Hyperbolic approach in order to study the heat transport in metallic wire at high transitional regime

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The heat conduction through different metallic wires, which are excited by a modulated thermal source, is studied under the framework of Maxwell-Cattaneo equation or hyperbolic heat equation. Using the numerical solution under the radial direction of cylindrical coordinates. It has been shown in the high frequency regime that remarkable oscillations of the temperature amplitude are obtained, this amplitude depends also on the type of the wire used. Additionally the metallic wires are studied using the parabolic and hyperbolic equation of heat, under tow successive Gaussian excitations. In small gap time the response of the hyperbolic heat equation shows specific behaviour when the relaxation time is quite smaller than the gap time between the successive Gaussian excitations. The obtained numerical results are supported by theory and experiments which constitute an unmistakable character of hyperbolic behaviour.

1. Introduction

During the last years, with the appearance of novel disciplines that require work in areas of high frequency and high transitional regime such as the technology of ultrasound in gases, nuclear collisions, heating of solids by laser pulses [1- 2]. It has become necessary to review the behaviour of some laws at high transitional regime and very short time scales. Forever the heat transport phenomena have been studied based on the well-known Fourier's law which brings us back into the parabolic equation of heat. This equation has provided extensive and successful results in the study of heat transport and also is supported by many experiments that agree with the theory in most experimental conditions encountered in various disciplines of physics and engineering industry [1-2-3]. One of the most simple and accepted approach that can overcome the limitations of Fourier's law at high frequencies, is the Maxwell-Cattaneo equation, which is a hyperbolic partial differential equation, this type is called also the telegraph equation [3-4]. Many research groups carrying out experiments showing that the effect of hyperbolic heat can be easily observed [5-6-7]. But, other authors have criticized the methodology of these experiments and they showed other experimental alternatives that denied the existence of a hyperbolic heat effect [8-9]. Thus, there are few experiences that consider that the hyperbolic heat equation is adequate for studying the case of high frequencies [6-10-11].

One of the most powerful techniques that have shown to provide useful results consists in exciting the physical system using a modulated thermal excitation [10-11].

The easiest way to find a general equation of evolution of heat in a rigid body is to generalize the classical equation of heat. Intuitively, the most appropriate model for this situation is the classical Fourier law [3]

$$\varphi = -K\nabla T \qquad (1)$$

By substituting (1) in the energy balance equation:

$$\rho \frac{\partial u}{\partial t} = -\nabla \varphi \qquad (2)$$

With: $\qquad du = C_v dT \qquad (3)$

Where $\varphi, u, \nabla T, K, \rho$ $\varphi, u, \nabla T, K, \rho$ and C_v are respectively the heat flux [w.m^{-2}], internal energy per unit mass [J.kg^{-1}] the temperature gradient [K. m^{-1}], thermal conductivity[w.m^{-1}.K^{-1}], density [kg.m^{-3}], and heat capacity per unit mass at constant volume [J.kg^{-1}.K^{-1}]. By substituting (1) in the law of energy balance in the absence of external sources.

$$\rho C_v \frac{\partial T}{\partial t} = K \Delta T \qquad (4)$$

This equation shows admirable agreement with experiments for most practical cases, but it has some anomalies at very small time intervals and in regime of high frequencies [3]. Onsager [12] noted that the Fourier model suffers of some contradictions with the principle of microscopic reversibility, but this contradiction '… is removed when we recognize that Fourier's law is only an

approximate description of the process of conduction, neglecting the time needed for acceleration of the heat flow'. In another way, Fourier's law has unphysical properties and it lacks inertial effect. Its main drawback comes from the fact that Fourier law predicts an infinite speed of heat propagation, such that a thermal disturbance in any part of a sample results in an instantaneous perturbation anywhere else in the sample. This fundamental problem is due to the fact that Fourier law establishes explicitly that the both the temperature gradient and heat flux start instantaneously when one of them is imposed over a sample.

