Initial stage of acceleration process of electrons and protons in laser and maser beams

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Concepts drawn from dynamic relativistic equations are used to gain insights into the initial stage of acceleration process of an electron in laser and a proton in maser beams with additional constant co-axial magnetic field. It has been shown that the particle is subjected to the action of the electric field with delayed phase due to the relativistic effect. The delay grows with the particle's velocity increase. It is shown how the delay at a synchronic conditions ensures the continuous acceleration of the particle. The illustrations explain the difference in the acceleration process of an electron and a proton at synchronic conditions.

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1. Introduction

Laser based acceleration are capable of producing high energy charged particles in much shorter distances than conventional accelerators due to the large electric fields associated with a laser beam. Many experiments and theoretical works have been shown the possibility of particles acceleration as a result of their interaction with the laser or maser beams of different parameters [1-8]. Recent years have seen important achievements in the development of laser particle accelerators [4]. The larger laser power the higher energy of the particle can be achieved. Now there are accessible lasers showing radiation power density of the level 10^{22} W/cm^2, which corresponds to the amplitude of electric field intensity of the order 10^{14} V/m [5]. The accelerated particles to the high energies are of interest in many applications, among others in fast ignition in inertial confinement fusion [9], proton cancer therapy, proton imaging and ion beam radiography [10].

It is firmly established existence the optimum value of the constant magnetic field at which the particle attains the maximum energy because of synchronic condition [1, 11-22]. The main purpose of this paper is to gain insights in a theoretical way into the basic effects occurring at the beginning of the particle acceleration in the laser or maser beams propagating in a vacuum**,** with an additionally applied external static co-axial magnetic field maintaining the synchronic condition. On the basis of the equation of motion of the charged particles in the laser beam and additionally applied magnetic field, the evolution of the acceleration process at the initial stage of the synchronic acceleration is shown. The results obtained for the case of the plain circularly polarized wave are presented in a graphical form.

2. Theory of acceleration of a charged particle in a laser beam and co-axial static magnetic field

We assume that the laser produces continuous, monochromatic, coherent, circularly polarized plane wave propagating in a vacuum and in an additional static co-axial magnetic field. The aim is to find the impact of this combined electromagnetic field on the trajectory and on the kinetic energy of a charged particle in the lossless conditions. The dynamical relativistic equation and the continuous equation for a charged particle in the electromagnetic and the static magnetic fields defining the trajectory and normalized energy γ in the lossless condition has the following form

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\left[\vec{\mathrm{v}} \times \left(\vec{B} + \vec{B}_z\right)\right]$$
$$\frac{d\gamma}{dt} = \frac{q}{m_0 c^2}\vec{\mathrm{v}} \cdot \vec{E} \tag{1a,b}$$

where $\vec{p}$, $\vec{\mathrm{v}}$, q and m_0 are the momentum, velocity, electric charge and the rest mass of the particle, $\vec{E}$ is the electric field intensity and $\vec{B}$ is the magnetic field induction of the electromagnetic wave, c is the velocity of the electromagnetic wave and B_z is the external static magnetic induction in the direction along the z coordinate (the laser beam axis) and

$$\gamma = \left(1-\beta^2\right)^{-1/2}, \vec{\beta} = \frac{\vec{v}}{c}, \quad \vec{p} = \gamma \cdot m_0 c\vec{\beta},$$

$$\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2, \quad \beta_{x,z,y} = \frac{v_{x,z,y}}{c}$$

We assume the clockwise circularly polarized electromagnetic wave is propagating in the *z* direction and a static co-axial magnetic field is applied. The electric field intensity, magnetic induction and the particle momentum vectors are respectably as follows

$$\vec{E} = \left[E_x, E_y, 0\right], \quad \vec{B} = \left[-B_x, B_y, B_z\right]$$

$$\vec{p} = m_0\gamma v_x\vec{i} + m_0\gamma v_y\vec{j} + m_0\gamma v_z\vec{k}$$

$$\frac{d\vec{p}}{dt} = \vec{i}m_0c\frac{d(\gamma\beta_x)}{dt} + \vec{j}m_0c\frac{d(\gamma\beta_y)}{dt} + \vec{k}m_0c\frac{d(\gamma\beta_z)}{dt}$$

with components

$$E_x = E\sin\phi, \quad E_y = E\cos\phi$$

$$B_x = \frac{E}{c}\cos\phi, \quad B_y = \frac{E}{c}\sin\phi, \quad B_z = \pm aB$$

with

$$\phi = \omega\left[t - \frac{z(t)}{c}\right], \quad B = \frac{E}{c} \tag{2}$$

where ϕ is the phase and ω is the angular frequency of the electromagnetic wave.

