Electromagnetic scattering from perfect electromagnetic conductor (PEMC) sphere placed in isotropic plasma

M. Z. YAQOOB[a*], A. GHAFFAR[b,c], I. SHAKIR[c], M. Y. NAZ[b], Q. A. NAQVI[d]
[a]*Department of Physics, G.C. University, Faisalabad, Pakistan*
[b]*Department of Physics, University of Agriculture, Faisalabad, Pakistan*
[c]*Department of Electrical Engineering, King Saud University, Saudi Arabia*
[d]*Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan*

The electromagnetic scattering from PEMC sphere placed in isotropic plasma medium is investigated theoretically. The analytical formulation of the problem is done in extended Mie theory. To secure the symmetry, all the electromagnetic fields i.e., incident and scattered are expanded in terms of spherical vector wave functions (SVWFs). The influence of plasma parameters i.e., plasma density and effective collision frequency on RCS is analyzed and presented graphically. It is concluded that, the plasma medium can be used to control the scattering amplitude and RCS. Further, the effects of plasma parameters on Co and Cross polarized components are also presented. The numerical results are compared with the published literature under special conditions and good agreement is found.

(Received November 30, 2014; accepted January 21, 2015)

Keywords: Scattering, Plasma, Perfect Electromagnetic Conductor (PEMC), Sphere, Radar Cross section (RCS)

1. Introduction

Plasma is a quasi-neutral, highly ionic state of matter comprising free electrons and ions [1]. Plasma, recently realized as metamaterial and found potential applications in the field of communication, defense technology, negative refractive index materials and plasma photonic crystals [2-4]. Further, the plasma based composites metamaterials excite many researchers and scientists by their extraordinary properties [5]. In addition to these, some of the researcher are engaged in the plasma coated perfectly conducting objects and reported that plasma coating helps to protect the target. The plasma coated conducting objects can be modeled as; when space crafts (missiles and spaceships) immersed in the ionosphere or come back to earth's atmosphere then plasma produced around them due to violent collisions, and modeled as plasma coated cylinder for missile or plasma coated sphere for spaceship [6]. Recently, the problem on scattering of electromagnetic radiation form anisotropic plasma coated perfect electromagnetic conductor (PEMC) circular Cylinder is formulated and reported that the plasma coated PEMC circular cylinder has more stealth capability as compared to perfect electric conductor (PEC) or perfect magnetic conductor (PMC) cylinder [7]. The PEMC is a newly realized metamaterial [8], and gives the generalization of all types of perfect conductors i.e., PEC and PMC [9-10]. It is characterized by the scalar admittance parameter i.e, M, if M is equal to zero or M approaches to infinity, it will behave as PMC or PEC respectively. The scattering from plasma coated PEMC cylinder problem is further extended as, electromagnetic scattering from PEMC cylinder placed in un-magnetized/isotropic plasma medium and analyzed that the radar cross section can be controlled by the appropriate plasma parameters as well as the type of perfect cylinder [11]. Yaqoob et al., studied the more general case of scattering i.e, scattering of electromagnetic radiation from plasma coated PEMC cylinder placed in chiral metamaterial and reported the effects on scattering amplitude due to different hot media and cylinder types [12]. In literature, very few research work has been done on the electromagnetic scattering form PEMC spherical geometries as compared to cylindrical. The electromagnetic scattering from coated and uncoated PEMC sphere has been discussed by Ruppin [13-14]. Ghaffar et al., extended the Ruppin's work and formulated the scattering problem for PEMC sphere placed in chiral medium [15]. It was reported that the radar cross section strongly depends upon the chirality, for strong chirality the radar cross section can be minimized. Up till now, no research work has been done on the electromagnetic scattering from plasma hosted PEMC sphere, to fill this gap, the scattering of electromagnetic radiation from PEMC sphere placed in plasma is discussed. The influences of plasma parameters i.e., plasma density, collisional frequency on scattering amplitude of Co and Cross polarizations are studied and presented. This work has potential applications in plasma based composite metamaterials, target protection, space sciences, which shows the novelty of work. The organization of the paper is as follow; in section 2, the physical modeling in the frame work of extended Mie theory is presented while the section 3 contains the numerical solution and graphical description of the analytical formulation. At the end, conclusion of this research work is discussed in section 4.

