Effect of initial velocity of accelerated charged particles on laser resonant acceleration

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The theoretical and numerical analysis are presented concerning the conditions at which the charged particles of different initial velocity can be accelerated to the significant kinetic energy in the circularly polarized laser or maser beams and a static coaxial magnetic field. The studies are carried out using the analytically derived equations for the particles dynamics and theirs kinetic energy. The presented illustrations enabled interpretation of the complex motion of the particles and possibilities of their acceleration. At the examples of an electron the kinetic energy and trajectory as a function of the acceleration time at the resonance condition are illustrated in the appropriate graphs. The increase of the particles initial velocity results in the reduction of the coaxial magnetic field intensity at the resonance condition. This field intensity reduction rises with the velocity of the preaccelerated particles.

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1. Introduction

The accelerated particles can make an essential influence in many areas of technology and science [1-7]. Many experiments have shown the possibility of particles acceleration as a result of their interaction with the laser or maser beams of different parameters [8-29]. There are known presentations of the new types of charged particle accelerators and a different approach leading to acceleration of charged particles to relativistic energies [30-35]. In the objective of simplification of calculations the zero initial conditions are usually applicable by putting the particle in a location (0, 0, 0) and with its initial velocity (0, 0, 0). Taking into consideration the fact that such initial conditions are not very real, we performed analysis and calculations at the other initial conditions. The study results show that the change in the location of the particle's initial conditions does not change the shape of the trajectory, only it changes its starting point [34-41]. However the variation in the initial velocity of the particle, impacts the shape in the trajectory and in the total velocity of the particle accelerated as a result of the electromagnetic and magnetic fields actions. At conditions far away from the resonance one, projection of the trajectory onto the (x, y) plane when $v_{0x} \neq 0$, $v_{0y} \neq 0$ and $v_{0z} \neq 0$, is subjected to the destabilization. It means that the route of the stationary trajectory appearing at certain non resonance conditions, at the zero initial velocity was found to be disrupted. In this case at subsequent cycles of the trajectory the (x, y) projection curves at certain not resonance conditions, were found to be located at different positions, while at zero velocity initial conditions only the z coordinate changes as the particle is moving along the trajectory. While the (x, y) projection is maintained during the acceleration process. The energy does not fall periodically to the zero, but to the value of the initial energy. It also results from calculations that non-zero components of the initial velocity of the particle at resonance conditions, impact on the increase rate of the kinetic energy provided the resultant velocity is greater than zero in the direction of the particle motion.

2. Motion of initial accelerated charged particles

Salamin et al in [37, 38] studied the dynamics of an electron initially moving parallel to the laser propagation direction at the velocity $v_{z0} = c\beta_{z0}$ and $\beta_{x0} = \beta_{y0} = 0$, $x_0 = y_0 = z_0 = 0$. This choice of initial condition implies that $\gamma_0 = \left(1 - \beta_{z0}^2\right)^{-1/2}$. The authors obtained that the general shape of the electron trajectory in the case of initial motion at a velocity $v_{z0} = c\beta_{z0}$ along z axis is the same as for $v_{z0} = 0$, i.e. the helical trajectory. Due to the initial forward momentum, however, the electron in this case will describe a much longer helix over the same number of field cycles. The result for linear polarization for $v_{z0} = 0$ (calculated for over 1000 laser field cycles) and for $\gamma_0 = 10$, $(\beta_{z0} = 0.995)$ was shown there. For similar set of field parameters the results (calculated for over 100 laser field cycles only) were also demonstrated. The trajectory in presented figures was not a perfect helix. There are wiggles due to effect of the initial forward momentum. Only longitudinal extension of the electron trajectory (along z) depends mostly on the initial electron velocity β_{z0}. In our paper the trajectories of electron moving in a circularly polarized laser field have been analyzed. Also components of the scaled electron velocity are shown. In the case of circularly polarized field β_x and β_y exhibit beat structures, whereas β_z oscillates almost regularly between $\beta = 0$ and a maximal value close to unity.

Yousef I. Salamin et al [37] have shown for linear polarization that an electron injected initially with a kinetic energy of about 4.5 MeV is accelerated to a final energy of more than 1.5 GeV for $B_z = 30$ T, I = 10^{20} W/cm^2, $\lambda = 0.8$ μm after interaction with 15 field cycles. For example a 68.8 MeV electron may be accelerated to 30 TeV by circularly polarized pulse and for $B_z = 50$ T, over the distance of roughly 6.4 km. However, a maximum of about 15 TeV may be reached over almost 1.7 km for $B_z = 100$ T, but only if the electron is preaccelerated to $E_0 = 34.4$ MeV. The other results of these authors show that in $B_z = 1000$ T a 3.46 MeV electron may reach 1.5 TeV over a distance 17 m. They also show that MeV electron may be boosted to energies of up to several TeV by a plane wave of ultra high intensity laser field, provided it is injected parallel to the lines of a uniform axial magnetic field of strength currently available to laboratory experiment. However the calculation results published by the above mentioned authors assume the unrealistic time of acceleration which means application of the laser pulse interacting with the electron lasting of tents of microseconds. This time we can roughly estimate from the distance the electron penetrates and its velocity. In our opinion there do not exist lasers of such high intensities with microsecond pulses. In our paper we show simulations accomplished by the real laser parameters.

The optimum magnitude of the resonance magnetic field, under acceleration by a circularly polarized laser field, as were reported in [34, 40] is independent of the laser intensity and decreases with initial electron energy. The magnitude of the magnetic field at which resonance occurs has been found out for different initial electron energies and laser intensities. In the paper the initial position of the electron is assumed at (x_0, y_0, z_0) with initial momentum $(0, 0, p_{z0})$. $p_{z0} = \gamma_0 m_0 \mathrm{v}_{z0}$. The results have been obtained for different initial position, initial momentum, laser intensity and magnetic field. Presented figures show variation of optimum magnetic field as a function of p_{z0}. The magnitude of the optimum magnetic field is inversely proportional to p_{z0}. Presented figures also show that the energy gained by the electron increases with laser intensity and slightly decreases with initial electron energy. Acceleration gradient decreases with initial momentum p_{z0} and increases with laser intensity. Optimum magnitude of the magnetic field is very high for low initial electron energies. If such a strong magnetic field is not available at a laboratory, then the proposed scheme is useful for acceleration of pre-accelerated electrons because the optimum value of magnetic field at resonance conditions decreases with the initial electron energy.