The impact of the static magnetic field on the particle is included through the parameter $a = B_z/B$ for the constant magnetic field B_z directed along the *z* coordinate.

We can use the differential equations derived from Eqns (1a,b) describing dynamics of a charged particle in the relativistic case which have the following form:

$$\begin{aligned}
\frac{d(\gamma\beta_x)}{dt} &= -(1-\beta_z)f\sin\phi - fa\beta_y,\\
\frac{d(\gamma\beta_y)}{dt} &= -(1-\beta_z)f\cos\phi + fa\beta_x,\\
\frac{d(\gamma\beta_z)}{dt} &= -\beta_x f\sin\phi - f\beta_y\cos\phi,\\
\frac{d\gamma}{dt} &= -\beta_x f\sin\phi - f\beta_y\cos\phi
\end{aligned} \tag{3}$$

where

$$f = \frac{-qB}{m_0} \tag{4}$$

The result of solving Eqns (3) are expressions for trajectory coordinates for a charged particle initially at rest. The solving procedure of these equations are presented in [6, 11]. As a result we have obtained rather simple expressions for the Cartesian trajectory coordinates in the form of

$$\begin{aligned}
x &= \eta\sin\phi + (\eta-\xi)\sin\chi\phi,\\
y &= \eta\cos\phi - (\eta-\xi)\cos\chi\phi - \xi,\\
z &= \frac{c\vartheta^2}{\omega}\left[\phi - \frac{1}{1+\chi}\sin(1+\chi)\phi\right]
\end{aligned} \tag{5}$$

where $\eta, \chi, \xi, \vartheta$ are the constant parameters defined as

$$\eta = \frac{\chi\xi}{1+\chi}, \quad \chi = -\frac{B_z}{B_c}, \quad \xi = \frac{cB_c}{\omega B_z}, \quad \vartheta = \frac{-B}{(1+\chi)B_c} \tag{6}$$

The components of a particle velocity have a form

$$\begin{aligned}
v_x &= \frac{c\vartheta}{\gamma}(\cos\phi - \cos\chi\phi),\\
v_y &= -\frac{c\vartheta}{\gamma}(\sin\phi + \sin\chi\phi),\\
v_z &= \frac{c\vartheta^2}{\gamma}\left[1-\cos(1+\chi)\phi\right]
\end{aligned} \tag{7}$$

All solutions shown here are in the form provided the starting velocities of the particles are zero. One may found in our previous papers [6, 12, 22] the solutions with an arbitrary initial velocity. From Eqn 7, we can obtain the total velocity of the particle. This enables to calculate the relativistic kinetic energy using the following relativistic formula

$$E_k = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \tag{8}$$

The action on a particle of combined electric and magnetic fields in case of circular polarization is rather complicated to describe. The $\vec{E}$ vector of the electric field of the laser radiation rotates around the *z* axis. As a result of this, an acceleration process of the particle occurs. The particle starts moving along the direction of the circularly rotating $\vec{E}$ vector. From this moment on the particle will start to act the bending effect (without the acceleration effect) connected with the Lorentz force by the magnetic induction vector $\vec{B}$ of the laser field. The $\vec{B}$ vector will start to bend the particle trajectory in the co-axial direction (the *z* axis). This will create the v_z component of the particle's velocity and due to it the Lorentz force results the phenomena of rotation of the particle around the *z* axis. While the external static magnetic field B_z will push the particle in circular rotation. The particle will move along the helix winding around the z axis with rising the radius. The result of these combined actions is the helical trajectory with superimposed wiggles. The wiggles disappear at the synchronic conditions, which occurs if the field B_z will approach the so called cyclotron magnetic field induction $B_c = m\omega_0/q$.

At this moment, the important effect should be considered to clarify the further course of the acceleration. This is connected with the particle's velocity component v_z. Which means the particle while moving along the helix also is subjected to the shift with the velocity v_z in the same direction as the propagation of the laser wave. When the component v_z of velocity approaches the relativistic region, the electric field of the laser starts to act on the particle with a reduced frequency. The effect of this is connected with the relativistic shift of the laser frequency according to the equation $\omega_0(1-\beta_Z)$ in the particles frame of reference. The difference between the rotation rate of the $\vec{E}$ vector and in spite of the increasing relativistic mass the rotation rate of a particle around the axis begins to be reduced. As a result the accelerating force of the laser will act on the particle longer during a cycle. This effect is advantageous for the acceleration process since it results in an elongation of the time the electric field acts on the particle in the direction of its velocity causing the increased energy gain. Provided the static magnetic field approaches the cyclotron magnetic field intensity, eventually the synchronic effect is established. This effect will be clearly shown in the attached illustrations.

