Jiles-Atherton model in fitting hysteresis curves of magnetic materials

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This paper presents a method of fitting the magnetic hysteresis curves using a calculating algorithm within a computer programme. The calculating programme uses the equations of the Jiles-Atherton model to simulate the M(H) magnetization curve and aims at finding the values of the "a, α , k, c" parameters which are part of the J-A model once the regression curve for a major magnetization curve obtained experimentally is found. The constructed algorithm has calculating structures which modify, step by step, in plus or in minus (according to necessity) the values of these parameters. The parameters' modification takes place in such a way that the average square deviation between the points of the two graphs, also experimentally simulated, should decrease gradually and, in the end, it should be found below the preestablished percentage value in comparison to the previous value. The procedure for minimising the average square deviation has as a result the fact that, in the end, the simulated curve represents the regression curve. The calculating programme also has in view a second way of calculating the values of the "a, α , k, c" parameters by using the already found regression curve. This paper also presents the results obtained for the values of the J-A model parameters by fitting some cobalt ferrites obtained through sintering at different temperatures.

(Received March 1, 2011; accepted May 25, 2011)

Keywords: Computer programme, Magnetic hysteresis modeling, Magnetic materials, Parameters identification, Cobalt ferrites

1. Introduction

In this paper there is presented a calculation algorithm with two complementary modes of finding the regression curve in the case of major experimental magnetization curves. For this purpose the equations of the Jiles-Atherton model [1] are used. With the help of these equations the following simulated magnetization curves are traced: the M_{an}(H) anhysteretic curve, the M(H) first magnetization curve with H in the $[0, + H_{max}]$ interval and the major magnetization curve with H in the $[-H_{max}, +H_{max}]$ interval. The major curve is given by two equations, as it will be seen in section II, according to the way in which H increases or decreases in the mentioned interval. H_{max} is the maximum value of the magnetic field which magnetizes a ferro or ferrimagnetic sample given during the measuring of the magnetization curve. The equations of the J-A model present four parameters: a, α, k, c ". The consequence of finding the regression curve is finding the values of these parameters. Of course, attempts have been made to find the regression curve for a given experimental magnetization curve and to calculate the coefficients of the J-A model using different programming methods or different mathematical adjustments of the used equations to simplify the achievement of this objective [2]-[10]. Taking into consideration that one of the model's equations is a differential equation which does not have an analytical solution, the attempts of fitting experimental magnetization curves were based on numerical methods. Unlike the already taken actions, [5]-[8], what we suggest is a calculating algorithm with which the values of the J-A model's parameters are determined in two modes. The algorithm made to find out the regression curve is based on calculating structures which repeat in time and which gradually draws nearer the simulated major magnetization curve to the experimental one. This step-by-step approaching is based on a simple principle: the "a, α , k, c" coefficients are gradually modified in plus or in minus so that in the end the "distance" between the graphs should be minimum. Within the calculating programme the initial values of these parameters are aleatory. The average square deviation could be considered a measure of the distance between the simulated curve and the experimental one and is given by the equation:

$$S = \sqrt{\frac{\sum_{i=1}^{N} \left(M_i - M_{\exp i}\right)^2}{N}}$$
(1)

where N is the number of points on the experimental major magnetization curve, M_i represents the value of magnetization calculated by the computer programme for a field identical or very close to the experimental $H_{exp,i}$ and $M_{exp,i}$ is the value of magnetization experimentally determined for the $H_{exp,i}$ magnetic field. The calculating programme realises a levelled variation of the magnetic field and the calculated values of H do not exactly overlap the $H_{exp,i}$ experimental values but some of them could be "as close as possible". In other words, a subseries of the values for H has the terms very close to the series of the $H_{exp,i}$, i=1...N experimental values. The way in which this is done is detailed in section 3. The minimizing of S leads to obtaining a hysteresis loop having approximately the same shape and area as the experimental one, as it will be seen in section 4. The simulated curve obtained through minimizing the average square deviation is called regression curve and has the property that the sum of the squares of the distances from its points to the experimental ones is minimum, with the mention that these distances are measured along the ordinate. Thus the distance between two points on the two graphs, corresponding to the same magnetic field, is the difference of the magnetizations which appears in equation 1: M_i-M_{exp,i}. The value of the arithmetical average has a quality similar to the regression curve, i.e. the sum of the squares of the deviations from the value of the arithmetical average of the numbers which are introduced in its calculation is minimum; any other sum of the squares of the deviations of these numbers from a value which is different from the arithmetical average is higher. Therefore, the regression curve should be seen as an average curve which goes through the experimental points. The values of the J-A model's "a, a, k, c" coefficients for three cobalt ferrites obtained through sintering at different temperatures will also be shown in section 4.