Jing Guo-Liang et al. [41] describe the studies on acceleration of initially moving electron by co-propagation laser pulse in the vacuum. The electron is accelerated in the first step from the rest to a few MeV of energy using the laser pulses and then further accelerated in the second step close to a hundred of MeV by a co-propagation of ultra-short and ultra-intense laser pulse based on the chirped pulse amplification technique. For an electron initially at the rest its maximum energy at the peak of circularly polarized and a planar laser pulse is merely 7 MeV. However, for an electron which is in motion with $\gamma_0 = 10$, $(5MeV)$, its peak energy gain at a laser pulse goes up to 130 MeV. It has been demonstrated by D.N. Gupta et al. using numerical simulations [35], that a single electron injected at certain angle to the axis of a plane-polarized laser beam may be captured and accelerated up to GeV energies [42,43]. Furthermore, the peak energy attained by the electron increases with the initial electron energy. Presented by the authors figures show the variation of the electron energy γ with time (normalized by $1/\omega_0$) for different initial electron momenta and λ=1 μm. They observed that the electron gains energies of about 40 MeV and 20 MeV for initial electron moment $p_0 = 0.1$ and $p_0 = 2$, respectively. They also observed that the accelerated electrons tend to get out of phase with the field. Therefore, they are decelerated and lose the energy. Eventually they cannot gain any more energy. However, the net electron energy gain can be enhanced by using a very high-intensity laser. The net energy gained by the electron during acceleration is sensitive to the initial electron energy. The electron can gain very high energy if it is pre-accelerated because the duration of the interaction between the laser and the electron increases with the initial electron energy. The authors show variation of the electron energy γ with time (normalized by $1/\omega_0$) for different initial electron kinetic energy $\gamma_0 = 1.5$ and $\gamma_0 = 5$ and for $\lambda_0 = 1$ μm and $\lambda_1 = 10$ μm (beat wave). The result indicates that the electron experiences a ponderomotive force due to the combined fields of both two counter-propagating waves of greatly different wavelength. The effect of an external magnetic field on the maximum energy gain is also investigated. If a very strong magnetic field is not available in the laboratory then an experiment can be carried out with the high initial electron energy and a high laser intensity, because the optimum magnitude of the magnetic field decreases with increasing the initial electron energy. Electron acceleration to relativistic energies has already been reported experimentally in [44-47].

3. Physics of the resonance acceleration

The energy transfer rate as it follows from energy equation of conservation

$$\frac{d\gamma}{dt} = \frac{q}{m_0 c}\vec{\beta} \cdot \vec{E}$$

is proportional to $\vec{\beta}\cdot\vec{E}$ where $\vec{\beta}c$ is the particles' velocity vector, p, q and m_0 are the momentum, charge and rest mass of the particle, $\vec{E}$ and B are the vectors of the electric and magnetic field strengths of the electromagnetic wave, c is the velocity of the electromagnetic wave. The equation shows [48,50] that the energy transferred to the particle will be maximized if the vectors $\vec{\beta}$ and $\vec{E}$ are parallel, and retain their orientation over many oscillations. $\vec{E}$ oscillates so rapidly that $\left|\vec{\beta}\cdot\vec{E}\right|$ nearly averages to zero over any macroscopic time scale. The key to achieving large energy transfers is to make $\vec{\beta}$ rotate rapidly (with $\vec{E}$) for long time. If the radiation is circularly polarized, it is possible to choose parameters such that $\vec{\beta}$ and $\vec{E}$ maintaining their relative orientation through many oscillations and allow significant energy transfer to occur. For most choices of parameters, the energy is an oscillatory functions of time. The particle alternately gains and losses energy. If the parameters are such that $\vec{\beta}$ and $\vec{E}$ rotate synchronously, the particle's energy is not a periodic function but increases or decreases monotonically. In this region efficient coupling between the particle and radiation field can occur and the interaction is resonant. Such a synchronization can be achieved by combination of static longitudinal magnetic induction and the laser radiation frequency. The sign of the energy equation of conservation depending on the initial orientation of $\vec{\beta}$ and $\vec{E}$ i.e. on the angle $\theta(t)$ between the rotating electric field vector $\vec{E}$ and the rotating perpendicular component of the particle's velocity vector $\vec{v}_\perp$

$$\vec{\beta}_\perp\cdot\vec{E}=\beta_\perp E\cos\theta$$

As the orientation of $\vec{\beta}$ and $\vec{E}$ vectors changes in time, the energy of the particle must also change. In such a case we will observed a periodic function of particle's kinetic energy in time. Only in the resonance case, when $\theta=0$, ($\cos\theta=1$), or $\theta=\pi$ for electron, ie as the rate of rotation of $\vec{\beta}$ approaches to that $\vec{E}$, the particle can get the maximum energy from the radiation electric field and efficient energy transfer is possible. In the spatial case, if B_z is chosen such that $\Delta\omega=\dfrac{qB_z}{\gamma_0 mc}-(1-\beta_{z0})\omega=0$ (the resonance condition), θ does not oscillate, but (for electron) evolves monotonically toward π. So if a homogeneous, mono energetic beam of electrons is injected into the interaction region with $\Delta\omega=0$, the energy change of a particular electron depends on its initial position within of wavelength of light: those with θ_0 between $\pi/2$ and $3\pi/2$ are accelerated, the others are decelerated. If $\theta_0=\pi$ and $\Delta\omega=0$, than $\theta(z)$ is constant, and $\vec{\beta}$ and $\vec{E}$ rotate at exactly the same rate. In this case the electron initially continues the greatest acceleration. Therefore, if two particles are injected with the same energy but different angles, the particle with the smaller angle θ_0 will be accelerated longer and to the higher energy. So if the charged particle is injected into the resonance interaction region with velocity $\vec{v}_\perp$, the energy of the particle depends on its initial velocity direction in the (x, y) plane.