In this way, the heat flux and temperature gradient are simultaneous and therefore there is no difference between the cause and the fact of the heat flow [3]. Even still it does not describe the phenomena are very fast or very steep as (light scattering in gases, neutron scattering in liquids, ultrasound propagation, heat propagation at low temperature ...) [3]. Cattaneo in 1948 [13] gives a solution to this problem and proposed to introduce in the Fourier law a relaxation term as.

$$\tau \frac{\partial \varphi}{\partial t} = -(\varphi + K \Delta T) \qquad (5)$$

Introducing (5) into (2), we obtain:

$$\Delta T - \frac{1}{\mu} \frac{\partial T}{\partial t} - \frac{\tau}{\mu} \frac{\partial^2 T}{\partial t^2} = 0 \qquad (6)$$

Were, τ, $\mu = (K/\rho C_v)$ are respectively the thermal relaxation time [s] which represents the time necessary for the initiation of the heat flux after a temperature gradient has been imposed and thermal diffusivity [m^2.s^{-1}] of the material [14]. If the relaxation time becomes negligible the equation (6) becomes (4). The second order derivative of time shows the heat flow propagates as wave and the first order derivative corresponds to the diffuse character of heat [3-12]. Numerous authors [15-16-17-18] estimated the relaxation parameter for metals, superconductors and semiconductors is in the order of picoseconds to microseconds. In the case of values beyond the microsecond there is unphysical meaning of this term. In this case the law of Fourier becomes more suitable for describing the phenomena [18-19-20].

To study the response of the system subject to a general perturbation due to an energy supply term g(r,t), we rewrite the equation (1), expanding the heat flux in Taylor series around $\tau = 0$, and approximating at first order in τ

$$\varphi + \tau \frac{\partial \varphi}{\partial t} = -K \Delta T \qquad (7)$$

On the other hand, energy conservation equation is given by [1]

$$\nabla \varphi + \rho C_v \frac{\partial T}{\partial t} = g(r,t) \qquad (8)$$

Where $g(r,t) = g$ [w.m^{-3}] is the source term which represents the power rate per unit volume at which the heat flux is generated. Combining equations (8) and (7) the hyperbolic heat conduction equation is obtained [3, 18]

$$\Delta T - \frac{1}{\mu} \frac{\partial T}{\partial t} - \frac{\tau}{\mu} \frac{\partial^2 T}{\partial t^2} = -\frac{1}{K}(g + \tau \frac{\partial g}{\partial t}) \qquad (9)$$

2. Excitation of the wire with a thermal sinusoidal signal of high frequency

In this paper we inspect the behaviour and the propagation of heat in a several types of metallic wires subjected to different types of excitations, in hyperbolic approach. The wire with defined dimensions is excited with periodic thermal power source, delivering a harmonic amount of heat. We excite the wires with a modulated heat source at a frequency f, [18-19-20].

$$g = Q(1 + \cos(\omega t)) \qquad (10)$$

Where $\omega = 2\pi f$ [rad.s^{-1}] is the pulse of the modulated heat source and Q [w.m^{-3}] is the spatial distribution of deposit energy over the metallic wires per unit volume. Inserting equation (10) in to (9) and we obtain

$$\Delta T - \frac{1}{\mu} \frac{\partial T}{\partial t} - \frac{\tau}{\mu} \frac{\partial^2 T}{\partial t^2} = -\frac{Q}{K}(1+$$
$$\cos(\omega t) - \tau \omega \sin(\omega t)) \qquad (11)$$

With the following boundary and initial conditions

$$\begin{cases} T(r,0) = 0; T_t(r,0) = 0 \\ 0 \leq r \leq 20 \times 10^{-6} \end{cases} \qquad (12)$$

3. Results and analysis

The result in Fig. 1 illustrates the normalized amplitude as a function of frequency which is the numerical solution of (11) with boundaries and initials conditions (12). The wires used have the following features, same radius (r =20μm) and the same length (L=2cm). At 20 C° the values of the thermal diffusivity are:

$$\mu_{silver} = 144,88 \times 10^{-6} m^2.s^{-1},$$
$$\mu_{Gold} = 128,72 \times 10^{-6} m^2.s^{-1} \text{ and}$$
$$\mu_{Platinum} = 25,67 \times 10^{-6} m^2.s^{-1}.$$

We obtain this result by sweeping the frequency between 10^3 -10^6 Hz by solving (11) numerically [21] for each frequency value. We observe that for small frequencies $\omega \tau \square 1$, the hyperbolic approach converges

for all types of metallic wires to the same results as predicted by the classical law of Fourier. In contrast, in the case of $\omega\tau \square\ 1$, the curves follow an oscillatory regime for the considered metal and predict the oscillatory behaviour of radial temperature amplitude. So each type of material has typical behaviour in high frequency and it's depending on his relaxation time τ and thermal diffusivity μ. This typical behaviour constitutes a classical results founded analytically by [18-22]. Our results show that each wire has a specific oscillation witch not depending on the modulate source, knowing that all the wires are subjected to the same modulate source. One can conclude from the values of thermal diffusivity and Fig. 1 that more the thermal diffusivity is smaller more the amplitude of oscillations is higher. Also a strong enhancement of heat transport is observed when the relaxation time becomes closer to thermalization time τ_m defined by $\tau_m = (r^2/4\mu)$[19-20-22] where r is the radius and μ is the thermal diffusivity of the wire.

