

# Effects of varying strong nonlocality on the propagation and interaction of bright solitons in nonlinear media

HONG LI<sup>a,b</sup>

<sup>a</sup>*School of Mechanical and Electronic Engineering, Huangshi Institute of Technology, Huangshi, Hubei, China, 435003*

<sup>b</sup>*School of Physics and Electronic Science, Hubei Normal University, Huangshi, Hubei, China, 435002*

Dynamics of bright solitons are investigated by the numerical simulation in nonlinear media with varying nonlocality, and effects of the varying randomly and periodically nonlocalities on the soliton propagation and interaction are analyzed. The strong nonlocality is expanded as the fundamental nonlocality and the second-order nonlocality, and effects of the fundamental nonlocality is different from those of the second-order nonlocality. The constant nonlocality stabilizes propagation of the bright soliton if the nonlocality is strong enough, and suppresses the interaction. But the varying nonlocality leads to disintegration of the bright soliton and enhances the interaction. The effects of the varying randomly nonlocality strictly depend on the stochastic strength, and the effects of the varying periodically nonlocality become completely reliant on the period length.

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## 1. Introduction

Optical solitons in fibers are formed by a balance of group dispersion and Kerr non-linearity, and the spatial optical soliton is a beam which propagates in a nonlinear medium without changing its structure. If this balance can be maintained dynamically the soliton exists as a robust object withstanding even strong perturbations. Numerical simulation and experiments have demonstrated solitons can propagate an extended distance without distortion, so they maybe become the ideal message carrier in long distance communication [1, 2].

Solitons have been typically considered in the context of so-called local nonlinear media, where nonlocality of the nonlinearity is a property exhibited by many nonlinear optical materials. In such media the refractive index change induced by an optical beam in a particular point depends solely on the beam intensity in this very point. Nonlocality is thus a feature of a large number of nonlinear systems leading to novel phenomena of a generic nature. For instance, it may promote modulational instability in self-defocusing media, and suppress wave collapse of multidimensional beams in self-focusing media [3~5]. Stable dark or bright solitons were observed only recently in nonlocal nonlinear media, and relevant recent works cause a renewed interest in this area including the interaction between two neighboring bright solitons. Several reviews on the development of the nonlocal spatial solitons have been presented, and their rich phenomena and potential applications in communication and signal processing were investigated. It is shown theoretically that stable spatial bright soliton states can exist in self-focused nonlocal media and Gauss-function-like bright soliton states can exist in self-focused strongly nonlocal media. Experiments have

also revealed fundamental and vortex-ring solitons supported by the strong nonlocality, as well as soliton steering, and theoretical analyses predict the stabilization of other self-trapped modes [6~9].

Stabilization of localized waves is greatly enhanced in nonlocal nonlinear media, and the nonlocality often results from certain transport processes. It has been shown that nonlocality may affect modulational instability of the soliton pulse. Recent numerical and analytical theoretical studies demonstrated both stable and unstable evolution of the bright soliton in the nonlocal nonlinear media, and show that the nonlocality plays a crucial role in the physical features of the bright solitons, such as the modulation instability. The nonlocality governs the diffusion strength of the refractive index in the nonlocal nonlinear material, and the physical features exhibited by spatial optical soliton propagation can be addressed in the nonlocal media by Gaussian-shaped response and exponential-decay response.

The practical nonlocal media can be considered as a medium with the randomly varying nonlocality that is a nonlocal function of the incident field, which is stochastic function which fluctuate around its mean value. This randomly varying nonlocality induced by a wave with the intensity can be presented in path stochastic strength form. Interestingly, the response function can also be periodic for parametric interaction, and the nonlocality satisfies varying periodically one along the transmission line. The varying nonlocality plays a crucial role in the physical features of the bright soliton. In this paper, the effects of the varying nonlocality are investigated on the soliton propagation and interaction in nonlocal nonlinear media, and some novel results are obtained.

## 2. Theoretical formula

Media are considered with nonlinearity that is a nonlocal function of the incident field, the wave-packet propagates along the  $z$  axis within the nonlocal nonlinear media, and the envelop of the wave-packet can be described by the nonlinear Schrödinger equation [9]

$$j \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + \eta \left[ \int R(x-x') I(x') dx + \int L(x-x') I^2(x') dx \right] u = 0, \quad (1)$$

where  $u$  is normalized wave-packet function.  $z$  and  $x$  are distance coordinate and transverse coordinate.  $\eta$  is a material constant,  $\eta = 1$  corresponds to the self-focusing media and  $\eta = -1$  corresponds to the defocusing media.  $I(z, x) = |u(z, x)|^2$  is the pulse intensity. The field-intensity dependent change of the refractive index is characterized by two normalized symmetric response functions  $R(x)$  and  $L(x)$

$$\left( \int_{-\infty}^{\infty} R(x) dx = \int_{-\infty}^{\infty} L(x) dx = 1 \right), \quad \text{and}$$

$R(x) = L(x) = \delta(x)$  in a local Kerr medium.

The actual form of the nonlocal response is determined by the details of the physical process responsible for the general nonlocality. For all diffusion-type nonlinearities, orientational-type nonlinearities, and for the general quadratic nonlinearity describing parametric interaction, the response function may be Gaussian-shaped response or exponential-decay response, such as  $f = (2\sigma)^{-1} \exp(-|x|/\sigma)$  originating from a Lorentzian in the Fourier domain. For instance, the nonlinear contribution to refractive index  $n(x)$  can be given by

$$n(x) = \int R(x-x') I(x') dx + \int L(x-x') I^2(x') dx, \\ n(x) - \rho \frac{\partial^2}{\partial x^2} n(x) = |u|^2, \quad (2)$$

where  $\rho$  is the degree of the general nonlocality, which governs the diffusion strength of the refractive index in the nonlocal nonlinear material, and the corresponding nonlocality may determine the physical features of the bright solitons in the nonlocal nonlinear media