4. Equations for charged particle trajectories and kinetic energy

In this section we derive analytic expressions for trajectories of a free electron moving in a plane-wave laser field and an externally applied strong uniform magnetic field aligned in the direction of propagation of the laser beam. The electron dynamics resulting from interaction with these fields we have analyzed in the case of a circular polarized electromagnetic monochromatic wave in the lossless conditions [50, 52, 53, 54, 55, 56]. Dynamical relativistic equation and the continuous equation of normalized energy γ in this case have the following form

$$\begin{aligned}\frac{d\vec{p}}{dt}&=q\vec{E}+q\left[\vec{v}\times\left(\vec{B}+\vec{B}_z\right)\right],\\ \frac{d\gamma}{dt}&=\frac{q}{m_0c^2}\vec{v}\cdot\vec{E}\end{aligned}\tag{1}$$

where v is the vector of a particle's velocity and B_z is the external static magnetic induction in direction along the z coordinate, and

$$\begin{aligned}&\gamma=\left(1-\beta^2\right)^{-\frac{1}{2}},\quad \vec{\beta}=\frac{\vec{v}}{c},\quad \vec{p}=\gamma\cdot m_0c\vec{\beta},\\ &\beta^2=\beta_x^2+\beta_y^2+\beta_z^2,\quad \beta_{x,y,z}=\frac{\vec{v}_{x,y,z}}{c}\end{aligned}\tag{2}$$

The solving procedure of these equations are presented in [53]. As a result we obtained rather simple expressions for trajectory coordinates in the form of

$$\begin{aligned}x&=\eta\sin\phi+(\eta-\xi)\sin\chi\phi,\\ y&=\eta\cos\phi-(\eta-\xi)\cos\chi\phi-\xi,\end{aligned}\tag{3}$$

$$z=\frac{c}{2\omega\gamma_0^2(1-\beta_{0z})^2}\left\{\begin{aligned}&\left[1+2\vartheta^2-\gamma_0^2(1-\beta_{0z})^2\right]\phi-\\ &-\frac{2\vartheta^2}{1+\chi}\sin(1+\chi)\phi\end{aligned}\right\}\tag{4}$$

where η,χ,ξ,ϑ are constant parameters defined as

$$\eta=\frac{\chi\xi}{1+\chi},\ \ \chi=\frac{a\alpha}{\gamma_0\left(1-\beta_{0z}\right)},$$
$$\xi=\frac{c}{a\omega},\ \ \vartheta=\frac{\alpha}{1+\chi} \tag{5}$$

and the phase of the field ϕ stands for combination ωt - kz and has a form

$$\phi=\omega\left[t-\frac{z(t)}{c}\right], \tag{6}$$

The components of particle velocity have a form

$$\begin{aligned} v_x&=\frac{c\vartheta}{\gamma}(\cos\phi-\cos\chi\phi),\\ v_y&=-\frac{c\vartheta}{\gamma}(\sin\phi+\sin\chi\phi),\\ v_z&=c\frac{1+2\vartheta^2[1-\cos(1+\chi)\phi]}{2\gamma_0\gamma(1-\beta_{0z})}-\frac{1}{2\gamma}c\gamma_0(1-\beta_{0z}) \end{aligned} \tag{7}$$

The action of combined electric and magnetic fields in case of circular polarization is rather complicated to describe. The vector of the electric field E of the laser radiation rotates around the z axis. As a result of this an acceleration of the particle occurs. The particle starts moving along the direction of the circularly rotating E vector. From this moment on the particle will start to act the bending effect connected with the Lorentz force by the magnetic component B of the laser field. The component B will start to bend the particle trajectory in the co-axial direction (the z axis). This will create the v_z component of the particle's velocity and due to it the rotation of the particle around the z axis occurs. While the external static magnetic field B_z will push the circularly rotating particle rather in the radial direction. The particle will move along the helix winding around the z axis with rising the radius. The result of these combined actions is the helical trajectory with superimposed wiggles. The wiggles disappear at the resonance conditions.

5. The prove of resonance occurring at non-zero and zero initial conditions

We shall determine the extremes of the function $\gamma(\phi)$ for the case of the non-zero initial velocity $\beta_{0z}\neq 0$. The extremes we shall derive from the condition

$$\frac{d\gamma}{d\phi}=0$$

Accepting according to [52], that

$$\gamma=\frac{1+2\vartheta^2[1-\cos(1+\chi)\phi]}{2\gamma_0(1-\beta_{0z})}+\frac{1}{2}\gamma_0(1-\beta_{0z}) \tag{8}$$

From the above equations we get

$$\frac{d\gamma}{d\phi}=\frac{\vartheta^2(1+\chi)}{\gamma_0(1-\beta_{0z})}\sin(1+\chi)\phi=0$$

It appears from the last expression that extremes can be determined from the equation

$$\sin(1+\chi)\phi=0$$

and are falling in places for which

$$\phi_n=\pm\frac{n\pi}{1+\chi}$$

Taking into account the above and expression (8) it is possible to show that

$$\begin{aligned} \gamma_{\min}&=\frac{1}{2\gamma_0(1-\beta_{0z})}+\frac{1}{2}\gamma_0(1-\beta_{0z}),\ \ \textit{for even } n\ \ a)\\ \gamma_{\max}&=\frac{1+4\vartheta^2}{2\gamma_0(1-\beta_{0z})}+\frac{1}{2}\gamma_0(1-\beta_{0z})\ \ \textit{for odd } n\ \ b) \end{aligned} \tag{9}$$