2. Analytical formulation

2.1. PEMC sphere placed in Isotropic plasma medium

In this section, the scattering problem is physical modeled by mean of analytical formulation i.e., extended Mie theory. The figure 1 shows the schematic view of the scattering problem. The region I represents the plasma region. In which, the plasma medium is considered as an isotropic, homogenous and un-magnetized medium. While, the region II represents the PEMC sphere of radius $\boldsymbol{r} = \boldsymbol{a}$.

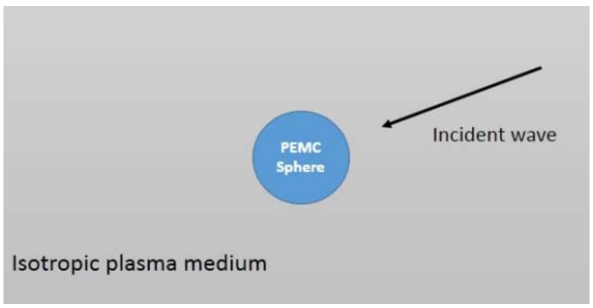

Fig.1 PEMC Sphere placed in homogenous isotropic plasma medium

The incident plane wave is expanded in terms of spherical coordinate system (r, θ, φ) and written in the spherical wave functions as [13-15]

$$\vec{M}^{(l)}_{\sigma mn\gamma} = \vec{\nabla} \times \left[\vec{r} Y_{\sigma mn}(\theta,\varphi) R^{(l)}_n(kr)\right] \quad (1)$$

$$\vec{N}^{(l)}_{\sigma mn\gamma} = \frac{1}{k}\left[\vec{\nabla} \times \vec{M}^{(l)}_{\sigma mn\gamma}\right] \quad (2)$$

The spherical harmonic $Y_{\sigma mn}(\theta,\varphi)$ is characterized by the subscript $\sigma = e \ or \ o$. For $\sigma = e\ (even)$ and $\sigma = o(odd)$ the spherical harmonic $Y_{\sigma mn}(\theta,\varphi)$ will be even and odd respectively. Where radial function $R^{(l)}_n(kr)$ presents a appropriate kind of spherical Bessel function $J_n(kr)$, spherical Neumann function $n_n(kr)$ and spherical Hankel function $h_n(kr)$ for l =1, 2 and 3 respectively. Here k is the wavenumber in plasma region and equals to $k = k_o(\varepsilon_p)^{+1/2}$ as given in [1,11] where k_o is the wave number in free space and $\varepsilon_p = 1 - \frac{i\omega_o^2}{(v+i\omega)\omega}$ is the relative permittivity of plasma medium, which is the function of (electron) plasma frequency $\omega_o = \left(\frac{ne^2}{m\varepsilon_o}\right)^{1/2}$, effective collision frequency v and the incident field frequency ω. The incident and scattered fields in each region is written below as [13]

$$\vec{E}^i = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(\vec{M}^{(1)}_{o1n0} - i\vec{N}^{(1)}_{e1n0}\right) \quad (3)$$

$$\vec{H}^i = -\frac{E_0}{\eta_p} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(\vec{M}^{(1)}_{e1n0} + i\vec{N}^{(1)}_{1n0}\right) \quad (4)$$

where $\eta_p = \sqrt{\frac{\mu_0}{\varepsilon_p}}$ is the impedance of isotropic plasma medium and scattered fields in region I are given as

$$\vec{E}^s = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(a^s_n \vec{M}^{(3)}_{o1n0} + c^s_n \vec{M}^{(3)}_{e1n0} - i b^s_n \vec{N}^{(1)}_{e1n0} - i d^s_n \vec{N}^{(1)}_{o1n0}\right) \quad (5)$$

$$\vec{H}^s = -\frac{E_0}{\eta_p} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(b^s_n \vec{M}^{(3)}_{e1n0} + d^s_n \vec{M}^{(3)}_{o1n0} + i a^s_n \vec{N}^{(1)}_{o1n0} + i c^s_n \vec{N}^{(1)}_{e1n0}\right) \quad (6)$$

where ${a_n}^s$ and ${b_n}^s$ are co-polarized scattering coefficients and ${c_n}^s$ and ${d_n}^s$ are cross polarized field scattering coefficient. The PEMC is realized as a perfect reflector [13], which does not permit the flow of electromagnetic energy through itself. So, the transmission through PEMC sphere is zero.