The strong nonlocality satisfies the well-known general power-law dependence on the incident intensity for local models with competing nonlinearities, the response functions can consider the expression as the second-order expansion of the model, and the strong

nonlocality can be calculated through following expansion of the refractive index [6]

$$n(x) \approx |u|^2 + \rho_1 \frac{\partial^2 |u|^2}{\partial x^2} + \rho_2 \frac{\partial^4 |u|^2}{\partial x^4}, \quad (3)$$

where  $\rho_1 = \frac{1}{2} \int x^2 R(x) dx (> 0)$  is the degree of the fundamental nonlocality, and  $\rho_2 = \frac{1}{24} \int x^4 L(x) dx (> 0)$  is the degree of the second-order nonlocality. The term  $\rho_2$  is the fourth-order quantification which is much smaller than the term  $\rho_1$  in the general nonlocal case, but the term  $\rho_2$  may approach or be larger than the term  $\rho_1$  in the strong nonlocality, and effects of term  $\rho_2$  on the soliton is not negligible compared with the latter one [6].

The self-focusing media ( $\eta = 1$ ) are considered where stable bright solitons are observed recently, and substituting Eq. (3) into Eq.(1) leads to the following evolution equation

$$j \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + [|u|^2 + \rho_1 \frac{\partial^2 |u|^2}{\partial x^2} + \rho_2 \frac{\partial^4 |u|^2}{\partial x^4}] u = 0. \quad (4)$$

The physical features exhibited by spatial optical soliton propagation can be addressed in the nonlocal media, where the nonlocality satisfies the well-known general power-law dependence on the incident intensity for local models with competing nonlinearities, and is induced by a wave with the varying intensity. The more realistic approach implies random or periodical behavior of the characteristic parameters in media, and they are considered to fluctuate randomly or periodically around their mean values in the focusing media. Two normalized symmetric response functions  $R(z, x)$  and  $L(z, x)$  fluctuate randomly or periodically around their mean values along the transmission line. Although Eq.(4) has a Hamiltonian structure, it is not exactly integrable because of the inhomogeneity (nonlocality,  $z, x$  dependent coefficient). Such a behave of the normalized wave-packet function  $u$  may be obtained as its response averaged over the inhomogeneity (nonlocality), which result in path-averaged soliton. However, taking a simple average of the inhomogeneity (nonlocality) fails to provide the proper response because of the correlations with variations of  $u$  and the inhomogeneity. So the effects of the inhomogeneity (nonlocality) are investigated by the direct numerical simulation to give proper response. In our calculation models, two sorts of the varying nonlocalities are considered below in the focusing media.

Nonlocality (a): a medium is considered with the randomly varying strong nonlocality that is a nonlocal function of the incident field, which is stochastic function fluctuating around its mean value. This randomly varying nonlocality induced by a wave with the pulse intensity

$I(z, x) = |u(z, x)|^2$  can be presented in general form:

$$\begin{aligned} \rho_1 &= A_0(1 + \sigma), & \langle \sigma(z) \rangle &= 0, & \langle \sigma(z)\sigma(z') \rangle &= D\delta(z - z'), \\ \rho_2 &= B_0(1 + \varepsilon), & \langle \varepsilon(z) \rangle &= 0, & \langle \varepsilon(z)\varepsilon(z') \rangle &= D\delta(z - z'), \end{aligned} \quad (5)$$

where  $A_0 (> 0)$  and  $B_0 (> 0)$  are the degrees of the path-average constant fundamental nonlocality and the second-order nonlocality.  $\langle \rangle$  denotes the path-average, and  $D (0 \leq D \leq 1)$  is stochastic strength (the standard deviation) of the varying randomly fundamental nonlocality and the second-order nonlocality.

Nonlocality (b): The response functions can also be periodic one along the transmission line for parametric interaction,  $R(z, x), L(z, x) \propto \sin(|z|/z_0)$  in certain regimes of the parameter space and satisfies [10]:

$$\begin{aligned} \rho_1 &= A_0(1 + \sigma), & \sigma(z) &= \sin(z/z_0), \\ \rho_2 &= B_0(1 + \varepsilon), & \varepsilon(z) &= \sin(z/z_0), \end{aligned} \quad (6)$$

where  $z_0$  is the normalized period length of the periodically varying nonlocality, and the effective standard deviation of the varying nonlocality is  $1/2$ .

### 3. Analysis of the numerical results

From the evolution equation (4), we can see the motion equation for the bright soliton is the standard nonlinear Schrödinger equation when there is no the nonlocality. Namely, the special case coincides with the optical bright soliton under the framework of Eq.(4) without the perturbation (the expanded nonlocality). The result means that the soliton

propagation and interaction are modulated by the expanded nonlocality.

We can perform a series of direct numerical simulations for the nonlinear Schrödinger equation (4) to discuss the effects of the expanded strong nonlocality on the dynamics of the bright solitons in the nonlocal nonlinear media. The incident one-bright soliton pulse is  $u(x, z=0) = \text{sech}(x)$ , or the initially input three-bright soliton pulses are  $u(x, z=0) = \text{sech}(x + \Delta) + \text{sech}(x) + \text{sech}(x - \Delta)$ , where  $\Delta$  is the separation between two neighboring solitons, and  $\Delta = 10$  (about 6 times of the initial soliton-width) is used in the below simulation. The special relativities are considered as  $A_0 \gg B_0$ ,  $A_0 > B_0$  and  $A_0 = B_0$  in the self-focusing media.

Fig. 1 is the normalized soliton intensity versus the propagation distance under the constant nonlocality. We can see that the bright soliton can propagate stably a very long distance when there is no the nonlocality ( $A_0 = 0, B_0 = 0$ ). The expanded nonlocality plays an important role in the evolution of the soliton in the self-focusing media. For example, the effects of the fundamental nonlocality on the propagation of the bright soliton is obvious under the weak nonlocality ( $A_0 = 2, B_0 = 0$ ), and the soliton can not propagate stably a long distance. The soliton can not be allowed to propagate under the large fundamental nonlocality ( $A_0 = 4, B_0 = 0$ ). The effects become different under the raised nonlocality ( $A_0 = 4, B_0 = 2$ ), and the strong nonlocality stabilizes the propagation of the bright soliton if the second-order nonlocality is strong enough. The effects of the second-order nonlocality may distinguish from those of the fundamental nonlocality, and the second-order nonlocality can effectively stabilize the propagation of the bright soliton ( $A_0 = 4, B_0 = 4$ ). So the effects of the second-order nonlocality on the solitons are not negligible in the media.

