Lattice reduction aided improved signal detection for MIMO system

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This paper presents an improved lattice reduction aided MMSE detection based on MIMO system. The proposed detection technique reduces the orthogonality defect of the MIMO channel matrix based on LLL algorithm and the Gram-Schmidt orthogonalization procedure. BER performance of MMSE detector with and without lattice reduction is analyzed using LLL algorithm for 4×4 and 8×8 MIMO system. The proposed detection technique can achieve more reliable estimation, compared to the conventional lattice reduction aided detection.

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1. Introduction

Multiple-input multiple-output (MIMO) communication systems have turned out as an attractive method to meet the higher data rates demanded by users for various applications. To utilize the advantage of achieving larger data rate in MIMO systems, the symbol detection problem must be solved. The conventional maximum-likelihood (ML) detector exhibits exponential complexities in both the number of transmit and receive antennas and the signal constellation. Minimum mean-square error (MMSE) detector offers reasonable receiver complexity with suboptimal error-rate performance. Thus, it is required to introduce symbol detection methods other than the ML detector. Lattice reduction aided detection (LRAD) methods provide an efficient solution to this symbol detection problem [1].

An efficient method for signal detection is introduced based on the mathematical theory of point lattices defined by periodic arrangements of discrete points. The basic idea is to consider the distortion introduced by the noise-free part of a MIMO channel as a representation of a lattice, then to perform suboptimal detection on an “improved" representation of the channel matrix derived from a “reduced" lattice. The suitably reduced lattice facilitates the search for the lattice point closest to the received vector, shifting most of the computational complexity to a pre-processing step before linear detection. Such LRAD based approaches to MIMO receiver design have significantly closed the gap between feasible yet high-performance MIMO detection and optimal ML detection [2].

Recently, the LRAD has been receiving attractive attention since it achieves high channel capacity in the MIMO systems. The LRAD transforms the column vectors of the MIMO channel matrix close to mutually orthogonal, followed by the estimation of the transmitted signals [3]. The most popular LRAD algorithm is the well-known LLL algorithm introduced by Lenstra, Lenstra, and Lovász [4]. Using this algorithm, the LRAD achieves highly reliable signal estimation and hence good bit error ratios (BERs). In particular, the LRAD in a 4×4 MIMO system achieves BER relatively close to that with the ML detector. In contrast, the LRAD in an 8×8 MIMO system does not achieve so good BER performance as the LRAD in a 4×4 MIMO system does, compared to BERs for the ML detector. This is because the signal transmitted from each antenna is interfered by more signals transmitted from the other antennas in the 8×8 MIMO system than in the 4×4 MIMO system. This fact implies that the detection scheme used for the 4×4 MIMO system is not directly applicable to the 8×8 MIMO system and that some adequate detection schemes should be needed for the 8×8 and large scale MIMO system [5].

In this paper, we propose an MMSE detection technique by combining the LLL algorithm and the Gram-Schmidt orthogonalization (GSO) procedure, to achieve the BER performance closer to the ML detection in the 4×4 and 8×8 MIMO system. Firstly, the column vectors of the channel matrix are reduced using the LLL algorithm. Secondly, we reduce LLL-reduced column vectors using the GSO procedure. Finally, the GS-reduced column vectors become mutually orthogonal and almost of equal length. Thus, the decision boundary becomes closer to the ML detection. As a result, the proposed scheme improves the BER performance, which is very closer to that of ML detection both in the 4×4 and 8×8 MIMO systems.

The remainder of this paper is organized as follows. Section 2 presents the proposed system model for MIMO system. Section 3 provides the detailed analysis of Lenstra-Lenstra-Lovanz (LLL) algorithm. In section 4, Gram-Schmidt Orthogonalization (GSO) procedure has been described. Section 5 defines Minimum Mean-Square Error (MMSE) Estimation. Section 6 gives the computer simulation results and discussion with the detailed analysis of improved signal detection using lattice reduction. Finally, we summarize and conclude the paper in Section 7.

2. System model

An n-dimensional lattice is defined as a discrete subset of R^n that has a group structure under ordinary vector addition. A complex lattice consists of all linear combinations of the set of linearly independent basis column vectors b_k, of the basis matrix B. A complex lattice formed from basis matrix B is therefore the set of points and is given as

$$L(B) = \left\{ \sum_{k=1}^{M} s_k b_k \,\middle|\, s_k \in Z[i] \right\} \qquad (1)$$

where, $Z[i] = \{a + ib \mid a, b \in Z\}$ is the ring of Gaussian integers. Finding a basis in which the basis vectors are reasonably short and almost orthogonal is known as lattice basis reduction.