Considering the fact, that in case of non-zero initial conditions the parameter

$$\vartheta=\frac{B_0}{\dfrac{B_z}{\gamma_0(1-\beta_{0z})}-B_c},\ \ \ B_c=\frac{m_0\omega}{q} \tag{10}$$

of equation (9b) can be presented in the form

$$\gamma_{\max}=\frac{2B_0^2}{\left[B_z-\gamma_0(1-\beta_{0z})B_c\right]^2}+\frac{1}{2\gamma_0(1-\beta_{0z})}+$$
$$+\frac{1}{2}\gamma_0(1-\beta_{0z})\ \ \ \textit{for odd } n$$

It follows, that when $B_z\to\gamma_0(1-\beta_{0z})B_c$, then $\gamma_{\max}\to\infty$. In case of zero initial conditions, i.e., when

$$\gamma_0(1-\beta_{0z})=1$$

the above expressions will obtain the form of the derived earlier equations i.e.:

$$\begin{aligned} \gamma_{\min}&=1\\ \gamma_{\max}&=1+2\vartheta^2 \end{aligned} \tag{11}$$

Described here analysis shows at non resonance conditions that the kinetic energy of the particle is fluctuating according to the phase ϕ and approaches repeatedly the equal maximum values corresponding to the normalized γ_{max} values. It also results from the last equation, that in case of an increase of the constant magnetic induction of the co-axial B_z magnetic field, also a value of the ϑ parameter as well as a value of the γ_{max} factor is growing, when $B_z \to B_c$, is heading for the infinity, $\gamma_{max} \to \infty$. Using the extremes of the function $\gamma(\phi)$, it is possible to show that the phase distances between subsequent extremes, as in the case of zero initial conditions can be determined from the following equation

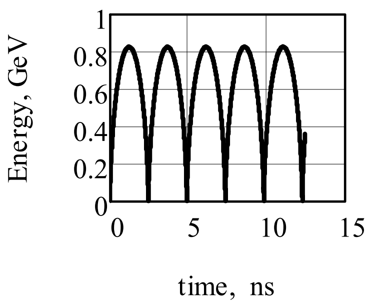
$$\Delta\phi = \phi_n - \phi_{n-1} = \frac{\pi}{1+\chi} = \frac{\pi}{1 - B_z / B_c}$$

From this equation it results, that when the B_z magnetic induction grows, also the distances between the subsequent phase extremes of the function $\gamma(\phi)$ grow. And if B_z approaches the resonance value, $B_z \to B_c$, these distances are becoming infinitely large, that is $\Delta\phi \to \infty$.

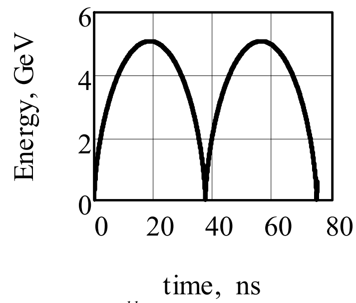
From (10) we can obtain the equation for resonance initial velocity:

$$\mathrm{v}_{0z}^{rez} = \frac{B_c^2 - B_z^2}{B_c^2 + B_z^2} c = \frac{1 - B_z^2 / B_c^2}{1 + B_z^2 / B_c^2} c \qquad (12)$$

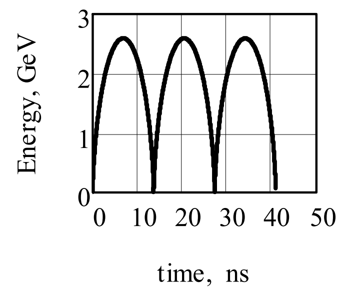
The examples are shown in the set of graphs for λ = 1μm and 10 μm. It is demonstrated in what a way the v_{oz} initial velocity of the electron influences the parameters of its motion, in particular, what these parameters should be, if the initial velocity is approaching the resonance velocity determined by the expressions (12)



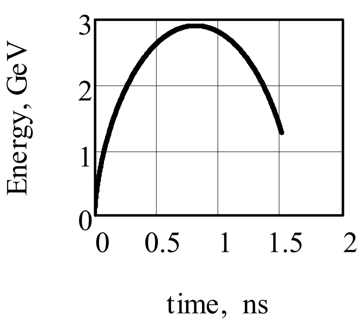
a) E = 10^11 V/m, λ = 10 μm, B_z = -357 T, v_0 = 0.796c ($\mathrm{v}_{0z}^{rez} = 0.8c$)

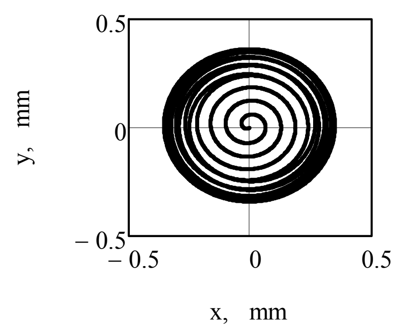


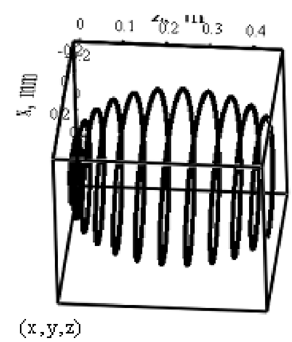
b) E = 10^11 V/m, λ = 10 μm, B_z -357 T, v_0 = 0.798c ($\mathrm{v}_{0z}^{rez} = 0.8c$)



c) E = 10^11 V/m, λ = 10 μm, -214 T, v_0 = 0.92c ($\mathrm{v}_{0z}^{rez} = 0.921c$)