We can find in the literature the similar explanation of this effect however suggesting that to achieve the synchronic condition the electron should be injected into the laser beam with rather a significant velocity. A particle does not rotate at such a speed as the electric vector does in the laboratory frame of reference. Due to the relativistic effect, actually rotation rate of the vector $\vec{E}$ begins to be slower in the particles frame of reference. Our paper is mainly devoted for analyzing this basic effect.

Eqns (5, 7 and 8) completely describe all the components of a position and velocity of the accelerated particle, as well as its total velocity and the kinetic energy under the interaction of the laser and the static magnetic field. The results in a graphic form have been obtained using the original above derived analytical expressions. The authors of this paper were not able to find the similar types of analytical expressions in the literature, which could immediately confirm correctness of the solutions presented above. It has been decided to verify the above expressions by numerical integration of the output of differential equations (3). For this purpose, in order to solve Eqns (3) the well known Runge-Kutta method has been chosen. It should be emphasized that due to the lack of appropriate analytical expressions the vast majority, in particular the recent theoretical publications are based on the use of this numerical method. It is especially useful for more complicated laser beams than the planar wave. The results obtained by using the both numerical and analytical methods proved to be identical. It can be concluded that using the analytical formulas one can obtain the reliable results. The primary reason for selection of the analytical models to study the phenomena is the possibility of getting more transparent results showing the influence of various parameters on the course of the acceleration phenomena.

The electrical field intensities E = 5x10^{10} and 10^{12} V/m of the electromagnetic field have been used in the calculations. The quantitative illustrations of the calculation results presented in a graphical form enable the discussion of the influence of many parameters on the acceleration process of these particles. Generally speaking, the increase in the laser beam intensity results in the increase of the particle's energy. However, the increase of the external magnetic field causes shrinking of the helical trajectories and the rise of laser beam intensity has the opposite action. If a sufficiently high magnetic field is applied it enables to keep the particle inside the laser beam. However, it is reasonable the application the strictly defined value of the magnetic field which ensures the synchronic effect. Limits in the achievable energy of the accelerated particles arise from the limits in the available at present the laser beam intensity, the laser beam diameters, the pulse duration and the static magnetic field intensity.

The energy gained by the charged particle due to the interaction with the laser beam and the static magnetic field has been calculated using the relativistic formula (8). In order to maintain the acceleration, one should be sure that during the circulation the particle in its helical motion should not escape from the laser beam. If the acceleration process is to be continued, especially for protons the magnetic field intensity at synchronic condition should be extremely high. If such a high field is not available, the particles energy will oscillate and leave the beam with the energy which may be sufficient for some purposes. The intensity of electric field E has an impact on the duration at which the charged particle gains the expected energy. The larger the intensity the less time is required to obtain the defined level of the kinetic energy. For most choices of initial parameters, the energy gained by a particle is an oscillatory function of time. The particle alternately gains and losses energy. At the synchronic conditions the oscillations of the velocity disappear and the energy starts to rise continuously during the acceleration process.

The acceleration process of the charged particles is closely connected with the distance that the particle has to travel in order to get a desired level of the kinetic energy.

Since the component v_z of the electron velocity through almost the whole acceleration process is close to the light velocity *c* the distance *z*(*t*) was found to be the simple product *ct*. It is not the case for a proton, since its velocity may remain through the significant part of the acceleration time rather far away from the light velocity In the calculations the starting value of the v_z component as well as the remaining components of the velocity have been chosen to be zero in all presented cases.

3. Results of simulation of the acceleration process of electron and proton

In the acceleration process the most important item to discuss is the process of the kinetic energy gaining by the charged particles. By performing the selected studies using the above presented analytical equations (5, 7, 8) it have been possible to establish the course of the particles motion and its energy gaining under the action of the laser radiation field and the additionally applied constant co-axial magnetic field. The results are presented in the graphical form. The special attention will be paid for the initial period of acceleration. The attached illustrations show some features concerning the electron and proton acceleration process resulted by the action of the mentioned fields.