We consider a MIMO wireless communication system with n_T transmit and n_R receive antennas. The complex baseband model for this MIMO system is

$$y = Hx + n \qquad (2)$$

where, *y* is the received vector, *H* is the channel matrix, *n* is the channel noise, and *x* is the vector of transmitted symbols, as shown in Fig. 1. The task of the MIMO receiver is to recover *x* from *y*, based on knowledge of both the channel matrix *H* and the channel noise variance σ^2. We restrict our attention to transmit symbols drawn from finite sets of points, known as constellations, drawn from a square grid, and in particular 16-QAM constellations as shown in Fig. 2(a). We do not consider non-rectangular constellations, such as 16-PSK, due to an inherent incompatibility with the lattice-theoretic framework exploited by lattice reduction aided detection, and also the limited applicability of non-rectangular constellations in emerging wireless communication standards.

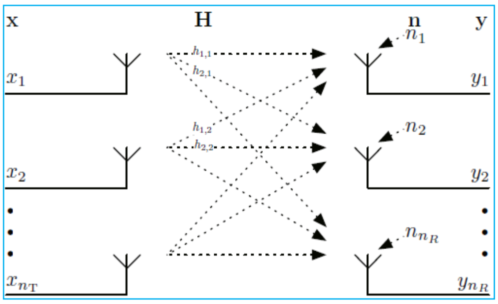


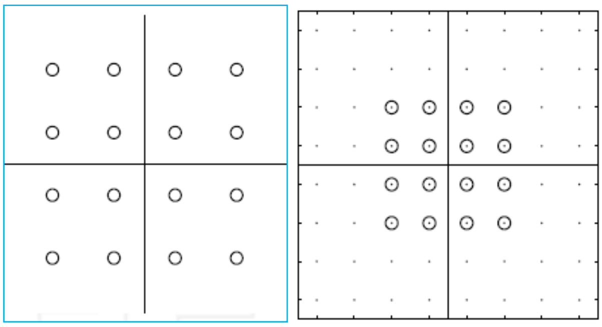
Fig. 1. MIMO wireless channel.

The constellations considered are formed from a subset of scaled and shifted Gaussian integers. We restrict attention to the subsets shown in Fig. 2(b). We refer to constellations formed in this manner as Gaussian integer constellations. The constellation A_n employed at the n^{th} transmit antenna is given as

$$A_n = \frac{X_n}{\sqrt{C_n}} \qquad (3)$$

Where, C_n is the average energy of X_n. Dividing each element of X_n by C_n ensures that A_n has unity average energy and is referred to as normalized constellations. For square QAM constellations such as those in Fig. 2(a), $C_n = (|X_n| - 1)/6$. In summary, the transmitted symbols x_n are formed by scaling of the elements as

$$X_n = \sqrt{\frac{E_{sn}}{C_n}}\,\bar{x}_n \qquad (4)$$



(a) (b)

Fig. 2. Signal space diagram a) 16-QAM constellations b) Scaled constellation.

The error probability of a detector is determined by the distance of constellation points (mapped by H) from the associated decision boundaries. The essential idea of LRAD is to obtain a “more orthogonal” representation for the channel realization *H*, before detection using a low-complexity (sub-optimal) receiver.

3. Lenstra-Lenstra-Lovanz (LLL) algorithm

The Lenstra-Lenstra-Lovász (LLL) algorithm was originally published as a lattice reduction algorithm operating on real-valued matrices. Many works use the real decomposition of the complex-valued MIMO transmission model. Lattice reduction methods can operate on both real and complex integer lattices and in particular the LLL algorithm has been extended for complex lattice reduction.

The complex LLL (CLLL) algorithm can be summarized as follows. We make the following definitions:

I. H_i is the squared Euclidean norm of the orthogonal vectors produced by the Gram-Schmidt orthogonalization (GSO) of H.

II. μ_{ij} is the ratio of the length of the orthogonal projection of the i^th^ basis onto the j^th^ orthogonal vector and the length of the j^th^ orthogonal vector.

III. H_i^L and T^i represent the values of the reduced basis and transform after the i^th^ step of the LLL algorithm.

IV. Initially, $H^0{}_L = H$ and $T^0 = I_{nT.}$

V. k is the index of the current column of H being processed such that $2 \leq k \leq n_T$ and δ satisfying ¼ $< d < 1$ is a scale factor.