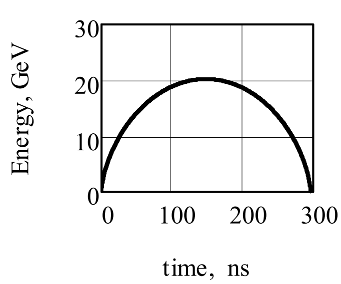




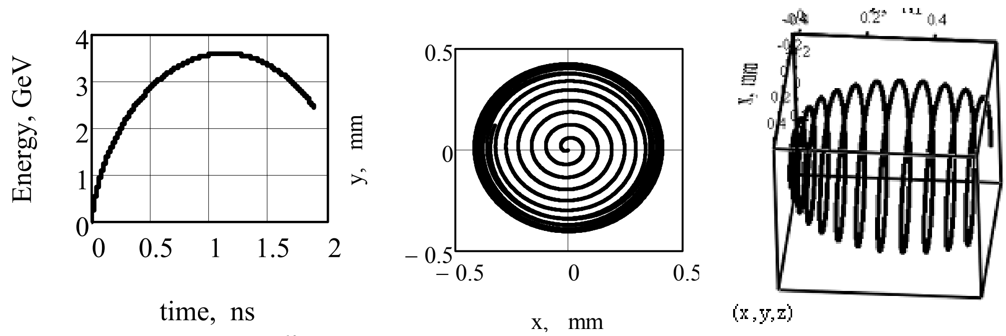


d) E = 10^12 V/m, λ = 10 μm, B_z = -500 T, v_0 = 0.62c ($\mathrm{v}_{0z}^{rez} = 0.642c$)

Fig.1. Illustration of the influence of the electron initial velocity. Velocity smaller than the resonance one v_{0z}^{rez}.



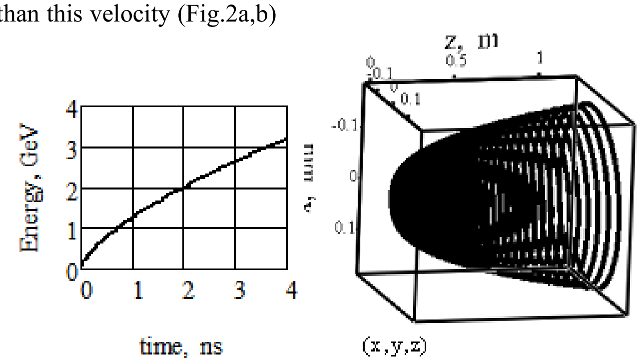
a) E = 10^11 V/m, λ = 10 μm, B_z = -357 T, v_{0z} = 0.85c ($\mathrm{v}_{0z}^{rez} = 0.8c$)



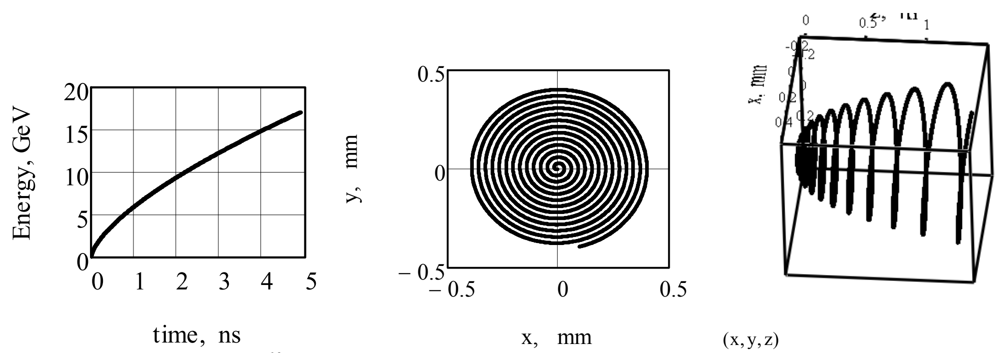
b) E = 10^12 V/m, λ = 10 μm, B_z = -500 T, $v_0 = 0.66c$ ($v_{0z}^{rez} = 0.642c$)

Fig.2. Illustration of the influence of the electron initial velocity. Velocity larger than the resonance one v_{0z}^{rez}.

From Figs.1 and the equation (12) it results that for the selected value of the B_z magnetic field strength and frequency of the laser light ω only one value of the initial velocity of the v_{0z} particle exists, at which the particle is subjected to the extreme acceleration. We will be calling this velocity the resonance velocity v_{0z}^{rez}. Both smaller initial velocity than the resonance initial velocity (Fig.1a,b,c,d) as well as larger than this velocity (Fig.2a,b) cause the degeneration of conditions of the acceleration. The velocity v_{0z}^{rez} may be obtained from equation (12). The graphs were carried out for the B_z value selected as an example to be -357 *T*, the *E* field strength is 10^{11} *V/m* and the wavelength of the laser radiation is 10 *μm*. In case of presented illustrations for the chosen B_z, the resonance initial velocity of the particle $v_{0z}^{rez} = 0.8\,c$ (Fig.1a,b).



a) E = 10^11 V/m, λ = 10 μm, B_c = B_z = -1071 T, $v_0 = 0$, ($v_{0z}^{rez} = 0c$)

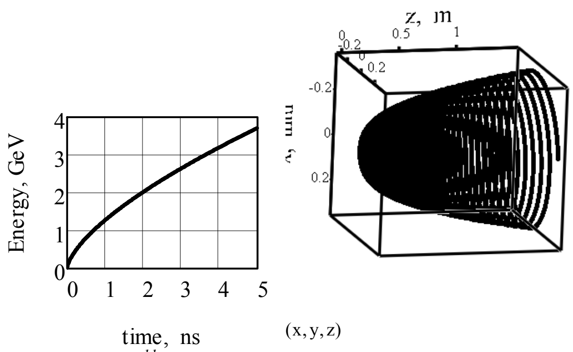


b) E = 10^12 V/m, λ = 10 μm, B_c = B_z = -1071 T, $v_0 = 0$ ($v_{0z}^{rez} = 0c$)

Fig.3. Illustration of the influence of the E on the resonance acceleration of electron.

