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Field intensity distribution by a cylindrical Plano convex lens placed in a homogeneous un-magnetized plasma meida

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A theoretical investigation has been carried out to analyze the optical focusing electromagnetic field by a Plano convex lens place in an isotropic un-magnetized plasma environment. Derived geometrical optics field is valid everywhere except at focal point where it gives infinite value. Singularity of the field at focal point is addressed using Maslov's method. The effect of the physical parameters such as the plasma electron density, the angle of incidence and the effective collisional frequency have been observed to have an evident influence on the levels of the transmitted field-intensities along the focal region. The problem discussed in this paper has been, also, solved using Kirchhoff's approximation and the results of the two methods are in a good agreement.

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1. Introduction

Focusing of electromagnetic waves into dielectric media is a subject of considerable current interest due to its applications in hyperthermia, microscopy, and optical data storage. During last few decades, considerable effort has been put to the study of electromagnetic fields in metamaterials [1-3]. It is known that when high velocity space vehicles with antennas, as lens antennas for example, encounter plasma effect when entering into the Earth's atmosphere. Thus, usually antennas of space vehicles are in surface contact with a plasma layer that affects the radiation characteristics of such antennas[4, 5]. Similarly, the existence of plasma on boundary of targets changes the reflected and scattered wave energy. Such type of analysis is very significant to find out the proper parameters of the plasma, which affect the reflection, absorption, and transmission of the electromagnetic energy especially in the study of the interaction of intense electromagnetic waves with curved surfaces [6-12]. That is basic reason for motivation to consider the problem of transmission of electromagnetic waves through a Plano convex lens placed in isotropic un magnetized plasma environment.

In this study, transmission of a plane wave through a cylindrical Plano convex lens placed in an un-magnetized collisional plasma medium is considered. The analysis is restricted to a general oblique incidence plane-wave with perpendicular polarization. The field transmitted by lens is focused at caustic point or focal point. The field at focal point is obtained by geometrical optics technique which shows singularity at this point [13]. To over come this disadvantage of GO Maslov's method is used to obtained analytical field in the caustic region. According to Maslov's method, the field expression near the caustic can be constructed by using the geometrical optics information, though we must perform the integration in the spectrum domain in order to predict the field in the space domain[14-17]. This method has been success-fully applied to predict the field in the focal region of focusing systems by many researchers [18-21].

The effects of some physical parameters such as the plasma electron density, the angle of incidence and the effective collisional frequency on the transmitted field intensity distribution by along the focal region in isotropic un-magnetized plasma environment from the cylindrical Plano convex lens have been studied. The accuracy of the presented formulations has been confirmed by another solution using Kirchhoff's approximation. The time-harmonic$(i\omega t)$ dependence is adopted and suppressed in what follows.

2. Formulations

Consider a Plano convex lens with aperture diameter a and focal length f is placed in un magnetized plasma environment as depicted in Fig.1. The plasma medium is considered as an un-magnetized, incompressible and isotropic medium. The density of the medium is kept uniform. The motion of electrons in plasma is only considered because mass of ion is much greater as compared to the mass of electron $(M \gg m)$. The surface of the Plano convex lens is described by following equation

$$\zeta = g(\xi) = \frac{n}{n+1} f - \frac{1}{\sqrt{n^2-1}} \sqrt{\xi^2 + \frac{n-1}{n+1} f^2} \qquad (1)$$

where (ξ, ζ) are Cartesian co-ordinates, n is the refractive index of dielectric lens and f is the focal length of the lens. The refractive index of collisional homogeneous plasma is defined as

$$n_p = \sqrt{1 - \frac{j\,\omega_p^2}{(V + j\omega)\omega}} \tag{2}$$

where ω_p is the plasma frequency which satisfy the condition of under-dense plasma $\omega_p < \omega$. The wave numbers in an un-magnetized plasma media is $k_p = n_p k_0$.