Knowing the mechanisms which are the basis of magnetization processes has risen a particular interest especially now when the field applications are widely spread. There is not a unique theory to thoroughly describe the entire range of hysteresis curves' shapes for ferromagnetic materials. The J-A Model describes the sigmoid shapes of these loops quite well. The aim of this model is to find the dependence of magnetization according to field H and which, when graphically represented, should overlap as well as possible the experimental hysteresis curves. This phenomenological model is a construction which is based on a series of hypotheses concerning the manifestation of magnetism at the micro level of matter and we are going to tackle these hypotheses further on. The first hypothesis considers the fact that each domains' magnetic moment is placed in an effective magnetic field which increases linearly with the magnetization of the sample. $H_e=H+\alpha M$, where α is a coefficient which can be interpreted as a measure of the coupling between the adjoining magnetic domains. The second hypothesis refers to the fact that the material has a series of anisotropies caused by impurities which can be in most of the cases nonmagnetic, faults, inhomogenities of mechanical tension, etc.- which are called pinning sites. Another hypothesis is that in the material there is a homogeneous spatial distribution of the pinning sites. It is known that the magnetization of the substance takes place through two mechanisms: the motion of the domain walls and the mechanism of magnetic moments' rotation. The ", pinning site" coining comes from the fact that the domain walls, during their motion under the action of the exterior magnetic field, remain temporary impeded in these zones and when the field increases with a minimum quantity, they will irreversibly move beyond the above mentioned zone. The pinning of these zones is made by losing energy. The model suggested by Jiles and Atherton [1] draws a balance of the energies involved in the magnetization process, their calculation on a certain basis, and, in the

end, an equation involving the magnetization and the field is obtained. This is a differential equation which can be directly integrated only if certain approximations are made, or in specific conditions. However, the equation can be numerically integrated. The dependence of the magnetization on the applied field, i.e. the classical hysteresis loop, can be represented with the help of the above mentioned equation.

2. The equations of the J-A model the calculation of the "a, α, k, c" parameters

The magnetization calculated according to the J-A model has two components, an irreversible one and a reversible one corresponding to the irreversible or reversible phenomena which take place within the magnetic material during the magnetization. The domain walls moving over the pinning sites represents irreversible movings and their movement between two successive pinning sites or the rotations of the magnetic moments which lead to the decrease of the magnetization at field inverting after achieving the saturation represent reversible phenomena. Thus: $M=M_{rev}+M_{irr}$.

The irreversible component of the magnetization is given by the following differential equation:

$$dM_{irr} = \frac{M_{an} - M_{irr}}{k\delta - \alpha (M_{an} - M_{irr})} dH , \qquad (2)$$

where M_{an} is the anhysteretic magnetization, k is a coefficient which appears as a factor of proportionality between the wasted energy when a domain wall crosses a pinning site and the variation of the sample's magnetization as an effect of this movement, and α is the already mentioned coefficient in section I. Within this equation also, δ =1 if dH/dt>0 and δ = -1 if dH/dt<0.