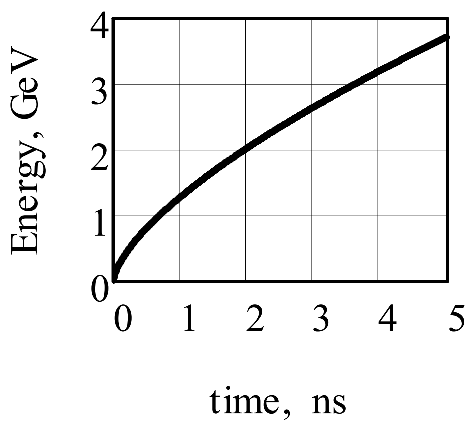
Fig.1a shows the acceleration process at resonance condition with initial velocity $v_{0z}^{rez} = 0.8c$ Fig.1a and b present the acceleration when the initial velocities are lower $v_{0z} = 0.796c$, $v_{0z} = 0.799c$ and Fig.2a when initial velocity is higher $v_{0z} = 0.85c$. It can be seen a strong growth of the maximum kinetic energy of the electron as well as a duration of the period of the oscillation of the function $E_k(t)$.

The graphs show that smaller initial velocity of the particle than the velocity 0.8c as well as grater than this velocity do not give the particle the guarantee of receiving possible largest the kinetic energy. The effect of action of both cases is similar. Only if the initial velocity falls in vicinity of the resonance velocity, the period of the function $E_k(t)$ grows significantly and the kinetic energy achieves the extreme values.



a) E =10^11 V/m, λ = 10 μm, B_z = -600 T, v_0 *= 0.522c*

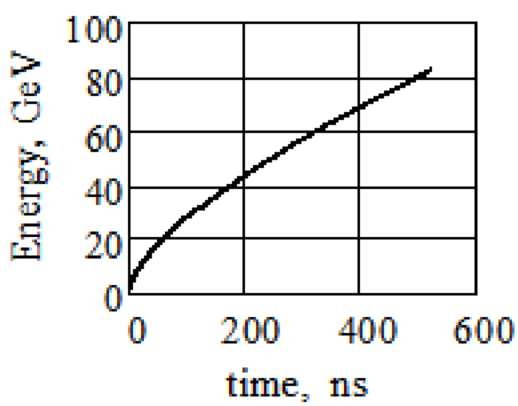
$(v_{0z}^{rez} = 0.522c)$



b) E =10^11 V/m, λ = 10 μm, B_z = -700 T, v_0 *= 0.401c*

$(v_{0z}^{rez} = 0.401c)$

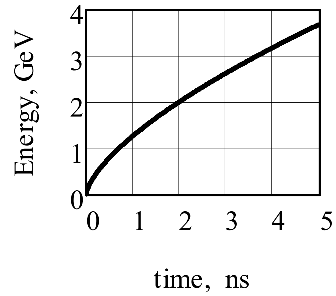
Fig.4. Illustration of the influence of the B_z on resonance acceleration of electron.



E =10^11 *V/m, λ =* 10 *μm, B_z* = -357 *T,* v_{0z} = 0.8*c*

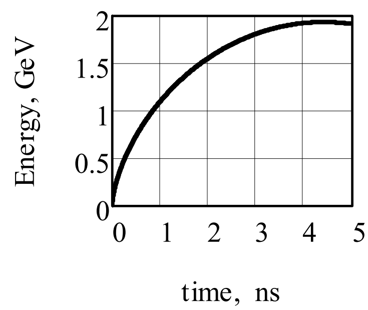
$(v_{0z}^{rez} = 0.8c)$

Fig.5. Illustration of the influence of the time at resonance acceleration on kinetic energy



a) E =10^11 V/m, λ = 1 μm, B_z = -5000T, v_0 *= 0.642c*

$(v_{0z}^{rez} = 0.642c)$



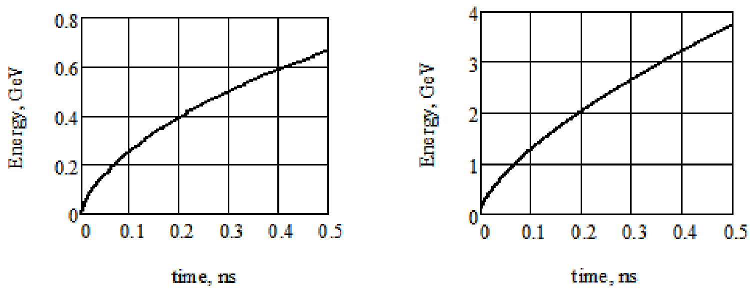
b) E =10^11 V/m, λ = 1 μm, B_z = -2000T, v_0 *= 0.933c*

$(v_{0z}^{rez} = 0.933c)$

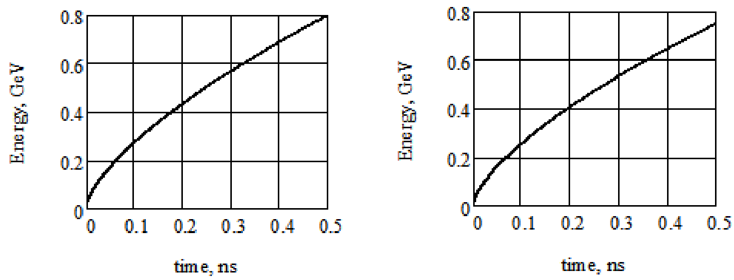
Fig.6. Illustration of the influence of the B_z on resonance acceleration of electron for λ = 1 μm