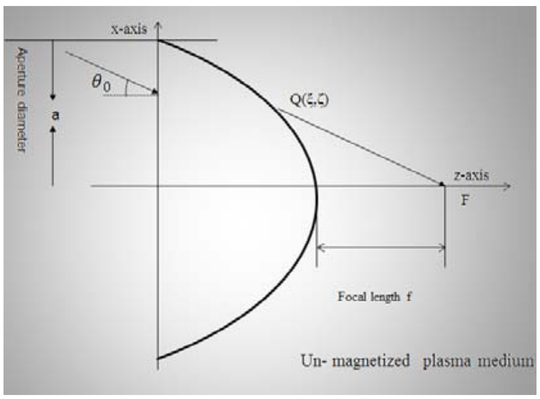

Fig. 1. A cylindrical plano convex lens. placed un-magnetized plasma environment.

Surface of the lens at $z = 0$ is excited a monochromatic electromagnetic wave polarized in the y-axis obliquely incident with an angle θ_0 in an un-magnetized collisional plasma layer as

$$\boldsymbol{E_{0i}} = E_i \hat{e}_y \exp[-jk_i(x \sin\theta_0 + z\cos\theta_0)] \tag{3}$$

The wave vector corresponding to the wave refracted from the lens is

$$\boldsymbol{q} = n\boldsymbol{p^i} + \sqrt{n_p - n^2 + n^2(\boldsymbol{p^i}.\boldsymbol{N})}\boldsymbol{N} - (\boldsymbol{p^i}.\boldsymbol{N})\boldsymbol{N} \tag{4}$$

Where $\mathbf{P^i}$ is wave number of incident wave and $\boldsymbol{N}$ is unit normal to the surface can be written as

$$\boldsymbol{N} = -\frac{g'(\xi)}{\sqrt{1+(g'(\xi))^2}}\hat{e}_x + \frac{1}{\sqrt{1+(g'(\xi))^2}}\hat{e}_z = -\sin\alpha\,\hat{e}_x - \cos\alpha\,\hat{e}_z \tag{5}$$

Thus $\mathbf{q}$ can be written as

$$\boldsymbol{q} = (\mathrm{K}(\alpha)\sin\alpha + n\sin\theta_0)\hat{e}_x + (n\cos\theta_0 + \mathrm{K}(\alpha)\cos\alpha)\hat{e}_z \tag{6}$$

where $\mathrm{K}(\alpha) = \sqrt{n_p - n^2\sin^2(\alpha - \theta_0)} - n\cos(\alpha - \theta_0)$

The GO transmitted field can be written as [16]

$$\boldsymbol{E}_t = \boldsymbol{E}^t \left[\frac{D(\tau)}{D(0)}\right]^{-\frac{1}{2}} e^{jk_t(S_0+\tau)} \tag{7}$$

where τ represents the parameter along the ray, $S_0 = n\zeta$ shows the initial value of the function. $D(\tau)$ represents the Jacobian of the transformation from Cartesian coordinates to ray coordinates. The Hamilton's equations for refracted field are

$$\frac{dx}{d\tau} = q_{\boldsymbol{x}}\,, \quad \frac{dz}{d\tau} = q_{\boldsymbol{z}} \tag{8}$$

$$\frac{dq_{\boldsymbol{x}}}{d\tau} = 0\,, \quad \frac{dq_{\boldsymbol{z}}}{d\tau} = 0 \tag{9}$$

For $x = \xi$, $z = \zeta$, $q_x = q_{x0}$ and $q_z = q_{z0}$, we obtain initial values for (x, z) and (q_x, q_z) as

$$x = \xi + q_x\tau \tag{10}$$

$$z = \zeta + q_z\tau \tag{11}$$

The Jacobian is evaluated as

$$D(\tau) = \frac{\partial(x,z)}{\partial(\xi,\tau)} = \mathrm{K}(\alpha)\sec\alpha + n(\cos\theta_0 + \sin\theta_0\mathrm{Tan}\alpha) + n_p\,\mathrm{K}(\alpha)\mathrm{P}\tau/\sqrt{n_p - n^2\sin^2(\alpha - \theta_0)} \tag{12}$$

$$J(\tau) = 1 + \frac{n_p\,\mathrm{P}\,\mathrm{K}(\alpha)\tau}{(\mathrm{K}(\alpha)\sec\alpha + n(\cos\theta_0 + \sin\theta_0\mathrm{Tan}\alpha))\sqrt{n_p - n^2\sin^2(\alpha - \theta_0)}} \tag{13}$$

where $\mathrm{P} = \dfrac{\frac{(n-1)^{\frac{3}{2}}}{\sqrt{n+1}}f^2}{(n^2\xi^2 + (n-1)^2 f^2)\sqrt{\xi^2 + \frac{n-1}{n+1}f^2}}$.