The anhysteretic magnetization is the magnetization of an ideal ferromagnet which does not have pinning sites and, as a result of this, its magnetization curve does not have a hysteresis, the magnetization being made only through the rotating mechanism of the magnetic moments. Langevin's function specific to magnetizing paramagnetic substances is used within the J-A model for anhysteretic magnetization, with the mention that the effective field within the He sample replaces the H magnetic field exterior to the sample. The equation of the anhysteretic magnetization is:

$$M_{an} = M_s \left(\coth \frac{H_e}{a} - \frac{a}{H_e} \right) , \qquad (3)$$

where $a=K_B \cdot T/\mu_0 \cdot m$ is another parameter within the model which is to be found as a result of the fitting process and *m* is the average magnetic moment of the volume unity corresponding to a magnetic domain. The J-A model has in view the fact that the value of the reversible component of the magnetization is a certain "c" percentage of the difference of the irreversible magnetization and the anhysteretic one:

$$M_{rev} = c \left(M_{an} - M_{irr} \right) \tag{4}$$

Finally, the value of the total magnetization is given by the following equation:

$$M = cM_{an} + (1 - c)M_{irr}$$
⁽⁵⁾

Jiles and Thoelke [2,3] calculate the "a, α , k, c" coefficients which appear in (2), (3), (4) or (5) equations by calculating some differential magnetic susceptibilities at different points of the anhysteretic magnetization curves, major or first magnetization.

These differential susceptibilities (dM/dH or dM_{an}/dH) are the derivates of the magnetization in relation to the H field at different points of the magnetization curve or at different points of the anhysteretic curve. The initial, remanent, coercive and saturation anhysteretic differential susceptibilities will be marked by χ'_{an} , χ'_{anmax} , χ'_{anc} , χ'_{anmax} . The differential susceptibilies on the major magnetization curve corresponding to the already mentioned areas will also appear in these equations: χ'_{in} , χ'_{max} , χ'_{c} , χ'_{m}

By serially developing the anhysteretic magnetization in the vicinity of H=0, M=0, we have:

$$M_{an} \cong M_s \frac{H_e}{3a} = \frac{M_s}{3a} (H + \alpha M) \tag{6}$$

By replacing (5) in (6) we have:

$$M_{an} = \frac{M_s}{3a} \left[H + \alpha \cdot c \cdot M_{an} + \alpha \cdot (1 - c) M_{irr} \right]$$
(7)

For H=0 we will have $M_{an}=0$. We have in view the fact that in this zone the irreversible component of the magnetization is null (for very small fields the domain walls perform reversible motions.). $M_{irr}=0$. What results from equation (2) is:

$$\frac{dM_{irr}}{dH}\Big|_{H=0} \cong 0$$
(8)

Taking into consideration equation (5) and (8) we may write:

$$\chi_{in}^{\cdot} = \frac{dM}{dH} \bigg|_{H=0} \cong c \frac{dM_{an}}{dH} \bigg|_{H=0}$$
(9)

As a matter of fact, equation (9) presents "c" as the relation between the differential magnetic susceptibilities calculated in the beginning zone of the curve of first magnetization:

$$c \cong \frac{\chi'_{in}}{\chi'_{an}} \,. \tag{10}$$

The following two equations are presented as they were calculated by Jiles and Thoelke [3]:

$$k = \frac{M_{an}(H_c)}{1-c} \left\{ \alpha + \frac{1}{\chi_{max}^{\prime} - \frac{c}{1-c} \cdot \frac{dM_{an}(H_c)}{dH}} \right\}, \quad (11)$$

$$M_{R} = M_{an}(M_{R}) + \frac{k}{\frac{\alpha}{1-c} + \frac{1}{\chi_{rem}^{\prime} - c\frac{dM_{an}(M_{R})}{dH}}}$$
(12)

M_R is the remanence magnetization.

The "c" coefficient being known from (10), "k" and " α " respectively can be found from (11) and (12).

By deriving equation (7) in H=0 and taking (8) into account, we have:

$$\frac{dM_{an}}{dH}\Big|_{H=0} \cong \frac{M_s}{3a} \left(1 + \alpha \cdot c \cdot \frac{dM_{an}}{dH}\Big|_{H=0}\right)$$
(13)

(13) leads to:

$$a \cong \frac{M_s}{3\chi'_{an}} + \frac{\alpha \cdot c \cdot M_s}{3}$$
(14)

In conclusion, if the graph of a major magnetization curve is known, including the anhysteretic and the first magnetization curves, the "a, a, k, c" coefficients can be calculated using the (10), (11), (12), (14) equations first calculating the susceptibilities involved in these equations.