From the figures it follows that the resonance acceleration of the particle occurs for different initial velocities, corresponding to the appropriate resonance values of the magnetic field strength B_z e.g. for v_{0z} = 0.522c B_z = -600T (Fig.4a) and for v_{0z} = 0.401c B_z = -700T (Fig.4b). This effect appears also at the other wave lengths of the laser radiation e.g. in the case when λ = 1 µm. In Fig.6a (v_{0z} = 0.642c B_z = -5000 T) and Fig.6b (v_{0z} = 0.933c B_z = -2000 T) and when λ = 10 µm and E_0 = 10^{12} V/m, B_z = -500T v_{0z}^{rez} = 0.642c (Fig.1d), v_{0z} = 0.66c, v_{0z}^{rez} = 0.642c (Fig.2b) and v_{0z} = 0.62c, v_{0z}^{rez} = 0.642c (Fig.1d). The presented results are confirming the possibility of considerable reducing the resonance magnetic field B_z by introducing the non-zero initial velocity of the particle of the magnitude determined by the expression (12). In several cases the courses of motion of the particle were illustrated by the examples of trajectories in the three-dimensional space. Figs.2b and 3a, show the resonance acceleration of the electron carried out at v_{0z} = 0 and at the resonance of B_z = -1071 T, when *E* = 10^{11} V/m, λ = 10 µm, (Fig.3a), λ = 10 *µm*, B_z = -1071 T and *E* = 10^{12} V/m (Fig.3b). Which indicates that the resonance magnitude of B_z does not depend of *E*.

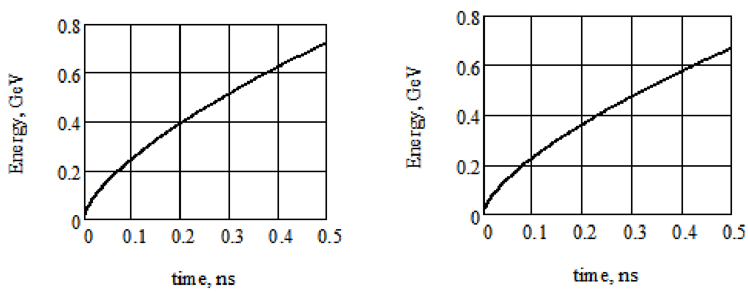
Fig. 7 were obtained for two electric field intensities determined by *E* = 10^{11} V/m and 10^{12} V/m, for the wavelength of the laser radiation 10µm and different for every graph the resonance B_z values and calculated from the equation (12) appropriate co-axial component v_{0z} of the initial resonance velocity.



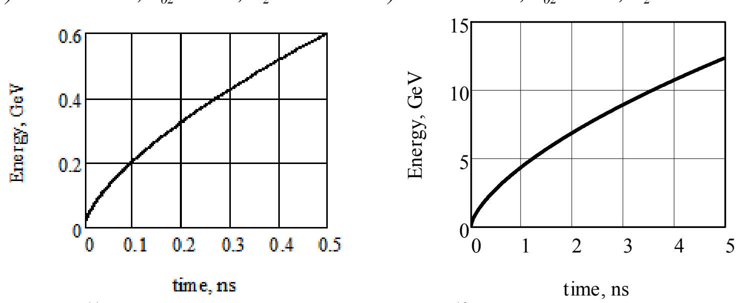
a) $E = 10^{11}$ V/m, $v_{0z} = 0$, $B_z = -1071$ T h) $E = 10^{12}$ V/m, $v_{0z} = 0$, $B_z = -1071$ T



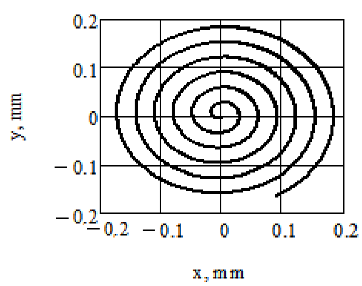
b) $E = 10^{11}$ V/m, $v_{0z} = 0.02c$, $B_z = -1050$T b) $E = 10^{11}$ V/m, $v_{0z} = 0.1c$, $B_z = -968$T



c) $E = 10^{11}$ V/m, $v_{0z} = 0.3c$, $B_z = -786$ T d) $E = 10^{11}$ V/m, $v_{0z} = 0.5c$, $B_z = -618$ T



d) $E = 10^{11}$ V/m, $v_{0z} = 0.7c$, $B_z = -450$ T, f) $E = 10^{12}$ V/m, $v_{0z} = 0.7c$, $B_z = -450$ T,



e) $E = 10^{12}$ V/m, $v_{0z} = 0$, $B_z = -1071$ T

Fig.7. The courses of the kinetic energy of an electron vs time. $E = 10^{11}$ V/m and 10^{12} V/m, wavelength of the laser radiation is 10 μm and the resonance values of the constant co-axial magnetic field strength and the appropriate values of longitudinal, initial resonance velocities of the electron are different for each of the graphs.

As it is to be expected, all graphs of the kinetic energy variation in time are characterized by the shape corresponding to the particle acceleration under conditions of the resonance. From the presented figures an important conclusion follows that applying the non-zero, the initial velocity v_{0z} not considerably differing from the resonance one, results changing the course of the kinetic energy $E_k(t)$ to a little extent. However such a way is giving the significant practical benefit relying that applying the non-zero value of the initial velocity it is possible to fulfill the conditions of the resonance by reducing B_z magnetic field strength and it is not important the achieving the exact value of the resonant B_z. It is enough to apply the B_z falling near its resonance value. In order to maintain the acceleration condition, the greater the initial velocity of the particle, the smaller B_z could be applied. In each of shown cases, in spite of different values of the sets of pairs (B_z, v_{0z}) the acceleration results are being achieved the same.