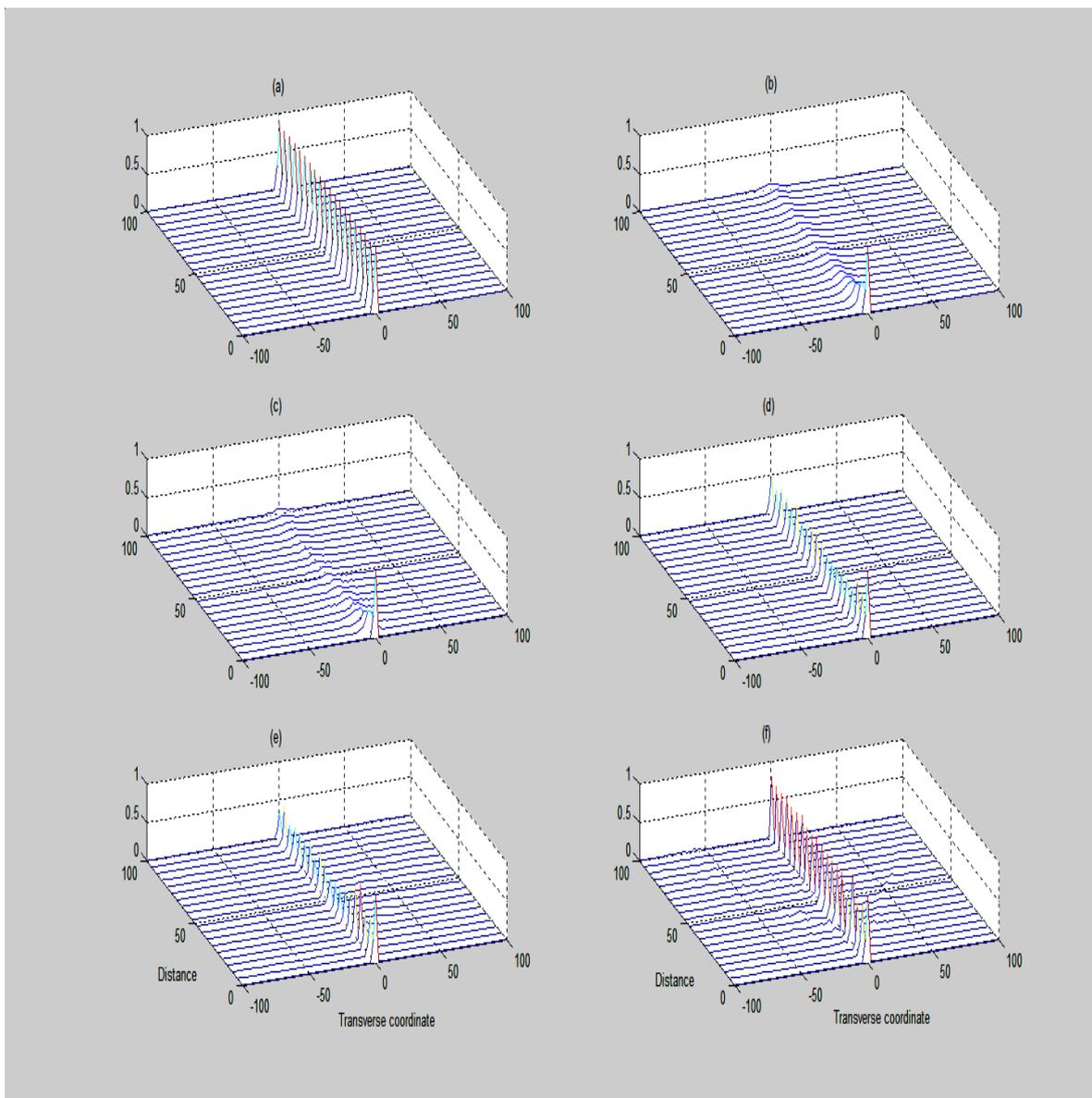


Fig. 1. The normalized one-bright soliton intensity versus the propagation distance under the constant nonlocality. (a)  $A_0 = 0$ ,  $B_0 = 0$ ; (b)  $A_0 = 2$ ,  $B_0 = 0$ ; (c)  $A_0 = 4$ ,  $B_0 = 0$ ; (d)  $A_0 = 2$ ,  $B_0 = 2$ ; (e)  $A_0 = 4$ ,  $B_0 = 2$ ; (f)  $A_0 = B_0 = 4$ .

Fig. 2 demonstrates the normalized intensity of three-bright solitons versus the propagation distance under the constant nonlocality. It is well known that when initial separation between neighboring solitons is larger than five times of the soliton-width in the general soliton system without the nonlocality, the interaction between neighboring solitons can be suppressed effectively [1, 11]. We can see the three solitons can propagate stably a very long distance when there is no the nonlocality. But the three solitons can not propagate stably a long distance even if the separation is larger than five times of the initial soliton-width under the weak nonlocality ( $A_0 = 2$ ,  $B_0 = 0$ ). The three solitons can not allowed to propagate under the large fundamental nonlocality ( $A_0 = 4$ ,  $B_0 = 0$ ). The combined role of

the fundamental nonlocality and the second-order nonlocality plays an important role in the soliton interaction, and the strong nonlocality changes the interaction distance where the interaction distance is defined as the distance where the timing shifts of the neighboring solitons exceed a half of their soliton-width. The interaction distance strictly depends on the fundamental nonlocality, the second-order nonlocality and the separation between two neighboring solitons. For instance, the three solitons can propagate stably a very long distance, and interaction is suppressed under the expanded strong nonlocality ( $A_0 = 4$ ,  $B_0 = 4$ ) and the large separation ( $\Delta = 10$ ).