3. The method of fitting the hysteresis curves

The structure of the calculating programme which fits the experimental magnetization curve has more calculating stages:

1)initialization of the physical units which are the object of the calculation;

2) calculation of the coordinates of the simulated magnetization curve;

3) calculation of the average square deviation S using equation(1);

4) step-by-step modification of the increasing or decreasing ",a, α , k, c" parameters in order to minimize S;

5) display the calculated physical units including the values of the "a, α , k, c" parameters, of the experimental and regression magnetization curves;

6) recalculation of the values of the "a, α , k, c" parameters using the (10), (11), (12), (14) equation and display of these values.

Detailing stage 1:

The experimental data are "read" first so that a set of variables corresponding to the magnetic field get the corresponding numerical values from a text file. Another set of variables receive the values corresponding to the magnetization. Using the second set of variables the value of the saturation magnetization Ms is determined through interpolation. Being an interactive programme there is the possibility for the user to establish the value of Ms from the very beginning. There is the same possibility to establish the variation step dH of the calculated magnetic field. This step should be much smaller than the average distance between two successive values of the intensity of the measured magnetic field. Thus the possibility of having calculated values of the magnetic field which are very close to the experimental ones is created. Let dHexp be the average distance between two successive values of the intensity of the measured magnetic field. In order to obtain calculated values of the magnetic field intensity as close as possible to the experimental values, it is necessary that dH should be much smaller than dH_{exp} . Thus the preestablished vicinity of a random experimental value of the H_{exp,i} magnetic field in which there is only one calculated value H of the magnetic field will be the interval V=($H_{exp,i}$ -dH/2, $H_{exp,i}$ +dH/2]. Within the calculating programme we can choose, for example, $dH=dH_{exp}/10$ or $dH=dH_{exp}/50$, in the second situation the calculation speed being approximately 5 times slower than in the first one. Despite all these inconveniences the programme duration in the latter situation does not exceed a few minutes if a 2GHz computer is used. If, for example, $dH=dH_{exp}/50$ is chosen, then there will be approximately 50 calculated values of the field between two successive experimental values, the same correspondence being also kept between the points of the simulated magnetization curve and the experimental one.

In the same stage the "a, α , c, k" parameters will receive random values. The maximum value of the calculated magnetic field H_{max} is initialized with the maximum experimental value of the magnetic field. Also, the initial values of the calculated magnetization M and of the calculated magnetic field H are null. It is preferable, in order not to get to undeterminations of calculation, that H should be very close to 0 but not 0 (for instance, we choose 0,001). During this stage also, other variables which play different parts within the calculating programme are initialized.

Detailing stages 2 and 3:

These stages are going to be described together because they are calculation processes which are considered simultaneously within the programme. Everything happens as it follows: immediately after the calculation of a pair of coordinates (H, M) a partial calculation of S takes place if H is in the vicinity V of an experimental value of the magnetic field $H_{exp,i}$, and at the end of the calculation of all the coordinates of the magnetization curve the calculation of S is also finalized. The fact that S is a sum of terms such as $(M_i-M_{exp,i})^2$ should be kept in mind and each term of this type is added to the sum when the programme calculates the magnetization M_i corresponding to a field very close to the one where $M_{exp,i}$ was measured. These stages form together a repetitive structure which has as a result the step-by-step calculation of the coordinates of all points of the simulated magnetization curve and, simultaneously, of the unit S.

The calculating programme makes the value of the magnetic field increase or decrease step by step together with the fixed quantity dH between the limits $0 \rightarrow +H_{max}$, $+H_{max}\rightarrow -H_{max}$ and $-H_{max}\rightarrow +H_{max}$. As a result, the calculated intensity H of the magnetic field goes through a series of values which form an arithmetical progression for each variation interval.