It is observed that the GO field becomes infinity at the Caustic points as is expected when $J(\tau) = 0$. We can derive the expression which is valid at the focal point using the Maslov's method. The exact location of focal or caustic point may also be obtained at $J(\tau) = 0$. By using Maslov's method the focal regions a valid field expression is as [14-17]

$$\mathrm{E}^r(\mathrm{r}) = \sqrt{\frac{\mathrm{k}}{\mathrm{j}2\pi}}\int_{-\infty}^{\infty}\left[\frac{D(\tau)}{D(0)}\frac{\partial(\mathrm{q_x})}{\partial(\mathrm{x})}\right]^{-\frac{1}{2}}\exp\{-\mathrm{jk}[\mathrm{S}_0 + \tau - x(qx,\mathrm{z})\; qx + qxx \mathrm{d}qx \tag{14}$$

The integrand and phase functions of the integral for the transmitted fields from lens into plasma material are obtained, respectively, as

$$\frac{D(\tau)}{D(0)}\frac{\partial \mathrm{q_x}}{\partial \mathrm{x}} = \frac{1}{D(0)}\frac{\partial(\mathrm{q_x},z)}{\partial(\xi,\tau)} = \frac{\mathrm{K}(\alpha)\mathrm{P}\, cos\alpha(\mathrm{K}(\alpha)cos\alpha+n\cos\theta_0)(cos\alpha\sqrt{4n_p+4\,n^2\sin^2(\alpha-\theta_0)}+2n\sin\alpha\sin(\alpha-\theta_0))}{2(n_p-n^2\sin^2(\alpha-\theta_0))} \tag{15}$$

$$\mathrm{S}=S_0+\tau-x(\mathrm{x},q_z)q_x+q_x x$$
$$\mathrm{S}=n\zeta-\xi p_x-\zeta q_z+q_x x\ +q_z z \tag{16}$$

where $\xi=\frac{(n-1)f\tan\alpha}{\sqrt{1-(n^2-1)\tan^2\alpha}}$, $\zeta=-\tan\alpha$, $x=r\cos\alpha$ and $z=r\sin\alpha$.

By using equations (14) and (15) into (13) and changing the variable q_x to α, given in (9), yields the final finite transmitted field expression valid along focal region as

$$\frac{E_y(x,z)}{E_i} = \sqrt{\frac{K}{2\pi j}}\int_{-\tau/2}^{\tau/2}\frac{2\mathrm{n}\cos\alpha}{\mathrm{n}\cos\alpha+\sqrt{n_p-\mathrm{n}^2\sin^2\alpha}}\sqrt{\frac{n_p-n^2\sin^2\alpha}{P\ q_z\cos\alpha}\frac{\partial q_x}{\partial\alpha}}\,e^{jk_0(n\zeta-\xi p_x-\zeta q_z+q_x x\ +q_z z)}d\alpha \tag{17}$$

3. Huygens-Kirchhoff's principle

To validate the formulation derived in the preceding section, the fields are obtained using the following formula implied by Huygens-Kirchhoff's principle [16]

$$E(x,z)=\frac{1}{4j\pi}\int_{-\infty}^{\infty}\frac{e^{-jR}}{R}E_0(\xi,\zeta)\exp(-\mathrm{jk}S)\,\mathrm{d}\xi \tag{18}$$

where $\mathrm{R}=\mathrm{q_x}(x-\xi)+\mathrm{q_z}(z-\xi)=\sqrt{(x-\xi)^2+f^2}$ and $E_0(\xi_0,\zeta_0)=J^{-\frac{1}{2}}$.