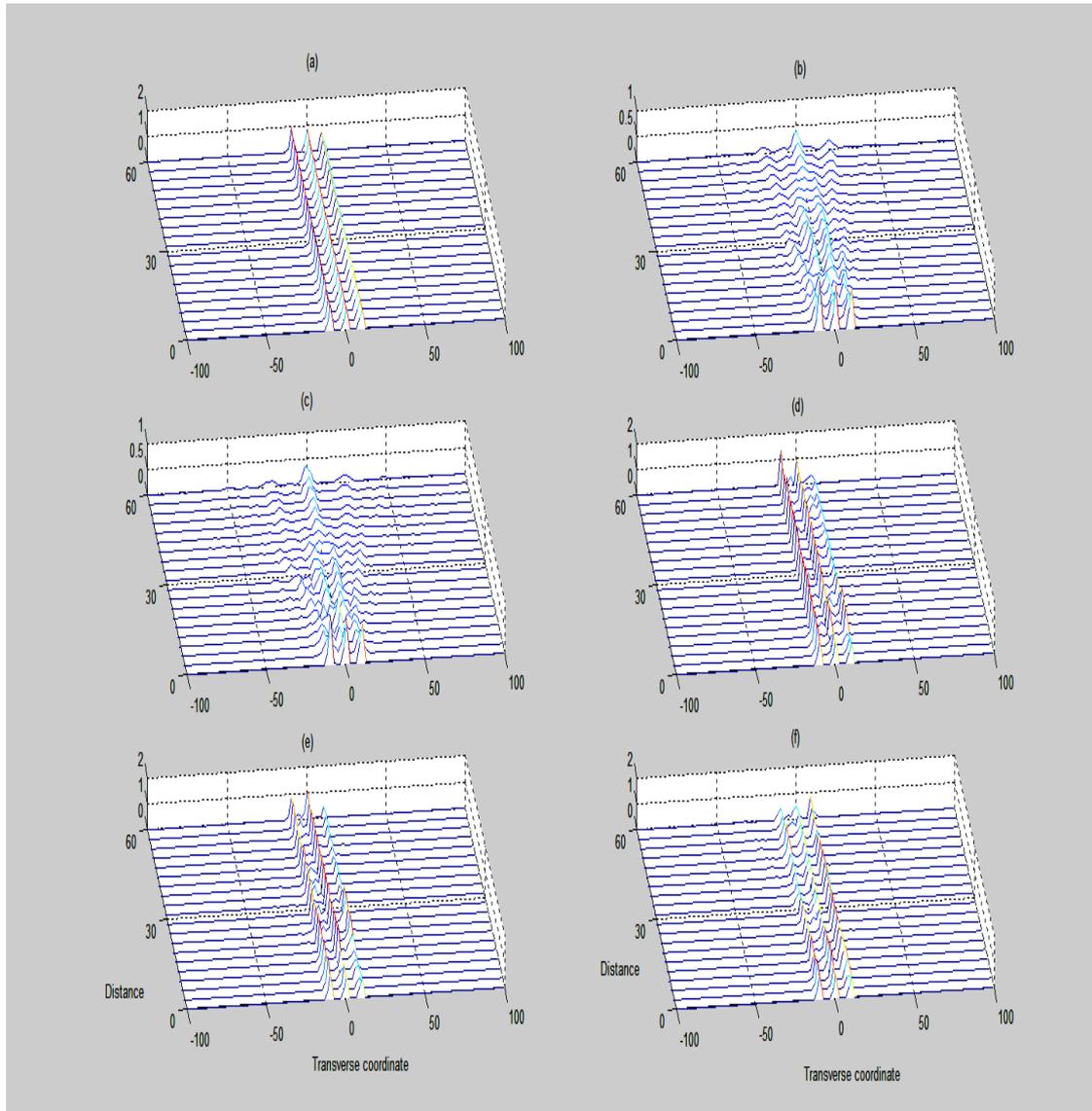


Fig. 2. The normalized three-bright soliton intensity versus the propagation distance under the constant nonlocality. (a)  $A_0 = 0$ ,  $B_0 = 0$ ; (b)  $A_0 = 2$ ,  $B_0 = 0$ ; (c)  $A_0 = 4$ ,  $B_0 = 0$ ; (d)  $A_0 = 2$ ,  $B_0 = 2$ ; (e)  $A_0 = 4$ ,  $B_0 = 2$ ; (f)  $A_0 = B_0 = 4$ .

Fig. 3 is the normalized soliton intensity versus the propagation distance under the varying nonlocality (a) with different stochastic strength, and Fig. 4 is the disintegration distance versus the stochastic strength. The random modulation with Gaussian random deviation is used in magnitude of the stochastic dispersion. Uniformly distributed random numbers from the interval  $\theta$  ( $\theta \in (-1, 1)$ ) were used, and random numbers are produced by the built-in generator `Random(x)` from the Matlab. Transformation of the uniformly distributed random numbers from the interval into Gaussian random numbers with properties in Eqs. (5) is performed using

$$\sigma = \sqrt{2D} \times \sin(2\pi\theta) \times \sqrt{-2 \ln \theta} \quad \text{and}$$

$$\varepsilon = \sqrt{2D} \times \sin(2\pi\theta) \times \sqrt{-2 \ln \theta} \quad [12].$$

As usual, an

average of a several of different sequences for random numbers has been used. We can see that the nonlocality plays an important role in the evolution of the bright soliton in the self-focusing media. For example, the constant nonlocality stabilizes propagation of the bright soliton if the nonlocality is strong enough. But the randomly varying nonlocality leads to disintegration of the bright soliton if the stochastic strength is large enough. The disintegration distance, which is defined as the propagation distance until soliton disintegration, strictly depends on the stochastic strength of the varying nonlocality. The varying randomly nonlocality amplifies the fluctuation of the soliton amplitude, and leads to disintegration of the bright soliton because the amplified fluctuation of the amplitude causes fluctuation of the line wave with small amplitude.

