1}} \xi}, \tag{43}$$

and

$$\psi = c_1 \frac{\alpha_1}{2 \alpha_3} e^{\sqrt{\frac{2 \alpha_3}{\alpha_1}} \xi} + c_2, \tag{44}$$

where c_1 and c_2 are constants of integration. Substituting Eqs. (43) and (44) into Eq. (35), we obtain the following exact solution to Eq. (1):

$$q(x,t) = \left\{ \begin{array}{c} \mp \sqrt{-\frac{\alpha_3}{\alpha_2}} \mp \sqrt{-\frac{2 \alpha_1}{\alpha_2}} \\ \times \dfrac{c_1 \sqrt{\frac{\alpha_1}{2 \alpha_3}} e^{\sqrt{\frac{2 \alpha_3}{\alpha_1}} (x - \gamma t)}}{c_1 \frac{\alpha_1}{2 \alpha_3} e^{\sqrt{\frac{2 \alpha_3}{\alpha_1}} (x - \gamma t)} + c_2} \end{array} \right\}$$
$$\times e^{i(-kx + \omega t + \theta_0)}. \tag{45}$$

If we set $c_1 = \frac{\alpha_1}{2 \alpha_3} e^{\sqrt{\frac{2 \alpha_3}{\alpha_1}} \xi_0}$, $c_2 = \mp 1$, we obtain:

$$q(x,t) = \left\{ \mp \sqrt{-\frac{\alpha_3}{\alpha_2}} \mp \tanh(\sqrt{\frac{2 \alpha_3}{\alpha_1}} (x - \gamma t + \xi_0) \right\}$$
$$\times e^{i(-kx + \omega t + \theta_0)}, \tag{46}$$

$$q(x,t) = \left\{ \mp \sqrt{-\frac{\alpha_3}{\alpha_2}} \mp \coth(\sqrt{\frac{2 \alpha_3}{\alpha_1}} (x - \gamma t + \xi_0) \right\}$$
$$\times e^{i(-kx + \omega t + \theta_0)}, \tag{47}$$

$$q(x,t) = \left\{ \mp \sqrt{-\frac{\alpha_3}{\alpha_2}} \mp \tan(\sqrt{-\frac{2 \alpha_3}{\alpha_1}} (x - \gamma t + \xi_0) \right\}$$
$$\times e^{i(-kx + \omega t + \theta_0)}, \tag{48}$$

$$q(x,t) = \left\{ \mp \sqrt{-\frac{\alpha_3}{\alpha_2}} \mp \cot(\sqrt{-\frac{2 \alpha_3}{\alpha_1}} (x - \gamma t + \xi_0) \right\}$$
$$\times e^{i(-kx + \omega t + \theta_0)}. \tag{49}$$

Eqs. (46) and (47) represent dark soliton and singular soliton solutions, respectively. These solitons are valid for $\alpha_1 \alpha_3 > 0$. Eqs. (48) and (49) also represent singular periodic solutions that are valid for $\alpha_1 \alpha_3 < 0$. The surface plot of soliton (46) is depicted in Fig. 1. The parameter values chosen are: $a_4 = 1$, $k = 1$, $\gamma = 1$, $\xi = 1$, $a_2 = 1$, $\omega = 1$, $b_1 = -3$, $a_5 = 1$, $b_2 = 1$ and $b_3 = 1$.

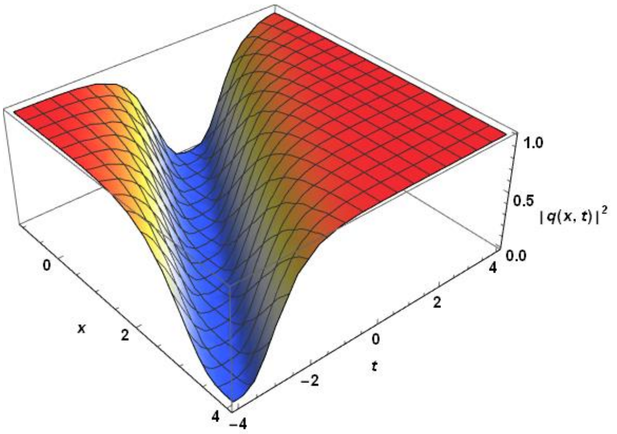

Fig. 1. Profile of dark soliton in optical fibers (color online)