2.2 Scattering Coefficients and Radar Cross section (RCS)

After the field expansion, the scattering coefficient are computed by applying the appropriate boundary conditions at the interface between PEMC sphere boundary and isotropic plasma. The generally PEMC boundary realized as

$$\left.\begin{aligned} \boldsymbol{n} \times (\vec{\boldsymbol{H}} + M\vec{\boldsymbol{E}}) = 0 \\ \boldsymbol{n} \cdot (\vec{\boldsymbol{D}} - M\vec{\boldsymbol{B}}) = 0 \end{aligned}\right\} \quad (7)$$

above, $\boldsymbol{n}$ is the unit normal and M is the scalar admittance parameter, which characterizes the PEMC. For tangential field components the boundary condition is

$$H^i_t + ME^i_t + H^s_t + ME^s_t = 0 \quad (8)$$

and the boundary condition for the radial field components is

$$\varepsilon_0 E^i_r - M\,\mu_0 H^i_r + \varepsilon_0 E^s_r - M\,\mu_0 H^s_r = 0 \quad (9)$$

By applying the tangential boundary condition on equation 3-4, the following scattering coefficients are obtained

$$a_n{}^s = -\frac{h_n(kr)[kr\ J_n(kr)]' + (M\eta_0)^2\ J_n(kr)[kr\ h_n(kr)]'}{(1+(M\eta_0)^2)\ h_n(kr)[kr\ h_n(kr)]'} \quad (10)$$

$$b_n{}^s = -\frac{J_n(kr)[kr\ h_n(kr)]' + (M\eta_0)^2\ h_n(kr)[kr\ J_n(kr)]'}{(1+(M\eta_0)^2)\ h_n(kr)[kr\ h_n(kr)]'} \quad (11)$$

$$c_n{}^s = \frac{iM\eta_0}{(1+(M\eta_0)^2)\ h_n(kr)[kr\ h_n(kr)]'} \quad (12)$$

and

$$d_n{}^s = \frac{iM\eta_0}{(1+(M\eta_0)^2)\ h_n(kr)[kr\ h_n(kr)]'} \quad (13)$$

In the above equations 12-13, the cross polarized scattering coefficient are equal to each other. To calculate the radar cross section for backward and forward scattering, the following formulas are used respectively.

$$\sigma(180°) = \left(\frac{2}{kr}\right)^2 \left[\left|\sum_{n=1}^{\infty} (-1)^n \left(n+\frac{1}{2}\right)(b_n{}^s - a_n{}^s)\right|^2 + \left|\sum_{n=1}^{\infty} (-1)^n \left(n+\frac{1}{2}\right)(c_n{}^s + d_n{}^s)\right|^2 \right] \quad (14)$$

$$\sigma(0°) = \left(\frac{2}{kr}\right)^2 \left[\left|\sum_{n=1}^{\infty} \left(n+\frac{1}{2}\right)(b_n{}^s + a_n{}^s)\right|^2 + \left|\sum_{n=1}^{\infty} \left(n+\frac{1}{2}\right)(c_n{}^s - d_n{}^s)\right|^2 \right] \quad (15)$$

3. Numerical Results and Discussion

In this section, some numerically calculated results of scattering of electromagnetic radiation from PEMC sphere placed in plasma medium, are presented. To get the better and more understanding about the physical aspects of the said scattering problem, the graphical description is used for these numerical results. In the whole results, the incident frequency is kept $f = 1\ GHz$ and the radius of the sphere is taken as $a = 0.2\lambda_o$. First of all, the above formulated scattering theory is verified by comparing with the already published literature, and good comparison is found. The figure 2 shows this comparison. If the plasma medium is replaced by the free space then the results of Co-polarized, Cross- polarized and total radar cross section (RCS) are same as found in [13]. Which shows the accuracy in our formulation and work.