3.1 Acceleration of an electron

Fig. 1 presents an electron acceleration process at synchronic conditions. In order to show the results of numerical simulation, we decided to use two values of the laser intensity laying within the achievable to day boundaries. We use the field intensity amplitude *E* expressed in *V/m* units. In Figs 1 and 3 $E = 5\text{x}10^{10}$ *V/m* and in Figs 2 the field intensity $E = 10^{12}$ *V/m* was used. Since in the publications on this subject the laser power density is rather used, the corresponding to the amplitude $E = 10^{12}$ *V/m* the laser power density lays within the magnitudes achievable to day (see Introduction). In Fig. 1 the 10 μm wavelength laser action on the electron, initially at rest, lasts only about 0.87 *ps*. This time corresponds to the 26 periods of the radiation wave in the laboratory frame of reference. A considerable reduction in the difference between the wave velocity and the electron velocity appears. Due to the acceleration an electron travels at a velocity close to the speed of light. In spite of the rather short time of acceleration and moderate laser intensity the electron velocity along the laser beam axis enters into the relativistic region (Fig. 1a) and the electron energy enters in the MeV region (Fig. 1b). This can be explained by continuous acceleration due to the electric field.

The rotation frequency of the $\vec{E}$ vector in the particles frame of reference can be found from the relativistic relation $\omega = \omega_0(1-\beta_z)$, where ω_0 is the frequency in the laboratory frame of reference. Fig. 1d shows the variation in time of the component E_x in the electron frame of reference. E_x component oscillation corresponds to the electric field vector $\vec{E}$ rotation in the electron frame of reference. While the $\vec{E}$ vector of the laser wave has rotated 26 times in the laboratory frame of reference, the $\vec{E}$ vector in the electron frame of reference during the same time performs only about 4 rotations. Comparison of Fig. 1c with Fig. 1d indicates that the electric field component E_x oscillates (the $\vec{E}$ vector revolves) synchronicly with the rotation of an electron around the beam axis. Due to this effect the number of rotations of the electric field vector $\vec{E}$ is found to be equal to the number of rotations of the electron around the beam axis. The above discussion explains why the electric field acts onto the electron continuously despite the rotation of $\vec{E}$. It should be reminded that this effect is achievable if the static magnetic field is near its synchronic value, which corresponds with a so called cyclotron magnetic field intensity $B_c = m\omega_0/q$. However, the synchronic static magnetic field may be reduced if the particle is pre-accelerated.

Due to the Lorentz force the electron starts to shift along the laser beam axis and the angle between the direction of the resultant electron velocity vector and the (*x, y*) plane rises (Fig. 1e). The rate of the angle rising drops with the acceleration time. This impacts on the energy gaining process by a particle (Eqn 1b). Due to the rather short time of acceleration (0.87 ps) and the strong static co-axial magnetic field the helical radius is very small as well as the distance the electron penetrates in the *z* direction (Figs 1c and 1f). Note that the v_z component is larger than the v_x and v_y components (compare Figs 1 a and 1g). The opposite is valid at the very beginning of the acceleration process. The 3D trajectory of an electron is depicted in Fig. 1h and its projection onto the (*x, y*) plane is shown in Fig. 1c.

In order to show the acceleration of electron to the energy of the GeV range, in Fig. 2 we illustrate the results obtained at the same the static magnetic field as in Fig. 1 and much stronger the laser intensity. Now the duration of the laser action on the electron is longer than in the case