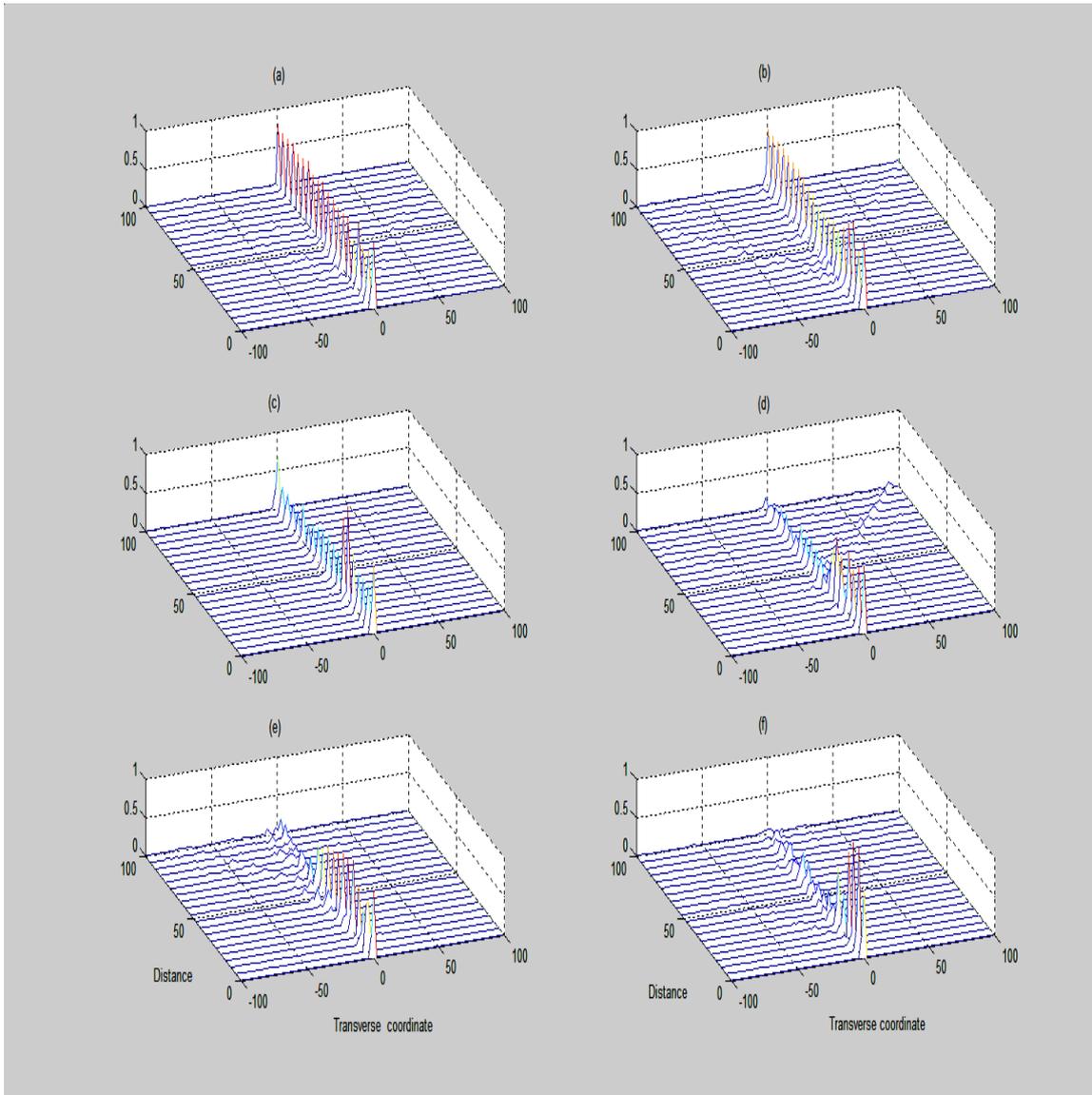


Fig. 3. The normalized one-bright soliton intensity versus the propagation distance under the nonlocality (a) with  $A_0 = B_0 = 4$ . (a) constant nonlocality; (b)  $D = 0.1$ ; (c)  $D = 0.2$ ; (d)  $D = 0.3$ ; (e)  $D = 0.4$ ; (f)  $D = 0.5$ .

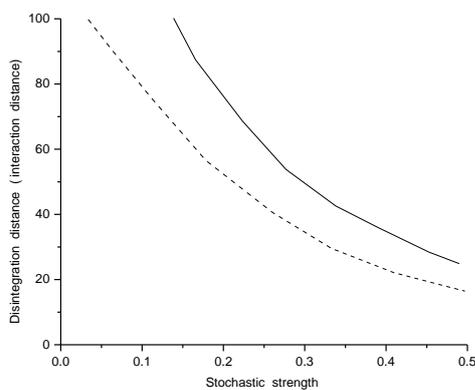


Fig. 4. The disintegration distance (interaction distance) versus the stochastic strength of the nonlocality (a) with  $A_0 = B_0 = 4$ . Solid line : disintegration distance; dashed line: interaction distance.

Fig. 5 demonstrates the normalized intensity of three bright solitons versus the propagation distance with different stochastic strength, and the interaction distance versus the stochastic strength is shown in Fig. 4. We can see the three solitons can not propagate stably a long distance even if the separation is larger than five times of the initial soliton-width (such as  $\Delta = 10$ ) and the nonlocality is strong ( $A_0 = 4, B_0 = 4$ ) under large stochastic strength. The varying nonlocality enhances the interaction, and reduces the interaction distance, which strictly depends on stochastic strength. For instance, the three solitons can propagate stably under the strong nonlocality ( $A_0 = 4, B_0 = 4$ ) in presence of the large stochastic strength ( $D = 0.5$ ).