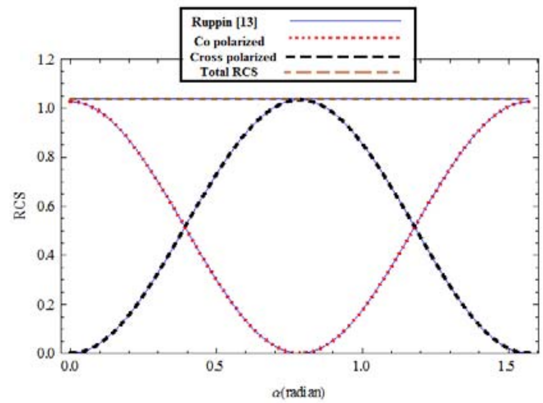

Fig.2 Comparison between total RCS, Co polarized and Cross polarized contribution to RCS

After the check of accuracy, the further analysis and effects on scattering amplitude with respect to plasma parameters are discussed. Figure 3 depicts the influence of plasma density (n) on the Co-polarized radar cross section of PEMC sphere placed in plasma medium, and one can clearly observe that with the increase of plasma density the RCS is reducing. Which shows that, by appropriate choice of plasma density, the Co-polarized component RCS can be controlled. Similar analysis is given in figure 4 for the Cross-polarized component of RCS for different plasma densities. It is observed that with the increase in plasma density, the cross polarized component of RCS is also increasing, which confirms the opposite behavior of Co and Cross polarized component of PEMC. Such behavior is also observed in [11] for a PEMC cylinder placed in un-magnetized/isotropic plasma medium, which further confirms the accuracy of our work.

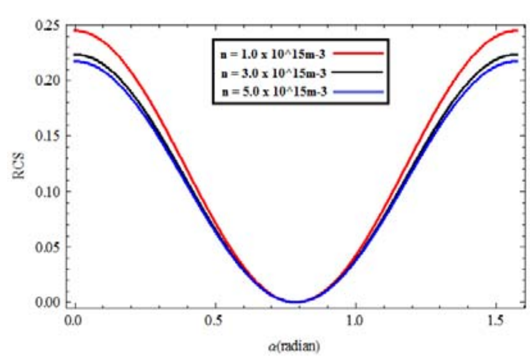

Fig. 3 Comparison between C0-polarized RCS components of PEMC sphere placed in plasma medium at different values of plasma density ($a = 0.2\lambda_0$, $f = 1\ GHz$ and $v = 1.0 \times 10^{10} Hz$)

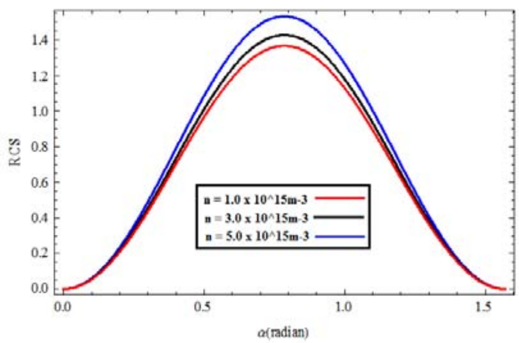

Fig. 4 Comparison between Cross-polarized RCS components of PEMC sphere placed in plasma medium at different values of plasma density $(a = 0.2\lambda_0,\ f = 1\,GHz\ and\ v = 1.0 \times 10^{10} Hz)$

In Figs. 5 and 6 shows the comparison between Co and Cross polarized components of RCS under different plasma effective collision frequencies and it is clear from these figures that with the increase in the collision frequency, the scattering amplitude also increases. Actually the plasma permittivity of plasma medium is $\varepsilon_p = 1 - \frac{i\omega_o{}^2}{(v+i\omega)\omega}$, which consists of two parts, one is real (transmission) and second one is imaginary (absorption) part. By increasing the value of effective collision frequency (v), the imaginary part (absorption) is decreasing and scattering amplitude is increasing.

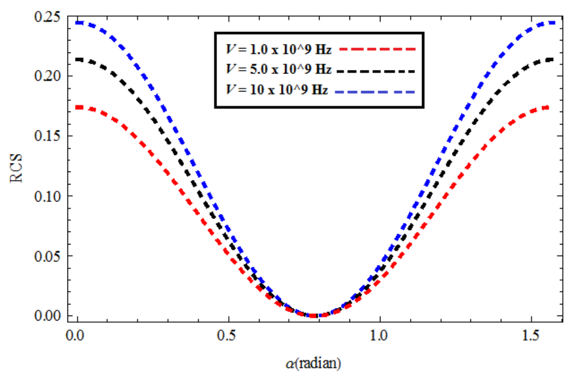

Fig. 5 Comparison between Co-polarized RCS components of PEMC sphere placed in plasma medium at different values of effective collision frequency $(a = 0.2\lambda_0, f = 1\,GHz\ and\ n = 1.0 \times 10^{15} m^{-3})$