The LLL algorithm consists of three basic steps:

a) H and μ are computed using a modified GSO procedure.

b) Size reduction aims to make basis vectors shorter and more orthogonal by asserting the condition that $|R(\mu_{k,j})| \leq 0.5$ for all j < k.

c) Basis vectors h_{k-1} and h_k are swapped if a so-called swapping condition is satisfied such that size reduction can be repeated to make basis vectors shorter.

4. Orthogonality defect

In application to MIMO system, we want a basis whose vectors are as orthogonal as possible. One metric for measuring the orthogonality of a basis is called the *orthogonality defect*. The orthogonality of a matrix H can be quantified using the orthogonality defect, defined as

$$\delta(H) = \frac{\prod_{k=1}^{nT} \|h_k\|}{\left|\sqrt{\det\left(H^H H\right)}\right|} \quad (5)$$

where, h_k is the k^{th} column of H, $\delta(H) \geq 1$ for all H and $\delta(H) = 1$ if and only if the columns of H are orthogonal. When the number of columns and rows of H are equal, the denominator can be simplified to |det(H)|. From equation (5), matrices with correlated columns or larger column norms will result in higher orthogonality defects. This also causes their inverse or generalized inverse to have larger row norms, leading to noise enhancement. The matrices with a lower orthogonality defect provide less noise enhancement in MMSE-based detectors and the probability of error is reduced.

To illustrate the impact of lattice reduction on orthogonality defect, we generate m×m random matrices by considering each element in the matrix uniformly between 0 and 1. The orthogonality defect was calculated using equation (5) both before and after lattice reduction. For dimension m from 2 to 8, the average orthogonality defect before and after the LLL reduction is shown in Fig. 3. The effect of lattice reduction on orthogonality defect is clearly apparent. The orthogonality defect is reduced by orders of magnitude. It is this improvement that reduces the error rate of lattice reduction based MMSE detector.

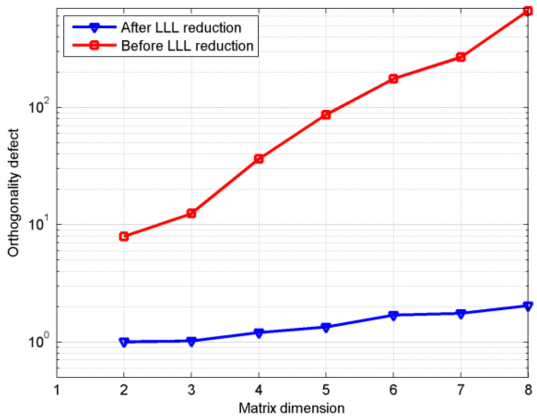


Fig. 3. Orthogonality defect before and after the LLL reduction.

5. Minimum mean-square error (MMSE) estimation

MMSE estimation acts to balance the reduction of the interference caused by H and the noise enhancement due to correlation of the columns in H. Rather than completely remove the effect of the MIMO channel, MMSE estimation works to find the estimate of the vector of transmitted symbols x_{MMSE} as follows:\

$$\hat{x}_{MMSE} = \arg\min \|W_{MMSE}\, y - x\|^2 \quad (6)$$

where, x_{MMSE} is found by independently rounding each element of ˜x to the nearest constellation point.

6. Lattice reduction aided improved signal detection for MIMO system

In general, the linear detection such as MMSE methods may increase the noise component in the course of linear filtering, thereby degrading the performance. Such noise enhancement problem becomes critical, especially when the condition number of channel matrix increases. Lattice reduction method can be useful for reducing the condition numbers of channel matrices. Fig. 4 illustrates two different sets of basis vectors that span the same space for two transmit antenna cases. Each vector corresponds to one of two columns in the channel matrix.

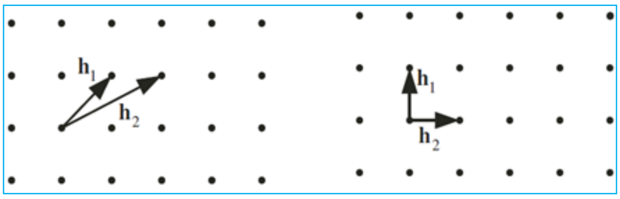


Fig. 4. Two sets of basis vectors that span the same space (a) basis vector set with a large condition number (b) orthogonal basis vector set.