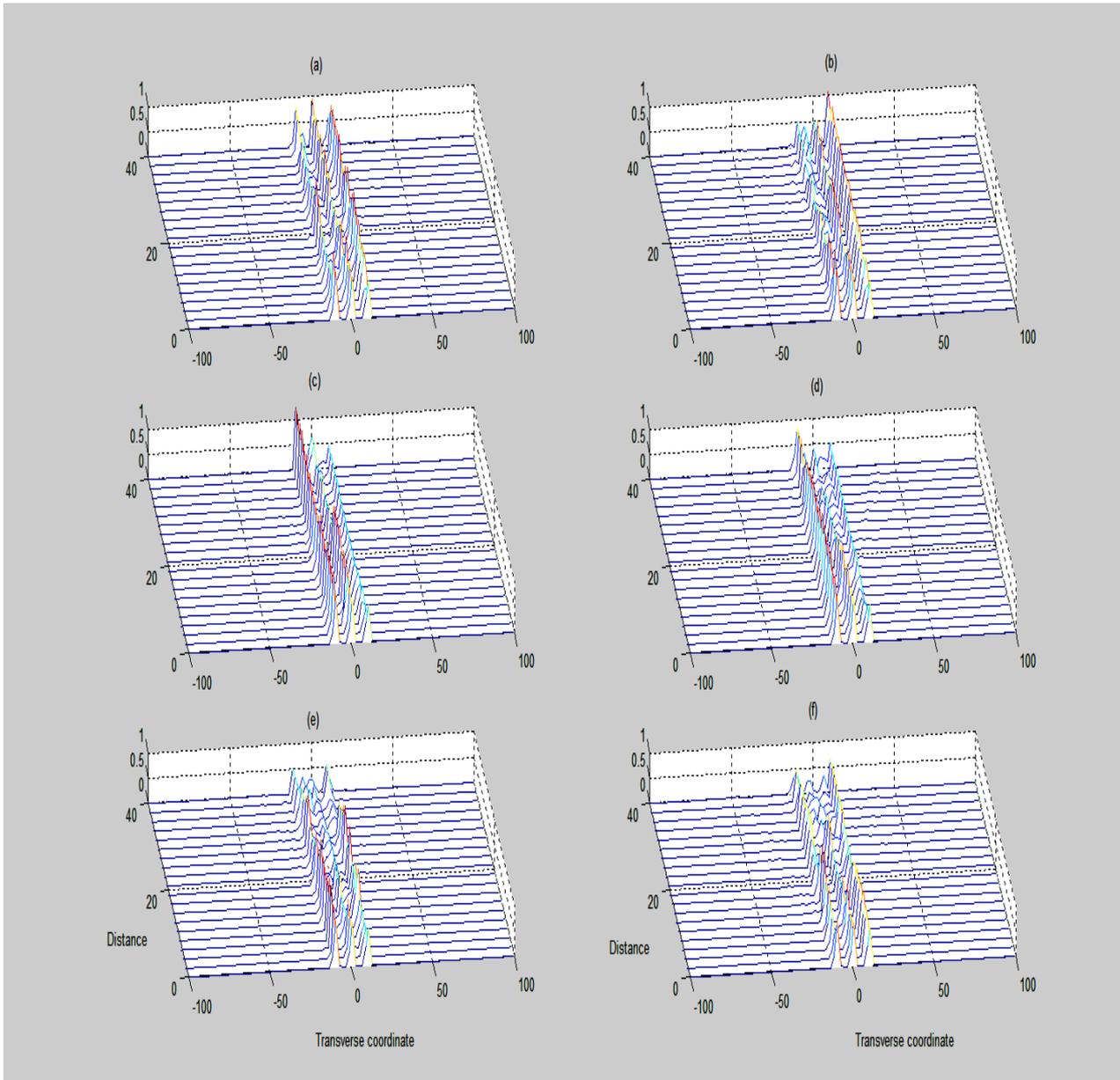


Fig. 5. The normalized three-bright soliton intensity versus the propagation distance under the nonlocality (a) with  $A_0 = B_0 = 4$ . (a) constant nonlocality; (b)  $D = 0.1$ ; (c)  $D = 0.2$ ; (d)  $D = 0.3$ ; (e)  $D = 0.4$ ; (f)  $D = 0.5$ .

Fig. 6 is the normalized soliton intensity versus the propagation distance under the nonlocality (b), and Fig. 7 is the disintegration distance versus the period length. For the period length  $z_0$ , three special cases on the period length of the varying nonlocality are considered as  $z_0 \ll 10$ ,  $z_0 \sim 10$  and  $z_0 \gg 10$ . We can see the periodically varying nonlocality as perturbation leads to soliton disintegration, and the disintegration distance relates to the period length of the varying

nonlocality. The period length of the varying nonlocality plays an important role in the soliton propagation under the nonlocality (b), there is a period length which is called as the worst period length at which the effect of the periodically varying nonlocality is the largest on soliton propagation, and the corresponding disintegration distance is the shortest. From the Figs. 4 and 5 we can see the worst period length is about 10.