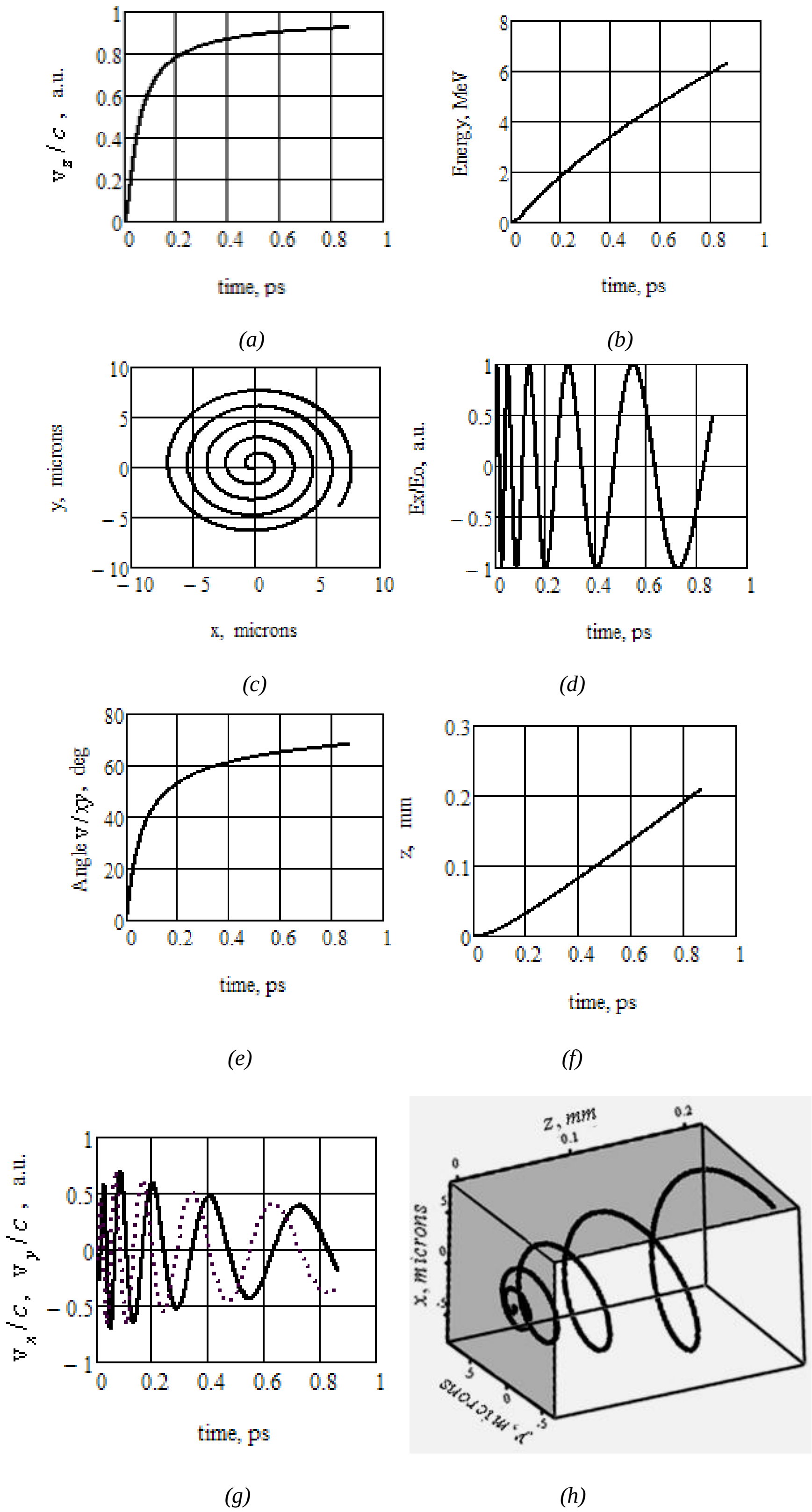

Fig. 1. *Acceleration process of* ***electron*** *at synchronic conditions by laser beam of wavelength* λ = 10 μm *during* 26 *cycles of the wave* (0.87 *ps in the laboratory frame of reference*). *Electric field intensity* E = 5x10[10] *V/m*, B_z = -1071 *T and* v_0 = 0. *Shown are variation in time of a*) *reduced* v_z *component b*) *the kinetic energy c*) *the trajectory projection onto the* (*x*, *y*) *plane*, *d*) *reduced* E_x *component*, *e*) *inclination of the velocity vector to the x*, *y plane*, *f*) *penetration along the axial direction*, *g*) *reduced* v_x (*solid line*) *and* v_y (*doted line*) *components*, *h*) 3*D trajectory.*

shown in Fig. 1. Since the difference between velocities of the laser wave and the electron is significantly reduced due to the relativistic effect, about four laser radiation cycles act on the electron in its frame of reference (Figs 2a and 2b). During the same time in the laser frame of reference the electric vector rotates 5000 times. This means that in spite of increasing relativistic mass (Fig. 2d) the electron is able to follow the rotation of the electric vector and to gain energy at a high rate. As a result of this, the electron energy enters into the GeV region (Fig. 2c). This can be explained by acceleration due to the strong electric field acting continuously on the electron. The inclination angle of the electron velocity vector with respect to the (x, y) plane achieves nearly 90 degrees (not shown in Fig. 2). Due to a strong static co-axial magnetic field the helical radius is very small, however the distance the electron penetrates in the z direction was found to be much extended with respect to that shown in Fig. 1f (it may be found from ct product).