Therefore

$$E(x,z)=\frac{1}{4j\pi}\int_{-l/2}^{l/2}\frac{J^{-\frac{1}{2}}}{R}\times exp[-jk(S+R)]\mathrm{d}\xi \tag{19}$$

4. Results and discussion

The transmitted electromagnetic waves due to a perpendicularly polarized obliquely incident plane wave on a cylindrical Plano convex lens place in isotropic an un-magnetized collisional plasma environment are depicted in this section. The frequency of the incident electromagnetic wave is taken in the microwave region. First, the results are compared with the Huygens-Kirchhoff approximation to check the correctness of the developed analytical formulations. Fig. 2 shows the comparison of the numerical results using Maslov's method (solid line) and using Kirchhoff's approximation (dashed line) which confirms that the agreement is good.

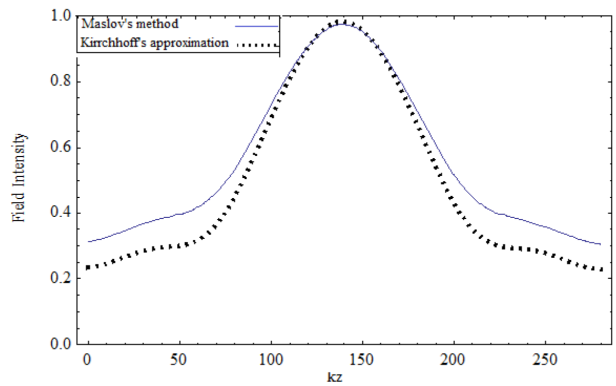

Fig. 2. The normalized filed intensity by Maslov's method (solid line) and by Kirchhoff's Principle (dashed line).

Figs. 3-5 demonstrate, respectively, the effects of the plasma electron density, the angle of incidence, and the effective collisional frequency on the transmitted field distribution from Plano convex lens in the un-magnetized collisional plasma environment. Fig. 3(a)-3(b) show the transmitted field intensity versus kx and kz, respectively for different values of the plasma electron density. These figures have been plotted for a constant focal length of the lens $kf=200$, length of aperture diameter of $ka=80$, an incident-wave frequency $\omega=1.5708\times10^8$Hz, effective collisional frequency $V=6\times10^9$Hz, and $\theta_0=1$ degree. It is clearly observed that if the electron density of the plasma layer increases then the value of the transmitted field intensity decreases along both axes and the location of the maximum transmitted field intensity slightly shifts away from the lens along the z-axis. At the values of the plasma electron density greater than $n_e=1\times10^{11}\,m^{-3}$, the field intensity pattern-symmetry around the maximum point along the z-axis is begin.

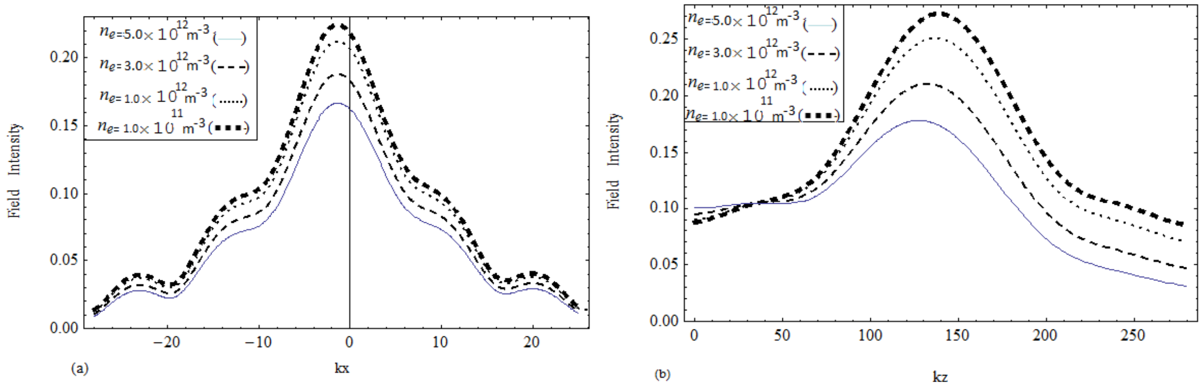

Fig. 3. The transmitted filed intensity distribution for different values of the plasma electron density (a) along the x-axis (b) along the z-axis.