The Figs. 2-a and 2-b show the normalized temperature as a function of the wire radius, in different frequency modulated heat source $f=10^6$ Hz for the Fig. 2-a and $f=10^4$ Hz for the Fig. 2-b. In Fig. 2-a we observe in the case of high frequency regime $\omega\tau \square\ 1$ strong oscillations, when the radius of the wire is smaller. Other than when the radius becomes bigger, we watch an attenuation of the amplitude oscillations as reported by [18]. From the values of the thermal diffusivity and figure (2-a), the one can observe the great relation between the oscillations frequency, amplitude of normalized temperature and the nature of wire and their thermal diffusivity. We conclude that more the thermal diffusivity is smaller more the amplitude and frequency of normalized temperature is higher, and more the thermal diffusivity is bigger more the frequency and amplitude temperature is lower.

In contrast in the case of low frequency regime $\omega\tau \square\ 1$ Fig. 2-b, all the wires show almost the same behaviour when the radius become bigger 4×10^{-5}m, however when the radius become closer to 0.6×10^{-5}m the amplitude of normalized temperature increase and each wire shows typical behaviour depending on the nature of the wire and their thermal diffusivity. Knowing that the wire used in figure (1) has radius of $r = 2\times10^{-5}$m we can easily prove that the values of normalized temperature given by Fig. 1 coincide with the values of normalized temperature in the case of $f=10^6$ Hz in Fig. 2-a and in the case of $f=10^4$ Hz in Fig. 2-b.

Undeniably is very important to take into account the thermal diffusivity of the wire in the high frequency regime or when the wire becomes smaller. On the other hand in the low frequency regime or when the radius of the wire grows up all the wires converge to the same values of normalized temperature.
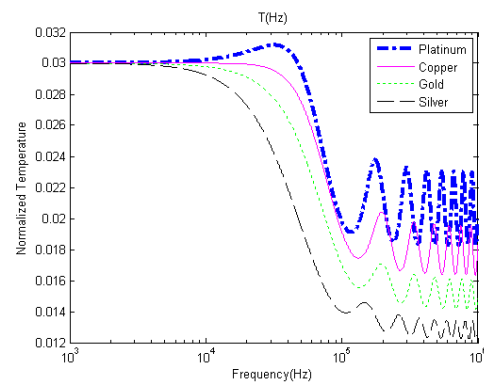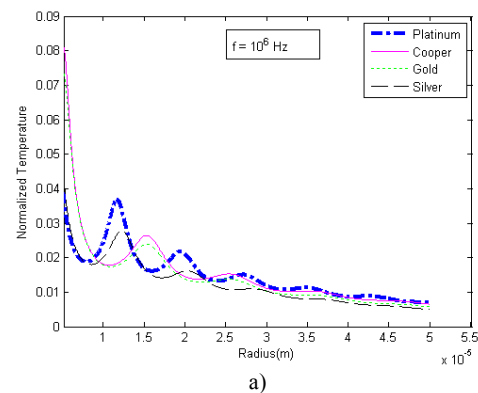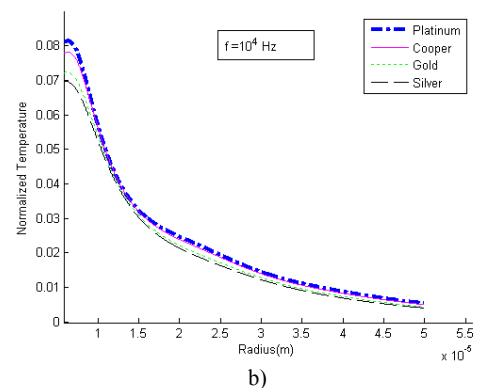

Fig. 1. The amplitude of the normalized temperature, in the radial direction, as a function of frequency. For wires of Gold, Platinum, Silver and Copper.

a)

b)

Fig. 2. a-b. The amplitude of the normalized temperature, in the radial direction, as a function of radius for wires of Gold, Platinum, Silver and Copper. a) Modulated frequency $f = 10^6$ Hz b) Modulated frequency $f = 10^4$ Hz.