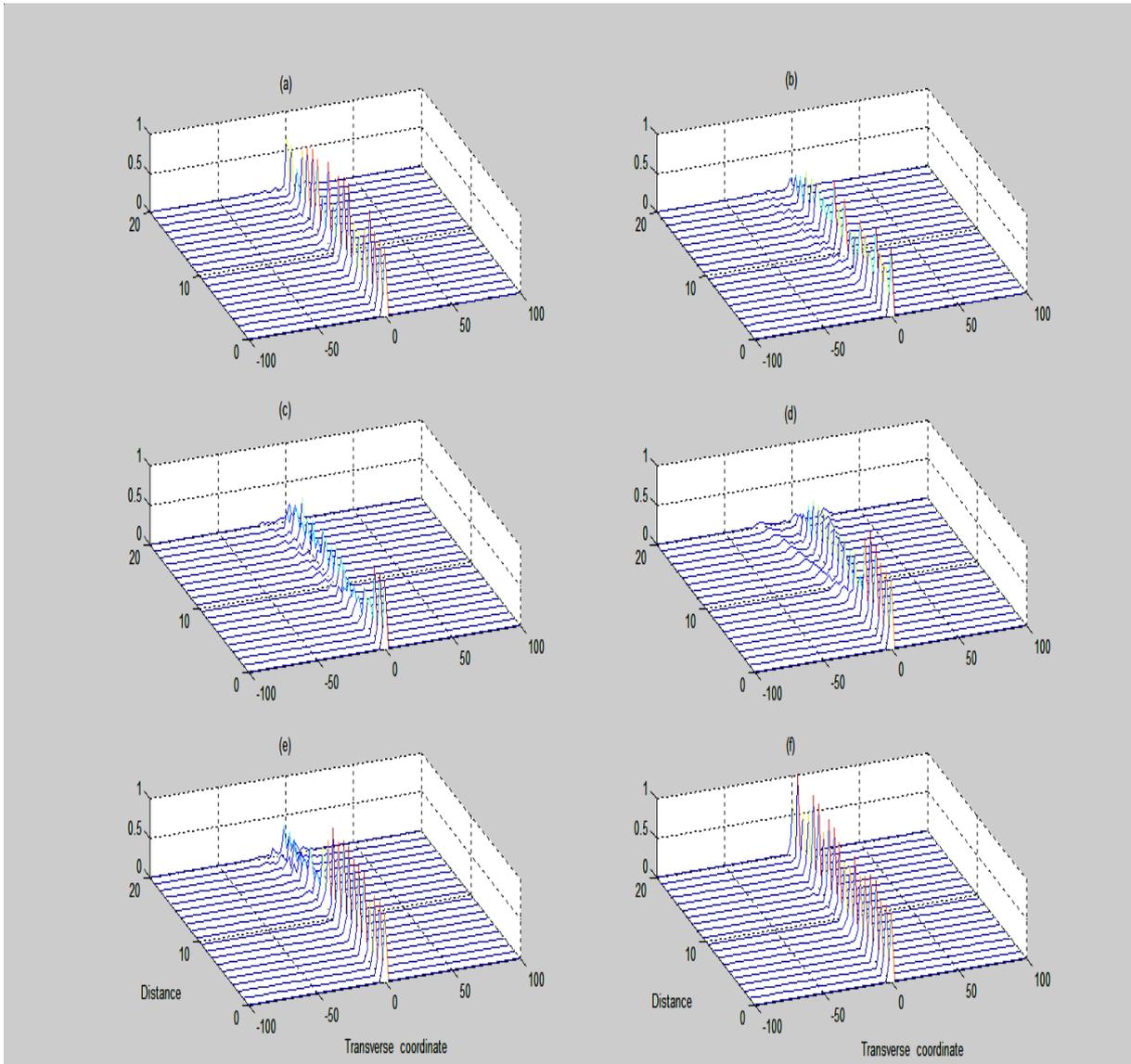


Fig . 6 The normalized one-bright soliton intensity versus the propagation distance under the nonlocality (b) with  $A_0 = B_0 = 4$ . (a) constant nonlocality; (b)  $z_0 = 1.0$ ; (c)  $z_0 = 5.0$ ; (d)  $z_0 = 10.0$ ; (e)  $z_0 = 20.0$ ; (f)  $z_0 = 40.0$  .

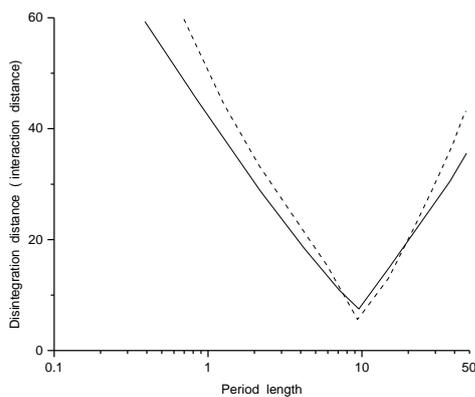


Fig . 7. The disintegration distance ( interaction distance) versus the period length of the nonlocality (b) with  $A_0=B_0=4$ . Solid line: disintegration distance; dashed line: interaction distance.

Fig. 8 demonstrates the normalized intensity of three bright solitons versus the propagation distance under the nonlocality (b), and the interaction distance versus interaction distance is shown in Fig. 7 . The three solitons may not propagate stably a long distance even if the separation is larger than five times of the initial soliton-width ( such as  $\Delta = 10$  ) and the nonlocality is strong ( $A_0 = 4, B_0 = 4$ ) under the varying periodically nonlocality. The varying periodically nonlocality reduces the interaction distance, and the interaction distance strictly depends on the period length. Also there is a period length which is called as the worst period length at which the effect of the periodically varying nonlocality is the largest on soliton interaction, and the corresponding interaction distance is the shortest. From the Figs. 7 and 8 the worst period length is about 10.