Fig. 4(a)-4(b) shows the transmitted field intensity versus kx and kz, respectively, for different values of the angle of incidence θ_0 of the incoming wave. The other parameters are; constant focal length of the lens $kf = 200$, length of aperture diameter of $ka = 80$, an incident-wave frequency $\omega = 1.5708 \times 10^8$Hz, effective collisional frequency $V = 6 \times 10^9$Hz, and $n_e = 1 \times 10^{11}\ m^{-3}$. It is noted that, if the incoming-wave angle of incidence θ_0 increases, the value of the transmitted field intensity increases along x-axis and the location of the maximum transmitted field intensity shifts slightly downward along the x-axis and slightly shift occur in the z-axis toward lens curved surface. The field intensity pattern is more sharp and symmetric at higher values of the incidence angle along z-axis.

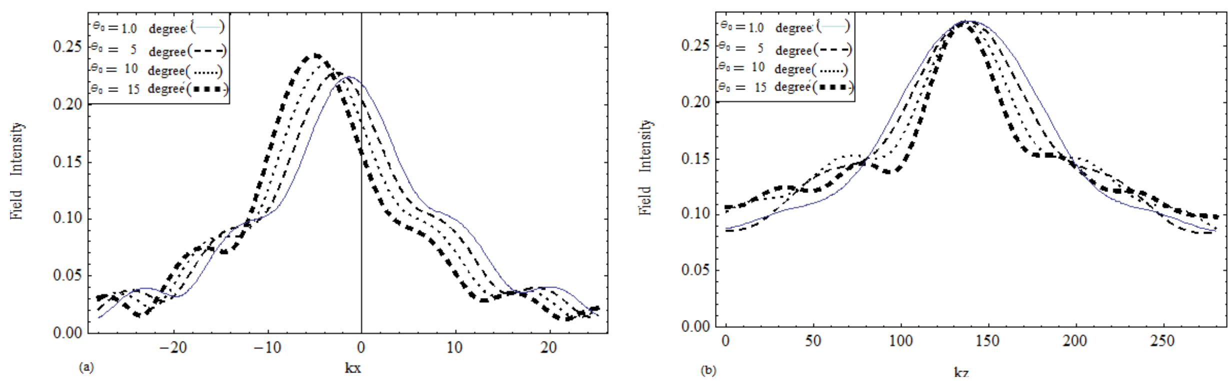

Fig.4. The transmitted field intensity distribution for different values of the incoming-wave incident angle (a) along the x-axis (b) along the z-axis.

Fig. 5(a)-5(b) shows the transmitted field intensity versus kx and kz, respectively, for different values of the effective collisional frequency V. The other parameters are; constant focal length of the lens $kf = 200$, length of aperture diameter of $ka = 80$, an incident-wave frequency $\omega = 1.5708 \times 10^8$Hz, $\theta_0 = 1$ degree and $n_e = 1 \times 10^{11}\ m^{-3}$. If the effective collisional frequency V increases, the value of the transmitted field intensity increases along both axes.

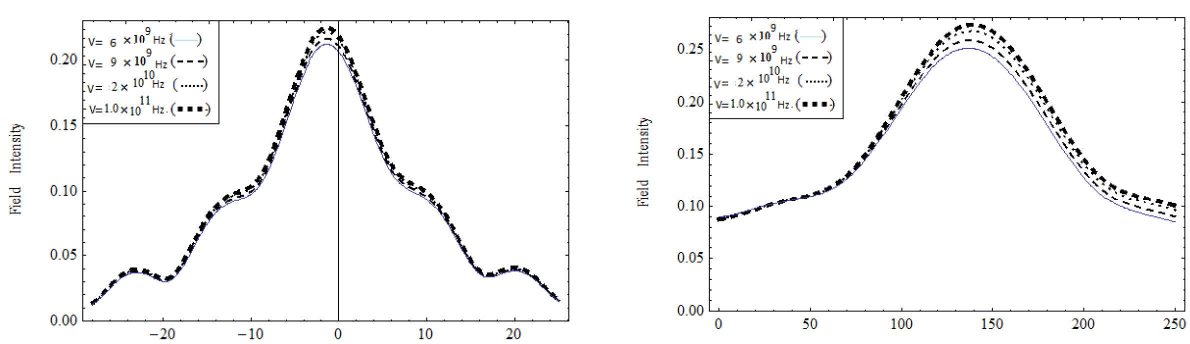

Fig.5. The transmitted field intensity distribution for different values of the effective collisional frequency (a) along the x-axis (b) along the z-axis.