4. Excitation of the metallic wire with a Gaussian heat source

The answer of a system to a thermal disturbance is practically interesting. Such situation is encountered when the metallic wire is suddenly and rapidly heated. The energy transfer in such fast processes can't be described

with Fourier's law and requires more involved description [3-12-13-18].

To study the behaviour of heat in two types of metallic wires made with Gold and Silver. The wires are excited by two successive identical sudden thermal pulses (Gaussian shape) separated by a very short time interval (Pico seconds). We excite and compare the responses in hyperbolic and parabolic models for both types of wires.

$$g = Q\left(e^{-\left(\frac{(t-\beta)^2}{\sigma^2}\right)} + e^{-\left(\frac{(t-\gamma)^2}{\sigma^2}\right)}\right) \qquad (13)$$

With σ[s] is the width of the Gaussian function and $t_i = \gamma - \beta$ [s] is the time separating pulses, Q represents the power rate per unit volume at which the heat flux is generated.

In this work the heat source take the form (13) and by inserting equation (13) into (9) we obtain the equation governing this system in the hyperbolic case:

$$\frac{\partial^2 T(x,t)}{\partial x^2} - \frac{1}{\mu}\frac{\partial T(x,t)}{\partial t} - \frac{\tau}{\mu}\frac{\partial^2 T(x,t)}{\partial t^2} =$$

$$\frac{Q}{K}\left\{1 + \left(\begin{array}{c}\dfrac{2(t-\beta)}{\sigma}e^{-\left(\frac{(t-\beta)^2}{\sigma^2}\right)} + \\ (1+\dfrac{2(t-\gamma)}{\sigma})e^{-\left(\frac{(t-\gamma)^2}{\sigma^2}\right)}\end{array}\right)\right\} \qquad (14)$$

With boundary and initial conditions

$$\begin{cases} T(x,0) = 0; 0 \le x \le 2\times10^{-2} \\ T_t(x,0) = 0; 0 \le x \le 2\times10^{-2} \end{cases} \qquad (15)$$

Figs. 3 shows the numerical response of the wire as a function of time, for hyperbolic (14) and parabolic (τ = 0) equations, in Cartesian coordinates following x direction. In two wires the first made with gold and second with silver previously used. The wires with the same radius and the same length as before, for Fig. 3,a $\beta = 1,5\times10^{-14}s$, $\gamma = 3,5\times10^{-14}s$ and for the Fig. 3,b. $\beta' = 3\times10^{-14}s$, $\gamma' = 13\times10^{-14}s$
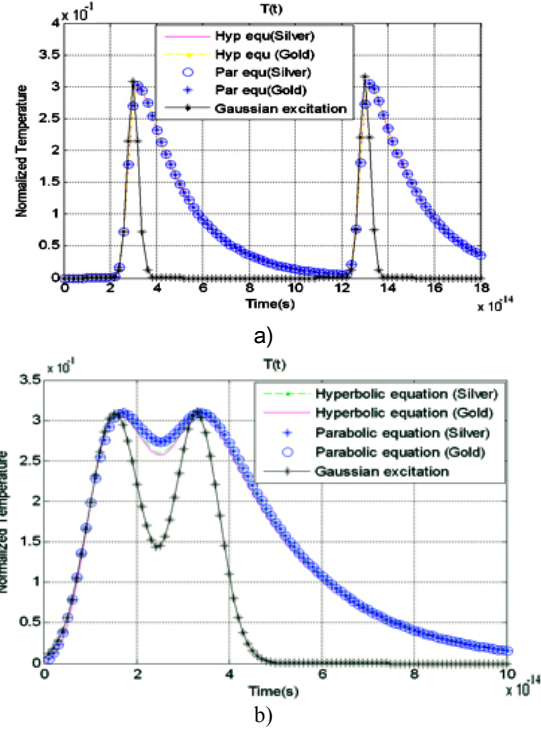

a)

b)