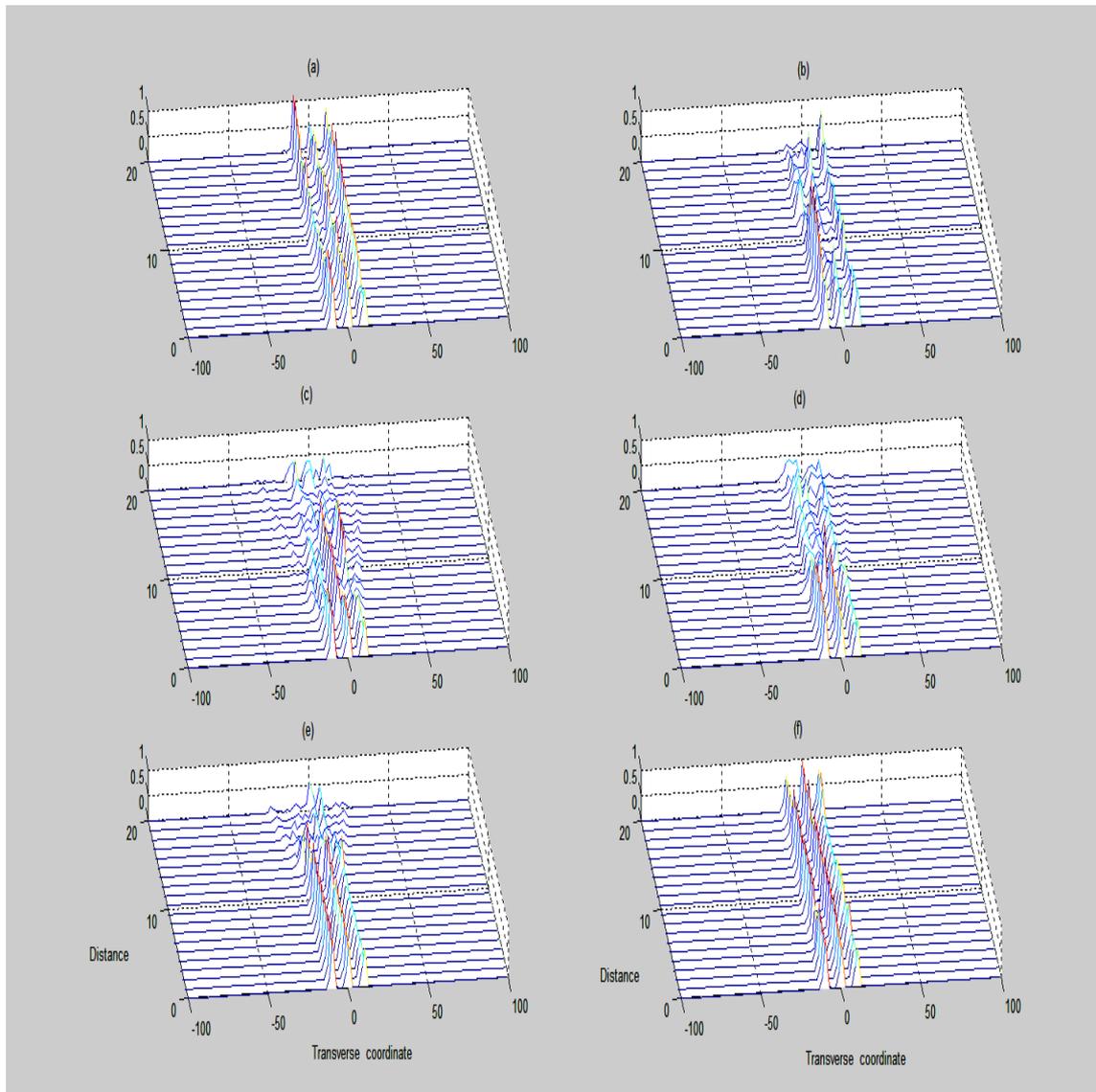


Fig. 8. The normalized three-bright soliton intensity versus the propagation distance under the nonlocality (a) with  $A_0 = B_0 = 4$ . constant nonlocality; (b)  $z_0 = 1.0$ ; (c)  $z_0 = 5.0$ ; (d)  $z_0 = 10.0$ ; (e)  $z_0 = 20.0$ ; (f)  $z_0 = 40.0$ .

The perturbation effects of the two varying nonlocalities are very different in the focusing media. From Figs. 3 ~ 8, we can see that the effects of the periodically varying nonlocality are larger than those of the randomly varying nonlocality because of the inhomogeneity effects resulting from the worst period length, and the corresponding propagation distance or the interaction distance is shorter under the same situation (the standard deviation of the varying randomly nonlocality equals the effective standard deviation of the varying periodically nonlocality).

#### 4. Conclusions

Dynamics of the bright solitons are investigated

by the numerical simulation in nonlocal nonlinear media with the varying nonlocality, and effects of the varying randomly and periodically nonlocalities on the soliton propagation and interaction are analyzed. The strong nonlocality is expanded as the fundamental nonlocality and the second-order nonlocality, and the varying nonlocality plays a crucial role in the physical features of the bright solitons. The constant nonlocality stabilizes propagation of the bright soliton if the nonlocality is strong enough, and suppresses the interaction. But the varying nonlocality destroys the bright soliton dynamics. The effects of the varying randomly nonlocality strictly depend on the stochastic strength, the nonlocality leads to disintegration of the soliton if the stochastic strength is large enough, and enhances the interaction. The effects

of the varying periodically nonlocality become completely reliant on the period length, there is a period length which is called as the worst period length at which the effect of the periodically varying nonlocality is the largest on soliton propagation or interaction, and the corresponding disintegration distance or the interaction distance is the shortest. Under the same situation, the effects of the periodically varying nonlocality are larger than those of the randomly varying nonlocality because of the inhomogeneity effects resulting from the worst period length, and the corresponding propagation distance or the interaction distance is shorter.

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## References

- [1] A. Hasegawa, Y. Kodama, Solitons in Optical Communications, In Clarendon Press, Oxford (1995).
- [2] H. Li, D. N. Wang, Journal of Modern Optics **54**, 807 (2007).
- [3] S. Skupin, O. Bang, D. Edmundson, W. Krolikowski, Physical Review E, **73**, 066603 (2006).
- [4] Y. Y. Lin, R. K. Lee, B. A. Malomed, Physical Review A, **80**, 013838 (2009).
- [5] Q. Guo, B. Luo, S. Chi, Optics Communications, **259**, 336 (2006).
- [6] N. Ding, Q. Guo, Chinese Physics, **18**, 4298 (2009).
- [7] D. M. Deng, Q. Guo and W. Hu, Physics Review A, **79**, 023803 (2009).
- [8] Q. Kong, Q. Wang, O. Bang, W. Krolikowski, Physics Review A, **82**, 013826 (2010).
- [9] Y. Y. Lin, R. K. Lee, Optics Express **15**, 8781 (2008).
- [10] M. A. Molchan, Symmetry, Integrability and Geometry: Methods and Applications (SIGMA) **3**, 083 (2007).
- [11] H. Li, T. Wang, D. Huang, Physics Letters A **341**, 331 (2005).
- [12] F. Kh. Abdullaev, J. C. Bronski, G. C. Papanicolaou, Physica D, **135**, 369 (2000).

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\*Corresponding author: lihong\_hust@yahoo.com