6. Dependence of v_{oz} of an electron and laser wavelength on the resonance magnetic field strength

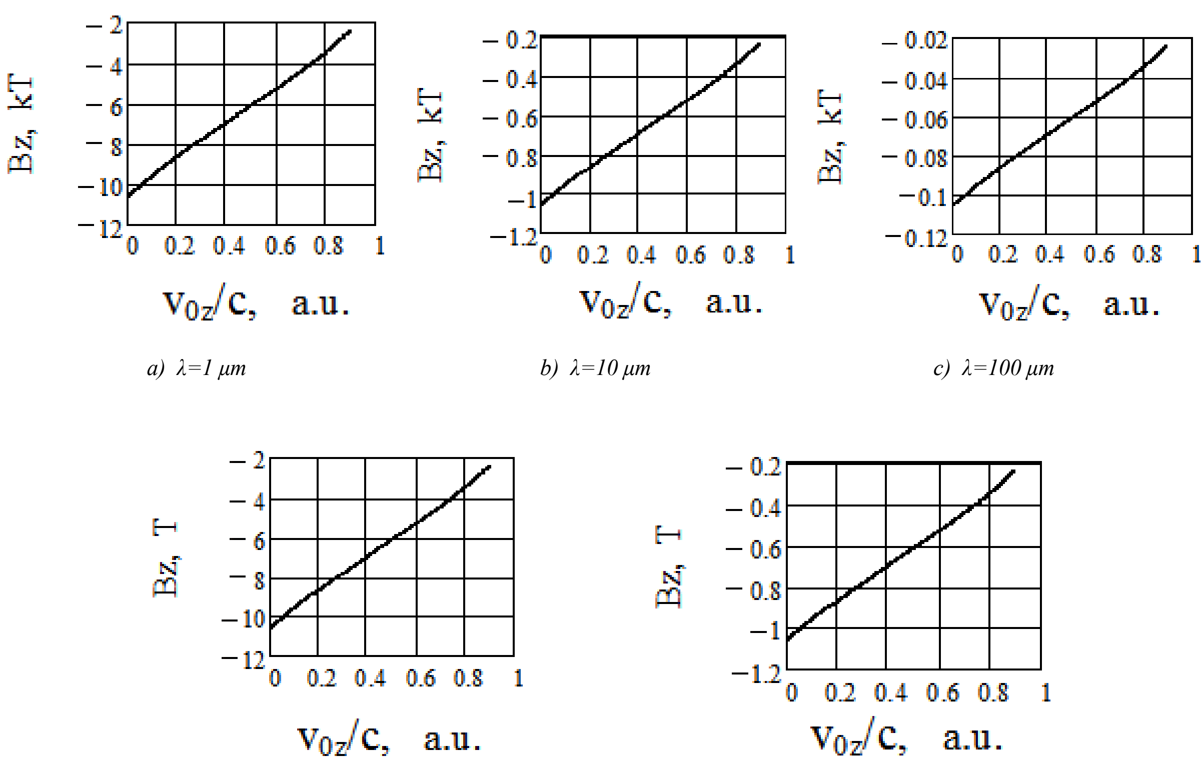
Using the resonance conditions (10)

$$\frac{B_z}{\gamma_0\left(1-\beta_{0z}\right)} \to B_c$$

can be easily find that

$$B_z = B_c\sqrt{\frac{1-\beta_{0z}}{1+\beta_{0z}}}$$

On the basis of the equation the following graphs were obtained:

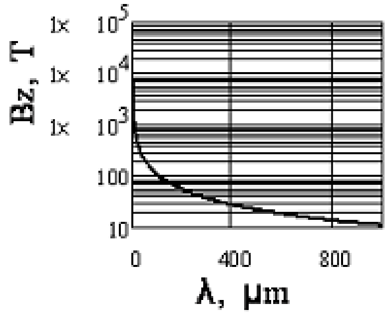


a) λ=1 μm *b) λ=10 μm* *c) λ=100 μm*

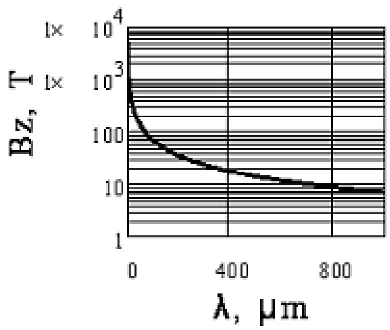
d) λ=1 mm *e) λ=1 cm*

Fig.8. Variation of the resonance B_z magnetic field strength in dependence on the particle initial velocity, for wavelengths of the laser or maser radiation 1 μm (a), 10 μm (b), 100 μm (c), 1 mm (d), 1 cm (e).

Having the B_z values for a selected wavelength it is possible easily to receive B_z for another wavelength through multiplying the previous B_z by the ratio of the wavelengths. It results from the equation (10) on B_c.



a)



b)

Fig.9 Variation of the B_z vs λ near the resonance, in dependence on different particle initial velocity v_{0z} *,* v_{0z} *= 0 (a),* v_{0z} *= 0.4c (b).*

Fig.8 allows to find the resonance value of v_{0z} for the defined intensity of the magnetic field and for the selected wavelengths of the laser or maser beams. And inversely, taking the value of the initial velocity v_{0z} and using Fig.9 it is possible to find the resonance value of the field B_z for the different wavelengths of the laser or maser radiation.

7. Summary

As a result of laser acceleration there exist some ways to increase the kinetic energy of the particle. We can mention the increase of the laser intensity, the increase of the acceleration duration, the increase of the radiation wavelength and the creation of conditions leading to the resonance. It is possible to obtain the resonance conditions by choosing the particle moving at the resonance value of the initial velocity with an appropriate adjusted the constant magnetic field B_z and with other methods. In the case considered in this paper the particle with non-zero initial velocity can fall in the resonance condition with reduced the constant resonance magnetic field strength. This conclusion was illustrated on the example of the electron in Figs.(1-6) for radiation wavelengths 1 μm, 10 μm, 100 μm, 1 mm and 1 cm in Figs.(7,8). The resonance values of the co-axial static magnetic field strength at the variation of the initial velocity depend on the wavelength of the radiation and rises with reduction of the radiation wavelength, Fig.9. From the presented figures it follows for the defined co-axial magnetic field intensity, the particle should be pre-accelerated to the defined velocity in order to achieve the resonance conditions. In other words, in order to create the possibility of the magnetic field B_z reduction, we suggest the increase of the initial velocity of the particle. Following the results shown above, we suggest the enhanced efficiency of the laser acceleration. The magnitude of the resonant B_z was found to depend on wavelength of the laser radiation and on the initial energy of the accelerated particle, while is independent of laser radiation intensity. We think that this fact significantly helps the experimental arrangements.

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