Fig. 3. a-b: Effect of the pulses in metallic wires with the interval time separating pulses, $t_i = \gamma - \beta$ in two extreme cases: a) $t_i = 10\times10^{-14}s$, $\sigma = 0.5\times10^{-14}s$
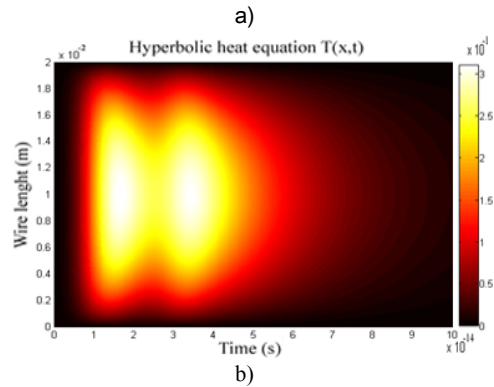b) $t_i = 2\times10^{-14}s$ $\sigma = 1\times10^{-14}s$.

a)

b)

Fig. 4. a-b. The evolution of the normalized amplitude of the temperature in Gold wire in the case of hyperbolic model, after two successive Gaussian excitations, separated by gap time for (a) $t_i = 10\times10^{-14}s$ and $t_i = 2\times10^{-14}s$ for (b).
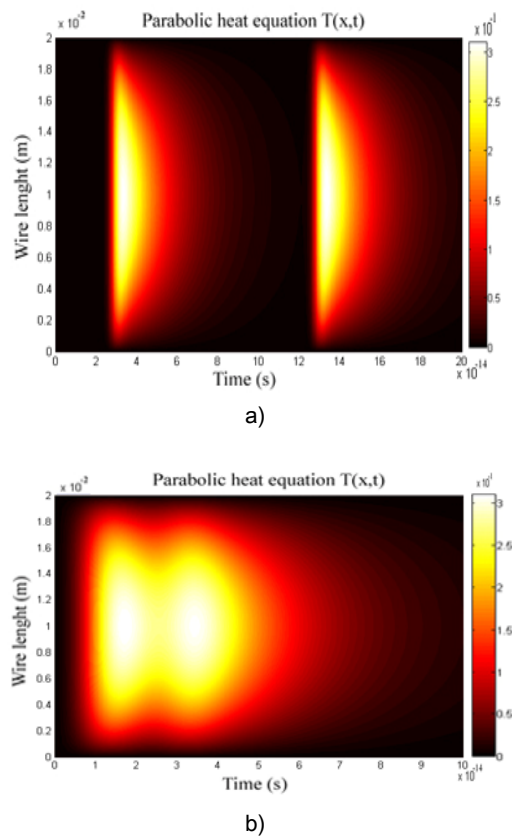
a)

b)

Fig. 5. a-b . Spatial and temporal evolution of the normalized amplitude of the temperature in Gold wire in the case of parabolic model, after two successive Gaussian pulses excitations, separated by an interval time for (a) $t_i = 10 \times 10^{-14} s$ *and* $t_i = 2 \times 10^{-14} s$ *for (b).*

5. Results and analysis

The numerical results given in Fig. 3-a, show that after the first and second peaks the responses of both gold and silver wires are similar to the both contribution hyperbolic and parabolic these results can be explained because the gap between two pluses is larger than the relaxation time. The Fig. 3-b shows that after the first excitation we have simultaneously an increase of temperature of the gold and silver wire which is legitimate for the both types of models.

However beyond the first peak localized at time $1,5 \times 10^{-14}$s, our system cools down in two different ways since the curve given by hyperbolic model shows faster cooling (3-b) of the gold wire than the silver one than those given by parabolic which are superimposed for the both types of wires gold and silver. Similarly for the second peak localized at time $3,5 \times 10^{-14}$s, the response given by gold wire in the hyperbolic case takes action quickly compared to that given by the silver wire in hyperbolic case. But in the parabolic case all the curves are similar. Beyond the second peak the four curves merge again and the system releases its heat in the same way for

both types of equations and for all types of wires. We can conclude that after the first excitation, if the system has sufficiently time to relax $\tau > 10 \times 10^{-14}$s, the both models have similar responses Figs. 3-a and 4-a and 5-a.

However in the case where the gap t_i time is inferior or closer to the relaxation time $\tau < 3 \times 10^{-14}$s, the system shows difference between both models Fig. 3-b Additionally we can see that the values of relaxation time of materials $\tau_{Silver} < \tau_{Gold}$ are significant to take in account in the case of high transitional regime. This typical behaviour is mainly due to the second order time derivative. As reported by (D. Jou, J. Casas-Văzquez, G. Lebon) [3]. We represent in Figs. 4-a and 5-a the evolution of the normalized temperature inside the gold wire as example to show the difference between the hyperbolic and parabolic models in the case when the gap time is bigger than the relaxation time. And the Figs. 4-b and 5-b represent the case when the gap time is quite smaller than the relaxation time of gold wire.