The basis vector set in Fig. 4(a) has a larger condition number than that in Fig. 4(b). A basis vector set with a small condition number reduces the noise enhancement in the linear MMSE detection. When the basis vectors are orthogonal as in Fig. 4(b), there is no noise enhancement at all in the process of linear filtering.

Lattice basis reduction reduces the orthogonality defect, thereby reducing noise enhancement. This is achieved by finding a closer to orthogonal set of basis vectors. This reduced lattice basis is found by optimizing the generating matrix, which in the present application is a MIMO channel matrix realization. This closer-to-orthogonal set is found using elementary operations on basis vectors. Complex integer linear combinations of the column vectors of H are taken to form the reduced matrix H^L.

$$H_L = HT$$
$$=> H = H_L T^{-1} \quad (7)$$

where, T is a uni-modular matrix with complex integer entries and det(T) = ±1, therefore T^{-1} also contains only complex integer entries.

Once the lattice reduced channel matrix is found, we then calculate the pseudo-inverse as would be done in MMSE detection. LRAD therefore operates using the following steps:

1. Find the reduced lattice basis
2. Use the pseudo-inverse of the reduced basis to form estimates
3. Quantize estimates to *x*
4. Transform and bound points to constellation points

The received vectors *y* is multiplied with the pseudo-inverse of the reduced basis H^L to find a soft estimate of the vector of transmitted symbols in the reduced domain. These symbols are then quantized to an integer grid to find an estimate of the vector of transmitted symbols.

Fig. 5 shows the BER performance of MMSE based 4×4 MIMO system with and without lattice reduction. For example, considering SNR 10 dB, we observe that MMSE with lattice reduction have the $P_e = 10^{-4}$ while without lattice reduction we have, $P_e = 10^{-1}$. Figure 6 shows the BER performance of MMSE based 8×8 MIMO system with and without lattice reduction. For example, considering SNR 10 dB, we observe that MMSE with lattice reduction have the $P_e = 10^{-7}$ while without lattice reduction we have, $P_e = 10^{-5}$. We can summarize that the MMSE detector takes the advantage of the diversity i.e. BER performance is improved when we increase the number of transmitting and receiving antennas in the MIMO system. But the trade-off is the computation complexity. In both cases lattice reduction provides significant improvement in BER performance. Lattice reduction based MMSE signal detection is a powerful pre-processing steps for a MIMO system.

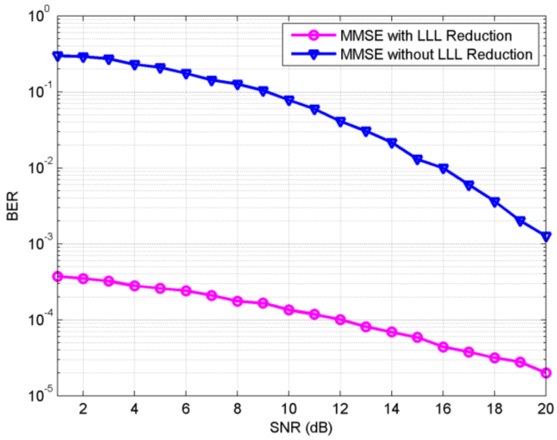


Fig. 5. BER vs E_b/N_o for MMSE with and without lattice reduction for 4×4 MIMO system.

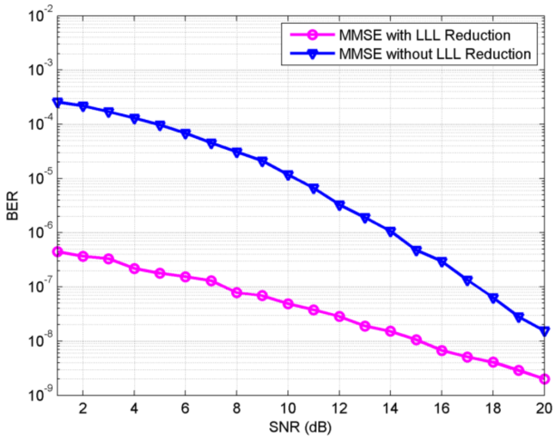


Fig. 6. BER vs E_b/N_o for MMSE with and without lattice reduction for 8×8 MIMO system.

7. Conclusion

Lattice reduction based MMSE signal detection for MIMO system was investigated and performance improvement in terms of BER was demonstrated. The proposed method improves the BER performances in both the 4×4 and 8×8 MIMO systems. This improved signal detection was achieved because the GS procedure creates the column vectors of the reduced channel matrix to be mutually purely orthogonal. Hence the decision boundary became closer to the ML detection.

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