4. Application to the model with power law nonlinearity

Let consider the transformation defined in Eq. (3) to split Eq. (2) respectively into real and imaginary parts as:

$$(a_1 - a_2\gamma + 3a_4k)\, u'' + (a_2\omega k - b_1 k - \omega - a_1k^2 - a_4k^3)\, u + [a_3 + \{a_5 - \{(2n+1)b_2 + 2n\, b_3\}\}k]\, u^{2n+1} = 0, \quad (50)$$

$$a_4\, u''' + \{a_2(\gamma k + \omega) - 3a_4k^2 - \gamma - 2ka_1 - b_1\}\, u' - [a_5 + (2n+1)b_2 + 2n\, b_3]u^{2n}u' = 0. \quad (51)$$

Integrating Eq. (51) with respect to ξ results:

$$(2n+1)a_4\, u'' - [a_5 + (2n+1)b_2 + 2n\, b_3]\, u^{2n+1} + (2n+1)\begin{Bmatrix} a_2(\gamma k + \omega) - 3a_4k^2 \\ -\gamma - 2ka_1 - b_1 \end{Bmatrix} u = 0. \quad (52)$$

Comparing Eq. (52) with Eq. (50), we get:

$$a_1 = a_2\gamma - 3a_4k + (2n+1)\, a_4, \quad (53)$$

$$a_3 = (k-1)[(2n+1)b_2 + 2n\, b_3] - a_5(k+1), \quad (54)$$

$$\gamma = \frac{\begin{pmatrix} a_2\omega k - b_1 k - \omega \\ -a_1k^2 - a_4k^3 \end{pmatrix} + (2n+1)\begin{pmatrix} 3a_4k^2 + 2ka_1 \\ +b_1 - a_2\omega \end{pmatrix}}{(2n+1)(a_2k-1)}. \quad (55)$$

Substituting Eqs. (53)–(55) into Eq. (52) gives:

$$\beta_1\, u'' + \beta_2\, u^{(2n+1)} + \beta_3\, u = 0, \quad (56)$$

where

$$\beta_1 = (2n+1)\, a_4, \quad (57)$$

$$\beta_2 = [a_5 - (2n+1)b_2 - 2n\, b_3], \quad (58)$$

$$\beta_3 = (2n+1)\begin{Bmatrix} a_2(\gamma k + \omega) - 3a_4k^2 \\ -\gamma - 2ka_1 - b_1 \end{Bmatrix}. \quad (59)$$

Balancing u'' with u^{2n+1} in Eq. (56), we get τ= 1/n. To obtain closed form solutions, we consider the transformation

$$u = V^{\frac{1}{n}}, \quad (60)$$

to reduce Eq. (56) to the below equation:

$$\beta_1[(1-n)V'^2 + nV\, V''] + n^2\beta_2\, V^4 + n^2\beta_3 V^2 = 0. \quad (61)$$

4.1. The csch method

The solution of Eq. (61) is expressed in the form of Eq. (14). Balancing $V\, V''$ with V^4 in Eq. (61) gives τ = 1. From Eqs. (14)-(16), Eq. (61) becomes

$$\beta_1\mu^2\left[(1+n)\text{csch}^4(\mu\xi) - (1-2n)\text{csch}^2(\mu\xi)\right] + n^2\beta_2A^2\text{csch}^4(\mu\xi) + n^2\beta_3\text{csch}^2(\mu\xi) = 0. \quad (62)$$

Collecting all coefficient of $\text{csch}^k(\mu\xi)$, for k = 2 and 4 to be equal zero, we have

$$\beta_1\,\mu^2\,(1+n) + n^2\beta_2\, A^2 = 0,$$
$$\beta_1\,\mu^2\,(1-2n) - n^2\beta_3 = 0. \quad (63)$$