6. Conclusions

In this work the hyperbolic approach of the heat conduction was studied through metallic wires, which are excited by a modulated thermal source, under the framework of Maxwell-Cattaneo equation or hyperbolic heat equation. It has been shown that the thermal responses of all the wires with the same radius is similar however in the high frequency regime the normalized temperature of wires show oscillatory behaviour depending on their thermal diffusivity and relaxation time. These numerical results are supported by theory and experiments [18-19-22] which constitute an unmistakable character of hyperbolic behaviour. In high frequency regime $\omega \tau \Box 1$ or when the radius becomes smaller we conclude that is so important to take into account the thermal diffusivity of the wire. On the other hand in the low frequency regime $\omega \tau \Box 1$ or when the radius of the wire grows up all the wires converge to the same values. In addition we used numerical solution for studying the response of gold and silver wires under two successive Gaussian excitations. We concluded when the gap time is quite smaller than the relaxation time of gold or silver the response of the wire under the hyperbolic approach gives more accuracy than the parabolic approach. The present study might provide us with an approach in addition to the analytical and other numerical methods, and this approach might also has the potential in analysis of other transport properties such as to predict the thermal relaxation time of nanofluides.

References

[1] H. S. Carslaw, J. C. Jaeger, Conduction of Heat in Solids, Second ed, Oxford University Press, London, 1959.

[2] A. N. Tikhonov, A. A. Samarskii, Equations of Mathematical Physics, Dover Books on physics and Chemistry New York (1990).

[3] D. Jou, J. Casas-Văzquez, G. Lebon Extended irreversible Thermodynamics 3ed Edition, Springer-Verlag Berlin Heidelberg New York 41-42-229-230 (2001).

[4] D. Y. Tzou, J. Heat Transfer **117** (1) 8-16(1995).

[5] M. N Ozisik, D. Y. Tzou, ASME J. Heat Transfer **116**(3), 526 (1994).

[6] K. Mitra, S. Kumar, A. Vedavarz, M. K. Moallemi, ASME J. Heat Transfer **117**(3), 568 (1995).

[7] L. Wang, X. Zhou, X. Wei, Heat Conduction: Mathematical Models and An- alytical Solutions, Springer-Verlag, Berlin, Heidelberg, 2008.

[8] A. Graβmann, F. Peters, Heat Mass Transfer **35**(4), 568 (1999).

[9] H. Herwig, K. Beckert, Heat Mass Transfer **36**(5), 387 (2000).

[10] W. Kaminski, ASME J. Heat Transfer **112**(3), 555 (1990).

[11] W. Roetzel, N. Putra, K. SaritDas, Int. J. Therm. Sci. **42**(6), 541 (2003).

[12] L. Onsager Phys. Rev. **37**, 405 (1931) and **38**, 2265 (1931).

[13] C. Cattaneo, Atti Seminario Univ. Modena **3**, 33 (1948).

[14] C. Eckart, Phys. Rev. **58** 267, 269, 919 (1940).

[15] A. F. Khadrawi, M. A. Al-Nimr, M. Hammad, International Journal of Thermophysics, **23**, 581 (2002).

[16] A. Vedavaraz, S. Kumar, M. K. Moallemi, ASME J. Heat Transfer **116**(1), 221 (1994).

[17] P. J. Antaki, int. J. Heat Mass Transfer **40**(13), 3247 (1997).

[18] J. Ordóñez-Miranda, J. J. Alvarado-Gil, International journal of Engineering Science **50**, 101 (2012).

[19] A. Mandelis, Diffusion Waves Fields: Mathematical Methods and Green Functions, Springer, New York, (2001).

[20] W. P. Leung, A. C. Tam, J. Appl. Phys. **56**(1), 153 (1984).

[21] G. Sewell, The Numerical Solution of Ordinary and Partial Differential Equations University of Texas-El Paso, A.JOHN WILEY & SONS, 2005.

[22] J. Ordóñez-Miranda, J. J. Alvarado-Gil, International journal of thermal sciences **48**, 2053 (2009).

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