The calculation of the coordinates (H, M) on the simulated magnetization curve takes place as it follows:

-for given H the effective He field is calculated using the equation He=H+ α M(initially H and M are null);

-the anhysteretic magnetization is calculated using equation (3);

-the variation of the irreversible component of dM_{irr} magnetization is calculated using equation (2), where dH is the variation step of the calculated magnetic field;

-the value of the irreversible component of the magnetization is found: $M_{irr}=M'_{irr}+dM_{irr}$, where M'_{irr} is the previous value of this magnetization corresponding to the previous point on the simulated magnetization curve;

-finally, the calculation of the total magnetization M takes place, including the irreversible component. Equation (5) is used for this;

-after calculating M for a given H field, the value of the field becomes H+dH on the $0\rightarrow H_{max}$, $-H_{max}\rightarrow +H_{max}$ intervals and H-dH on the $+H_{max}\rightarrow -H_{max}$ interval; after this modification of the field the sequence of operations described above starts again. The repetition of the calculation sequence stops after the values of the magnetic field "went" through the three intervals once and reached their extremes each time. The first interval corresponds to the calculation of the coordinates of the simulated first magnetization curve and the other two together correspond to the simulated major magnetization curve.

The calculation of the average square deviation S takes place simultaneously with the previously mentioned calculations.

The calculation of S takes place as it follows:

-at each repetition of the calculation sequence previously described, for each H value of the calculated magnetic field in the variation intervals $+H_{max} \rightarrow +H_{max}$ and $-H_{max} \rightarrow +H_{max}$ (the major magnetization curve), the fact that this value is in the vicinity V of an experimental value $H_{exp,i}$, i=1..N, N being the number of experimental data on the major experimental magnetization curve, is tested;

-if the previous condition is fulfilled, then the value of the calculated magnetization M(H) is memorized by a variable of the programme, Mi, and then the next modification of

the square of S takes place: $S^2 \rightarrow S^2 + [M_i - M_{exp,i}]^2$. If the condition is not fulfilled, S^2 keeps its value. Here S should be seen as a variable within the programme. Thus, the square of S increases with the $[M_i - M_{exp,i}]^2$ quantity. $M_{exp,i}$ is the measured magnetization corresponding to the measured magnetic field $H_{exp,i}$. It must be added that, initially, S=0.

When ending these calculations, after all the coordinates of the simulated magnetization curve have been step-by-step calculated, the value of S becomes $(S^2/N)^{1/2}$; thus the calculation of the average square deviation is finalized according to equation (1).

Stage 4 follows, which means the modification of one of the four ",a, α , c, k" parameters. This modification aims at obtaining a simulated curve closer to the experimental one, this fact being possible only if, as a result of the modification, the value of S is smaller than the previous one.

Detailing stage 4:

After modifying one of the mentioned parameters with a fraction "z" of its previous value, for example increasing, the calculation programme recalculates the coordinates of the simulated curve M(H) and recalculates the average square deviation S. In this way, a repetition of stages 2-3 takes place. If S's new value is smaller than the previous one the modification of that parameter continues increasing it again with the "z" fraction of its new value (for example, at the beginning z=50%). The repetition of this operation takes place until the value of S becomes bigger than the previous one. Then that parameter returns to the value before the last modification, and also S, after which the modification of the next parameter starts, etc. If from the very first increasing modification of one of the parameters, the decrease of S is not possible, then there is an alternative option to have a decreasing modification. If this does not have as a result the decrease of S, then the next parameter is taken into consideration. The decreasing modification takes place according to the same procedure as the increasing modification. Therefore, the step-by-step modifications of these parameters lead to the step-by-step decrease of S. After the succession of the modifications of all the parameters it is evaluated if the difference between the last two values of S is smaller than a certain percentage of S's last value (foe example 0,000001%). If this condition is not fulfilled, then the series of modifications of these parameters starts again. If this condition is fulfilled, then the "z" fraction decreases twice and the succession of the parameters' modifications resumes. The decrease of "z" implies a finer modification of each of the parameters which implies a more precise calculation of them. The decrease of the "z" fraction takes place until z<0,001%. It could be higher of course... but the programme will last longer.