Solving the system (63), we get:

$$A = \mp\sqrt{\frac{\beta_3(1+n)}{\beta_2(2n-1)}},\ \mu = \mp n\sqrt{\frac{\beta_3}{\beta_1(1-2n)}}. \quad (64)$$

The singular soliton solution to the model with power law of nonlinearity is therefore formulated as

$$q(x,t) = \mp\sqrt{\frac{\beta_3(1+n)}{\beta_2(2n-1)}} \times \text{csch}^{1/n}\left(n\sqrt{\frac{\beta_3}{\beta_1(1-2n)}}\,(x-\gamma\, t)\right) \times e^{i(-kx+\omega t+\theta_0)}, \quad (65)$$

where

$(1-2n)\,\beta_3\,\beta_1 > 0.$

4.2. The Tanh–Coth method

Substituting V, V' and V'' into Eq. (61) results the following:

$$\beta_1\left[\begin{array}{c}(1-n)(c_1-d_1Y^{-2}-c_1Y^2+d_1)^2\\ +n\left\{\begin{array}{c}(c_0+c_1Y+d_1Y^{-1})\\ \times\left(\begin{array}{c}-2c_1(Y-Y^3)+2d_1(Y^{-1}-Y)\\ +2d_1(Y^{-3}-2Y^{-1}+Y)\end{array}\right)\end{array}\right\}\end{array}\right]$$

$$+n^2\beta_2\,(c_0+c_1Y+d_1Y^{-1})^4$$

$$+n^2\beta_3(c_0+c_1Y+d_1Y^{-1})^2=0, \qquad (66)$$

where c_0, c_1 and d_1 are to be determined. Eq. (66) can be written as:

$$\beta_1(1-n)\begin{pmatrix}c_1(c_1-d_1Y^{-2}-c_1Y^2+d_1)\\ -d_1(c_1Y^{-2}-d_1Y^{-4}-c_1+d_1Y^{-2})\\ -c_1(c_1Y^2-d_1-c_1Y^4+d_1Y^2)\\ +d_1(c_1-d_1Y^{-2}-c_1Y^2+d_1)\end{pmatrix}$$

$$+\beta_1 n\begin{pmatrix}-2c_1\begin{pmatrix}c_0Y+c_1Y^2+d_1-c_0Y^3\\ -c_1Y^4-d_1Y^2\end{pmatrix}\\ +2d_1\begin{pmatrix}c_0(Y^{-1}-Y)+c_1(1-Y^2)\\ +d_1(Y^{-2}-1)\end{pmatrix}\\ +2d_1\begin{pmatrix}c_0Y^{-3}+c_1Y^{-2}+d_1Y^{-4}\\ -2(c_0Y^{-1}+c_1+d_1Y^{-2})\\ +c_0Y+c_1Y^2+d_1\end{pmatrix}\end{pmatrix}$$

$$+n^2\beta_2\begin{pmatrix}(c_0^2+2c_1d_1)\begin{pmatrix}(c_0^2+2c_1d_1)\\ +2c_0c_1Y+c_1^2Y^2\\ +2c_0d_1Y^{-1}+d_1^2Y^{-2}\end{pmatrix}\\ +2c_0c_1\begin{pmatrix}(c_0^2+2c_1d_1)Y+2c_0c_1Y^2\\ +c_1^2Y^3+2c_0d_1+d_1^2Y^{-1}\end{pmatrix}\\ +c_1^2\begin{pmatrix}(c_0^2+2c_1d_1)Y^2+2c_0c_1Y^3\\ +c_1^2Y^4+2c_0d_1Y+d_1^2\end{pmatrix}\\ +2c_0d_1\begin{pmatrix}(c_0^2+2c_1d_1)Y^{-1}+2c_0c_1\\ +c_1^2Y+2c_0d_1Y^{-2}+d_1^2Y^{-3}\end{pmatrix}\\ +d_1^2\begin{pmatrix}(c_0^2+2c_1d_1)Y^{-2}+2c_0c_1Y^{-1}\\ +c_1^2+2c_0d_1Y^{-3}+d_1^2Y^{-4}\end{pmatrix}\end{pmatrix}$$

$$+n^2\beta_3\begin{pmatrix}(c_0^2+2c_1d_1)+2c_0c_1Y\\ +c_1^2Y^2+2c_0d_1Y^{-1}+d_1^2Y^{-2}\end{pmatrix}=0. \quad (67)$$