5. Conclusions

This paper presents theoretical analyses of the electromagnetic fields in the focal region of a Plano convex lens placed in an un-magnetized collisional plasma environment under a general oblique perpendicular polarized wave incidence. Accurate analytical expressions for the field intensity distributions along the focal line were derived using Maslov's method, which is an efficient procedure based on the asymptotic ray theory (ART) and the Fourier transform method. The transmitted fields into the collisional plasma material were found to be in a fairly good agreement to those obtained using Kirchhoff's approximation. The effects of the plasma electron density, the angle of incidence of the incoming-wave, and the effective collisional frequency on the transmitted distributions were examined. The focal point is displaced slightly away from the lens curved surface along the z-axis at higher values of the plasma electron density and toward downward along the x-axis at higher values of the angle of incidence of the incoming-wave. The field intensity increases with the increase of the effective collisional frequency or the plasma electron density along both axes while with the increase of the incidence angle the field intensity increases along x- axis.

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References

[1] H. Minnett, B. Thomas, Electrical Engineers, Proceedings of the Institution of, **115**, 1419 (1968).
[2] H. Ling, S.-W. Lee, JOSA A, **1**, 965 (1984).
[3] W. Dou, Z. Sun, X. Tan, Progress In Electromagnetics Research, **20**, 213 (1998).
[4] D. R. Nicholson, D. R. Nicholson, Introduction to plasma theory: Cambridge Univ Press, 1983.
[5] V. Smilyanskii, Journal of Applied Mechanics and Technical Physics, **12**, 366 (1971).
[6] J. E. Rauch, W. N. Partlo, I. V. Fomenkov, R. M. Ness, D. L. Birx, R. L. Sandstrom, S. T. Melnychuk, "Plasma focus light source with tandem ellipsoidal mirror units," ed: Google Patents, 2003.
[7] M. Laroussi, J. Reece Roth, Plasma Science, IEEE Transactions on, **21**, 366 (1993).
[8] W. M. Manheimer, Plasma Science, IEEE Transactions on, **19**, 1228 (1991).
[9] I. Alexeff, T. Anderson, S. Parameswaran, E. Michael, J. Dhanraj, M. Thiyagarajan, Plasma Science, 2005. ICOPS'05. IEEE Conference Record-Abstracts. IEEE International Conference on, p. 350, 2005.
[10] J. Zhang, Z. Liu, International journal of infrared and millimeter waves, **28**, 71 (2007).
[11] A. Abdoli-Arani, B. Jazi, Waves in Random and Complex Media, **23**, 114 (2013).
[12] A. Abdoli-Arani, R. Ramezani-Arani, B. Jazi, S. Golharani, Waves in Random and Complex Media, **22**, 370 (2012).
[13] C. Thomson, C. Chapman, Geophysical Journal International, **83**, 143 (1985).
[14] R. W. Ziolkowski, G. A. Deschamps, Radio Science, **19**, 1001 (1984).
[15] K. Hongo, Y. Ji, Radio Science, **22**, 357 (1987).
[16] K. Hongo, Y. Ji, E. Nakajima, Radio Science, **21**, 911 (1986).
[17] K. Hongo, H. Kobayashi, Radio Science, **31**, 1025 (1996).
[18] A. Ghaffar, M. Shoaib, N. Mehmood, M. Naz, M. Azam, Q. Naqvi, International Journal of Applied Electromagnetics and Mechanics, **41**, 479 (2013).
[19] A. Ghaffar, M. Sharif, Q. Naqvi, M. Alkanhal, F. Khalid, S. Shukurullah, Applied computational electromagnetics society journal, **29**, 478 (2014).
[20] A. Ghaffar, M. Arif, Q. A. Naqvi, M. A. Alkanhal, Optik-International Journal for Light and Electron Optics, **125**, 1589 (2014).
[21] A. Ghaffar, M. H. Shahzad, M. Y. Naz, S. Ahmed, M. Sharif, Q. A. Naqvi, A. A. Syed, Journal of Electromagnetic Waves and Applications, **26**, 1007 (2012).

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