Stage 5:

Then the computer screen displays the experimental and the calculated magnetization curves, the ",a, α , k, c" coefficients, the remanent magnetization, the coercive field and even the susceptibilities mentioned in section II.

The susceptibilities mentioned in section II are the tangents to the magnetization curve at its different points,

and the programme calculates them simultaneously with the coordinates of the simulated magnetization curve.

The question is if the order in which the parameters' modification takes place matters. Experience shows that the calculating duration depends on that order as well but also on the initial values of the coefficients. The final values of the "a, α , k, c" parameters, within the limits of calculating errors, depend neither on the initial values or on the order in which the coefficients' modification takes place. However, for a fast rolling of the calculating programme it is better to initialize the values of these parameters with expected values and not with totally random values, thus $\alpha < 1$, c < 1 iar a > 1, k > 1.

Stage 6:

Immediately after finalizing the fitting, there is the possibility for the user to calculate the "a, a, k, c" parameters using the 10, 11, 12, 14 equations. By pressing a button on the interface the values of the parameters calculated in this way are displayed. These new values can be compared to those obtained through minimizing the average square deviation. Therefore, there are two complementary ways of calculating the parameters of the J-A model, mode 1 corresponding to the values directly obtained through fitting and mode 2 corresponding to the use of the above mentioned equations immediately after finding the regression curve. Mode 2 would not be possible without finding the regression curve because all the susceptibilities which are part of these equations are calculated using this curve. The repeated use of the programme for fitting more magnetization curves for a varied range of materials shows that close values of the J-A model's parameters are obtained through both modes of calculation, except for the value of "c".

4. Results

The experimental data used by the application are determined by a vibrating samples magnetometer (VSM). We shall further present the fitting results for three samples of $CoFe_2O_4$ which were obtained by coprecipitation method and were sintered for 5 hours in air atmosphere at temperatures of $1050^{\circ}C$, $1100^{\circ}C$ and $1150^{\circ}C$. The experimental and calculated values are presented in Table 1.

Table 1. The values of the Jiles coefficients for the three
samples of CoFe ₂ O ₄ obtained through mode 2 of
calculation.

	Sample1	Sample2	Sample3
Т	1050°C	1100°C	1150°C
ρ (g/cm ³)	4,89	4,92	5,01
Ms(A/m)	414864	424067	439668
M _{rem} (A/m)	174122	87667	77000
Hc (A/m)	18489	12615	7925
α	0,176	0,126	0,198
k (A/m)	18964	12755	7978
с	0,00009	0,00014	0,00023
a (A/m)	33661	36610	42860

Table 2. A comparison between the values of the Jiles
coefficients for sample 1 obtained through the two modes
of calculation

	a (A/m)	k (A/m)	α	с
Mode 1	33679	19083	0,178	1.10-6
Mode 2	33661	18964	0,176	0,00009



Fig.1. Fitting graph- sample 1

It can be noticed that in the case of the materials presented the values of the "a, k, α " parameters determined through the two modes of calculation are relatively close. The fact that "c" differs considerably in the two modes of calculation leads to the conclusion that in the case of the magnetization of these materials the reversible component of the magnetization is relatively unimportant in relation to the irreversible one, (c \rightarrow 0) and values different enough but close to value 0 of this parameter (table 2) will not have as a result an important modification of the field, the higher the precision of determining the regression curve, and implicitly, of the J-a model's parameters; on the other hand, the duration of the calculation increases.



Fig.2 Fitting graph – sample 2



5. Conclusions

The designed software based on J-A model can be used in fitting the experimental hysteresis loops of magnetic materials with high accuracy. The purpose of the fitting programme was the well-precise determination of the values of the model's coefficients for a certain material. This purpose was achieved. Moreover, this calculating programme can also be used for fitting the magnetization curves of samples under mechanical stress or under other interaction conditions and, in this way, the dynamic of the model's coefficients in the given situations can be observed. These coefficients, through their physical meaning, can provide useful information regarding the microstructure of magnetic materials and of the interactions at this level.

Acknowledgements

This research was possible with the help of POS DRU (The Sectorial Operational Programme for the development of Human Resources) in Romania.

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