Equating expressions at Y^i to zero, we have the following system of algebraic equations:

$$Y^{-4}:\beta_1(1-n)+n^2\beta_2 d_1^2=0,$$

$$Y^{-3}:\beta_1+2n\,d_1^2\beta_2=0,$$

$$Y^{-2}:\beta_1\,[-(c_1+d_1)-nd_1+3nc_1]$$

$$+2n^2\beta_2 d_1(3c_0^2+2c_1d_1)+n^2 d_1\beta_3=0,$$

$$Y^{-1}:\beta_1-2n\beta_2(c_0^2+3c_1d_1)-n\beta_3=0,$$

$$Y^0:\beta_1[(c_1^2+4d_1c_1+d_1^2)-nc_1^2-8nc_1d_1+nd_1^2]$$

$$+n^2\beta_2(c_0^4+12c_0^2c_1d_1+6d_1^2c_1^2)$$

$$+n^2\beta_3(c_0^2+2c_1d_1)=0,$$

$$Y:\beta_1-2n\beta_2(c_0^2+3c_1d_1)-n\beta_3=0,$$

$$Y^2:\beta_1[-(c_1+d_1)-nc_1+3nd_1]$$

$$+2n^2\beta_2 c_1(3c_0^2+2c_1d_1)+n^2c_1\beta_3=0,$$

$$Y^3:\beta_1+2n\beta_2c_1^2=0,$$

$$Y^4:\beta_1[1-n]+n^2c_1^2\beta_2=0. \qquad (68)$$

Solving the system of equations (68), we get:

$$c_0=\mp\frac{1}{n}\sqrt{\frac{6\beta_1-5n\beta_1-n^2\beta_3}{2\beta_2}},$$

$$c_1=d_1=\mp\frac{1}{n}\sqrt{\frac{\beta_1(n-1)}{\beta_2}},$$

$$\beta_1=\frac{n^2(3n^2-1)}{(6n-6+18n^2-15n^3)}\beta_3. \qquad (69)$$

The dark–singular soliton solution of the model is therefore introduced as below

$$q(x,t)=\mp\frac{1}{n}\left[\begin{array}{c}\sqrt{\frac{6\beta_1-5n\beta_1-n^2\beta_3}{2\beta_2}}+\sqrt{\frac{\beta_1(n-1)}{\beta_2}}\\ \times\{\tanh(x-\gamma t)+\coth(x-\gamma t)\}\end{array}\right]$$

$$\times e^{i(-kx+\omega t+\theta_0)}, \qquad (70)$$

where

$$(n-1)\beta_1\beta_2>0.$$

5. Conclusions

This paper retrieves new travelling wave solutions. The Csch method, extended Tanh–Coth method and modified simple equation method are applied to get results that are of great asset to the nonlinear Schrödinger–Hirota equation. They are used to carry out the integration of nonlinear Schrödinger–Hirota equation. The obtained solutions are under certain conditions and are very useful and may be important to explain some physical phenomena and find applications in the nonlinear evolution equations. The results of this paper will be of great future need. The results of those research findings will be available down the road.