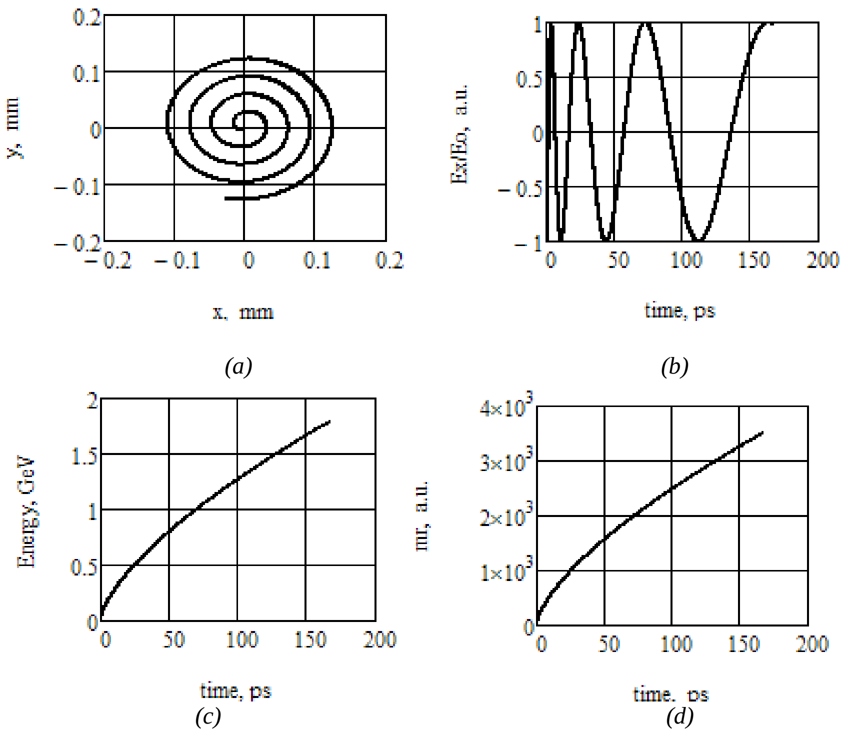

*Fig. 2. Acceleration process of **electron** at synchronic conditions by laser beam of wavelength* λ = 10 μm *during* 5000 *cycles of the wave* (166 *ps*) *which corresponds to about* 3.8 *laser radiation cycles in the electron frame of reference. Electric field intensity* $E = 10^{12}$ *V/m*, B_z = -1071 *T and* v_{0z} = 0. *Shown are variations in time of a) the trajectory projection onto the* (x, y) *plane, b) normalized* E_x *component acting onto the electron, c) the kinetic energy, d) relativistic mass.*

3.2 Acceleration of a proton

In order to find the behavior of a heavier particle than an electron at the initial stage of acceleration, the study of a proton acceleration process at synchronic conditions is performed (Fig. 3). Instead of a laser, the 1 *cm* wavelength maser has been selected for the simulation. The reason for using maser is connected with the intensity of the magnetic field which has to be used to achieve the synchronic condition. When a proton is accelerated by a laser, the intensity of the magnetic field at synchronic condition should be far away from achievable intensities to day, while for a 1 *cm* maser it is near to the upper limits of achievable magnetic field. However, it should be mentioned at this point that the synchronic magnetic field can be significantly reduced by using pre-accelerated particles.

Now the maser action on the proton, initially at rest, lasts considerably longer (about 67 *ps*) than the laser action on the electron shown in Fig. 1. This time corresponds to the two periods of the radiation wave in the laboratory frame of reference. Now a proton travels at a velocity much lower than the maser beam wave velocity (Fig. 3a). As can be seen in Fig. 3b and 3c, the rotation of the proton around the beam axis only slightly remains behind the rotation of the electric vector. The rate of oscillation of the electric field E_x (rotation of the vector $\vec{E}$) in the proton frame of reference (Fig. 3c) is only slightly smaller than the rate of rotation of this vector in the laboratory frame of reference. The velocity components v_x and v_y exceed values of 0.5c at the maximum (Fig. 3d) and the relativistic mass slightly rises (Fig. 3f). At the end of the process the proton gains the kinetic energy of 400 *MeV* (Fig. 3g) and penetrates 2.5 *mm* along the z axis (Fig. 3h).