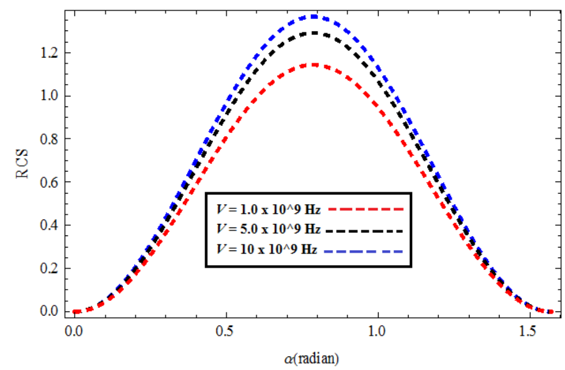

Fig. 6. Comparison between Cross-polarized RCS components of PEMC sphere placed in plasma medium at different values of effective collision frequency $(a = 0.2\lambda_0, f = 1\,GHz\ and\ n = 1.0 \times 10^{15} m^{-3})$

4. Concluding remarks

In this research work, the electromagnetic scattering from PEMC sphere placed in isotropic plasma medium is discussed. The following conclusions can be made by the previous numerical results;

- The Co and Cross polarized components always show opposite behavior for PEMC, and their behavior is independent to the geometry either cylindrical or spherical.
- The RCS of PEMC sphere can be controlled by the appropriate choice of the host plasma medium's parameters i.e., plasma density and effective collision frequency.
- The scattering response of electromagnetic radiation from PEMC spherical geometry is similar to the cylindrical one.
- Further, the plasma medium can be used to tune or control the electromagnetic scattering amplitude.

Acknowledgment

The authors would like to extend their sincere appreciation to the Deanship of Scientific Research (DSR) at King Saud University for its funding of this research through the Research Group Project noRGP-VPP-312.

References

[1] F. F. Chen, M. D. Smith, Plasma. John Wiley & Sons, Inc., (1984).
[2] O. Sakai, Tachibana, K. Plasma Sources Science and Technology, **21**(1), 013001 (2012).
[3] W. Li, Y. Zhao, R. Cui, H. Zhang, Frontiers of Optoelectronics in China, **2**(1), 103 (2009).
[4] J. G. Eden, IEEE Photonics J, **3**, 302(2011). .
[5] O. Sakai, In General Assembly and Scientific Symposium, 2011 XXXth URSI, 1 (2011).
[6] YL. I. Geng J of Antenna and Propogation, 1 (2011).
[7] A. Ghaffar, M. Z. Yaqoob, M. A. Alkanhal, M. Sharif, Q. A. Naqvi, AEU-International Journal of Electronics and Communications **68**(8), 767 (2014).
[8] I. V. Lindell, A. H Sihvola,. Journal of Electromagnetic Waves and Applications, **19**(7), 861 (2005).
[9] I. V. Lindell, A. H. Shivola, IEEE Trans. On Antenas and Propagat, **53**(9), 3012(2005).
[10] I. V. Lindell, A. H. Sihvola, IEEE Trans. on Antennas and Propag., **54**(9), 2553(2006).
[11] A. Ghaffar, M. Z. Yaqoob, M. A. Alkanhal, S. Ahmed, Q. A., Naqvi, M. A. Kalyar, Optik-International Journal for Light and Electron Optics. **125**(17), 4779 (2014).
[12] M. Z. Yaqoob, A. Ghaffar, I. Shakir, Y.M. Naz, S. Ahmed, A. Naqvi, Optoelectron. Adv. Mater. – Rapid Commun., **8**(5-6), 488 (2014).
[13] R. Ruppin, Journal of Electromagnetic Waves and Applications **20**(12), 1569 (2006)
[14] R. Ruppin, Progress in Electromagnetics Research Letters, **8**, 53(2009).
[15] A. Ghaffar, S. I. Ahmad, R. Fazal, S. Shukrullah, Q.A. Naqvi, Optik - International Journal for Light and Electron Optics, **124**(21), 4947(2013).

*Corresponding author: zeeshaan32@yahoo.com