References

[1] C. Sulem, P. L. Sulem, New York: Springer–Verlag, 1999.
[2] M. J. Ablowitz, Nonlinear Dispersive Waves, Cambridge University Press, 2011.
[3] M. J. Ablowitz, B. Prinari, A. D. Trubatch. Discrete and Continuous Nonlinear Schrodinger System, Cambridge University Press, 2004.
[4] I. K. Mylonas, C. B. Ward, P. G. Kevrekidis, V. M. Rothos, D. J. Frantzeskakis, Phys. Lett. A **381**, 3965, (2017).
[5] A. H. Ahmed, M. Z. Ullah, M. Asma, Q. Zhou, M. Mirzazadeh, A. Biswas, M. Belic, Optik **136**, 445 (2017).
[6] K. Hosseini, K. Sadri, M. Mirzazadeh, S. Salahshour, Optik **229**, 1 (2021).
[7] W. Li, L. Akinyemi, D. Lu, M. Khater, Symmetry **13**(6), 1085 (2021).
[8] E. A. Az-Zobi, W. A. Alzoubi, L. Akinyemi, M. Şenol, B. S. Masaedeh, Modern. Phys. Lett. B **35**(15), 2150254 (2021).
[9] L. Akinyemi, M. Senol, O. S. Iyiola, Math. Comput. Simulation **182**, 211 (2021).
[10] M. Senol, L. Akinyemi, A. Ata, O. S. Iyiola, Int. J. Modern Phys. B **35**(2), 2150021 (2021).
[11] D. Lu, Z. Zhang, Int. J. Nonlinear Sci. **24**(2), 96 (2017).
[12] D. Kumar, K. Hosseini, F. Samadani, Optik **149**, 439 (2017).
[13] A. Houwe, S. Yakada, S. Abbagari, Y. Saliou, S. Y. Doka, Eur. Phys. J. Plus **136**(4), 1 (2021).
[14] M. Shakeel, S. T. Mohyud-Din, Journal of the Association of Arab Universities for Basic and Applied Sciences **18**, 66 (2015).
[15] N. A. Kudryashov, Communications in Nonlinear Science and Numerical Simulation **17**(6), 2248 (2012).
[16] N. A. Kudryashov, Optik **206**, 163550 (2020).
[17] L. Akinyemi, K. Hosseini, S. Salahshour, Optik **242**, 167120 (2021).
[18] M. Senol, Revista mexicana de fisica **66**(3), 297 (2020).
[19] H. Jafari, H. Tajadodi, D. J. Baleanu, Comput. Nonlinear Dyn. **9**(2), 021019 (2014).
[20] A. Biswas, Y. Yildirim, E. Yasar, H. Triki, A. S. Alshomrani, M. Z. Ullah, Optik **158**, 399 (2018).
[21] L. Akinyemi, H. Rezazadeh, Q. H. Shi, M. Inc, M. A. Khater, H. Ahmad, A. Jhangeerj, M. A. Akbar, Results Phys. **29**, 104656 (2021).
[22] A. J. A. M. Jawad, M. J. Abu-AlShaeer, E. M. E. Zayed, M. E. Alngar, A. Biswas, M. Ekici, A. K. Alzahrani, M. R. Belic, Optik **223**, 165329 (2020).
[23] A. J. A. M. Jawad, A. Biswas, Q. Zhou, M. Alfiras, S. P. Moshokoa, M. Belic, Optik **178**, 172 (2019).
[24] A. J. A. M. Jawad, M. D. Petković, A. Biswas, Appl. Math. Comput. **217**(2), 869 (2010).
[25] A. Biswas, H. Rezazadeh, M. Mirzazadeh, M. Eslami, M. Ekici, Q. Zhou, S. P. Moshokoa, M. Belic, Optik **165**, 288 (2018).
[26] A. Biswas, H. Rezazadeh, M. Mirzazadeh, M. Eslami, Q. Zhou, S. P. Moshokoa, M. Belic, Optik **164**, 380 (2018).
[27] A. Biswas, M. O. Al-Amr, H. Rezazadeh, M. Mirzazadeh, M. Eslami, Q. Zhou, S. P. Moshokoa, M. Belic, Optik **165**, 233 (2018).
[28] H. Rezazadeh, Optik **167**, 218 (2018).

*Corresponding author: biswas.anjan@gmail.com