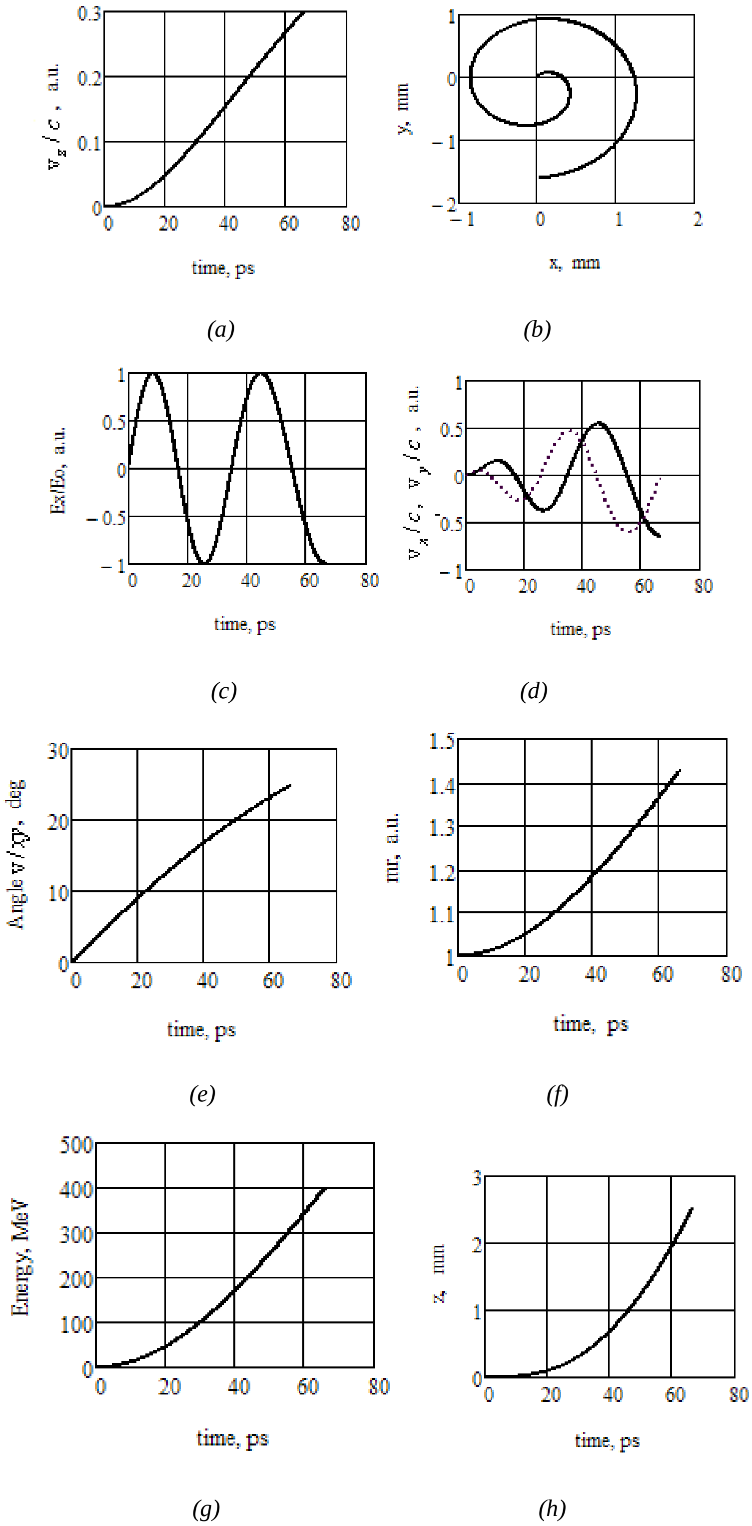

Fig. 3 Trajectory, velocity, energy and relative mass of a ***proton*** *at synchronic conditions accelerated by maser beam of wavelength* λ = 1 *cm during* 2 *cycles of the wave* (67 *ps*) *in the laboratory frame of reference. Electric intensity* $E = 5\text{x}10^{10}$ *V/m,* B_z = 1996 *T and* v_{0z} = 0. *Shown are variations in time of a) reduced* v_z / c *velocity component, b) the trajectory projection onto the* (*x, y*) *plane, c) normalized* E_x *component acting onto the proton, d)* v_x / c (*solid line*) *and* v_y / c *, e) inclination angle of* $\vec{\mathrm{V}}$ *with respect to the* (*x, y*) *plane, f) the relativistic mass increase g) the kinetic energy, h) the penetration distance along co-axial direction.*

4. Discussion

A particle (electron or proton) initially at rest is subjected to acceleration in the laser or maser radiation beam with additionally applied a constant co-axial magnetic field. The magnitude of it is selected to maintain the process of synchronic acceleration. A particle is accelerated in the direction of the electric field vector $\vec{E}$. In the case of circular polarization the value of the electric field vector of the radiation is constant and rotates around the *z* axis. In order to find the direction at which a particle moves we assume at the starting moment that the electric vector $\vec{E}$ of the radiation wave is directed along the *x* axis. Then the magnetic field *B* of the electromagnetic wave is created along the *y* axis. Between the particle velocity v_x and $\vec{B}$ at the starting moment is the right angle. Thus the Lorentz force is directed along the laser axis (*z* axis) causing the bending effect of the particles trajectory. As a result the particle's velocity has the components v_x and v_z. The vector $\vec{B}$ and component v_z are perpendicular to each other, hence the Lorentz force results not only bending of the particle to the *z* axis but also causes its rotation around the *z* axis. As a result the particle makes a helical trajectory.

It remains to discuss the role of the constant axial magnetic field B_z. The Lorentz force of B_z is coupled only with the component v_{xy} and will result the particle rotating around the *z* axis. It should be expected that at the beginning of acceleration the magnitude of magnetic field B_z has a decisive impact on the rotation of the particle around the *z* axis, since it is coupled with v_{xy}, which is much greater than v_z. If the process continues, the roles change and the field *B* takes the role of causing the rhythm of the particle rotation and the wave oscillations to be coincident. The appropriate magnitude (resonant) of the constant magnetic field B_z and the relativistic effect $(1 - \beta_z)\omega_0$, result the rotation periods of a particle and the $\vec{E}$ vector to be comparable. Then it is established the continuous resonant particle acceleration. In the case of the heavier particle (proton) a longer acceleration process is necessary in order to achieve the moment of equalization of the particle rotation frequency with that of the $\vec{E}$ vector.

5. Conclusions

An electron and a proton motion and theirs kinetic energies are analyzed within the initial stage of acceleration in a laser or maser beams. In this paper the acceleration process at solely synchronic condition is analyzed. It was found that to achieve this condition the co-axial magnetic field of a proper intensity should be applied. In order to quantitatively analyze the problem some illustrations are presented. An electron is subjected to acceleration in a 10 μm wavelength laser beam and a proton accelerated in a 1 *cm* wavelength maser beam. In the both cases an additional constant co-axial magnetic field is applied. During the acceleration process the particles move along the laser beam axis making a helical trajectory. The particle is subjected to the action of the electric field with delayed phase due to the relativistic effect. As it follows from the relativistic relation $\omega = \omega_0(1-\beta_z)$, where ω_0 is the frequency in the laboratory frame of reference, the delay grows with the particle's velocity increase. Due to the phase delay, during the same time, the number of the laser wave cycles in the particle's frame of reference begins to be reduced compared with this number in the laboratory frame of reference. It is shown that the phase delay at a synchronic conditions ensures the continuous acceleration of the particle. The attached figures show that the synchronic condition is achieved almost at the very beginning of the acceleration process, provided that a proper magnitude (or near to it) of the static co-axial magnetic field is applied. The presented illustrations show the difference in the acceleration process of an electron and a proton. Due to the much larger rest mass, a proton gains the kinetic energy at a slower rate than an electron. However, because used for a proton the lower frequency electric field *E* of a maser and higher intensity magnetic field B_z, at a synchronic condition the rotation of this particle, like an electron, remains in a coincidence with the rotation of the electric vector $\vec{E}$, almost at a very beginning of the process.

The energy gain rate of a particle depends on the $\vec{\mathrm{v}} \cdot \vec{E}$ product. This means that not only the values of velocity and electric field are important but also the inclination of the vector $\vec{\mathrm{v}}$ with respect to the (*x, y*) plane plays an important role [1]. The inclination angles for electron and proton are shown in the above presented figures. Rising the angles of the inclination as well as the velocity of the accelerated particles during the process of acceleration results in the decrease of the rate the kinetic energy is gained. Since the component v_z of the particles velocity does not play any role in the energy gaining rate, the v_x and v_y components are essential in this process. The angle between the vectors $\vec{E}$ and $\vec{\mathrm{v}}_{xy}$ plays the important role in the energy gaining process. Since this angle rises during the process, the energy gaining is gradually decreasing.

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