Left-handed structures for microwave applications

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In this paper, dispersion/attenuation diagrams are investigated for purely left-handed (PLH) and for composite right/left handed (CRLH) transmission lines. The magnitude and phase of the S parameters of these structures are analyzed in a broadband (1-7 GHz) range. The investigations on microstrip CRLH lines demonstrate the presence of the left-handed range, the stop-band (only for the unbalanced lines) and the right-handed range. By edge-coupling two such CRLH lines, devices with new properties, different from conventional microstrip devices were obtained. Symmetric and asymmetric 0 dB backward couplers working in the stop-band of the CRLH line were achieved. Very short 3 dB couplers (with only 2 or 3 unit cells) were also investigated.

(Received April 7, 2010; accepted April 26, 2010)

Keywords: Microwaves, Left-handed structures, Dispersion diagrams

1. Introduction

Metamaterials are a new class of artificial composite structures, which exhibit special electromagnetic properties. [1,2]. Metamaterials include (periodically) ordered homogeneities usually smaller than a tenth of the wavelength. However, metamaterials are not the only with electromagnetic composites characteristics significantly different than the properties of the matrix material. Electrically small "artificial molecules" have been used either to contributes to the electric flux density macroscopically by increasing the effective electric permittivity as for artificial dielectrics used in some microwave antennas [3,4], either to exhibit a generalized magnetic permeability function as for the artificial magnetics [5].

The propagation of an electromagnetic wave through a metamaterial can be associated with a left-handed triad between the **E**, **H** and **k** vectors and with the antiparallelism existing between the phase and group velocities. This is referred to as the purely left-handed (PLH) case.

The electromagnetic propagation phenomena in one direction through an effectively homogeneous material can be essentially modeled by a one-dimensional transmissionline. However, in practice, such a PLH transmission line is impossible to manufacture, so that a more practical model is the composite right- / left- handed (CRLH) model of line [6]. A microstrip CRLH line maybe obtained by chaining several unit cells. In this work, each unit cell contains a six-digit inter-digital capacitor and a shunt stub inductor, as shown in Fig.1. The symmetry plane AB shown in Fig. 1 will be used in the next Section for investigation of CRLH lines. Each stub inductor is terminated in a via-hole short circuit. The CRLH lines were investigated in microstrip technology on a 1.524 mm thick substrate with 3.02 dielectric constant and 0.0013 dielectric loss tangent.



Fig. 1. Unit cell of the microstrip CRLH line (ground plane not shown).



Fig. 2. The equivalent circuit of the microstrip single cell shown in Fig. 1.

For a large frequency band, the interdigital capacitor and the shunt inductor can be fairly described by equivalent circuits with lumped elements. In the Π equivalent circuit of the interdigital capacitor, the most important component is the series capacitance C_s^{ic} . On the other hand, the most import component in the T equivalent circuit of the stub inductor is the shunt inductance L_p^{si} . All the other elements in the equivalent circuit of the microstrip CRLH cell are parasitics. The S parameters of the interdigital capacitor and the stub inductor were investigated separately by full-wave electromagnetic analysis [7] and the parameters of the Π and T circuits were extracted. The results are presented in Fig. 3 and Fig. 4 for 5.03 mm length and 0.11 mm wide digits and for inductor stubs of 4.814 mm total length and 0.67 mm wide. The moderate dependence on frequency of these extracted parameters limits the viability of the Π and T circuits to a frequency band between 1 and 5 GHz.



Fig. 3. Dependence of the extracted capacitances of the Π and T circuits on frequency; a) $C^{ic}{}_{s}$, b) $C^{c}{}_{p}$, c) $C^{si}{}_{p}$.



Fig. 4. Dependence of the extracted inductances of the Π *and T circuits on frequency; a)* $L^{s_{i}}_{p}$ *, b)* $L^{s_{i}}_{s}$.



Fig. 5. Modified circuit of an unit cell for the CRLH line.

Certaintly, models with lumped component circuits will never perfectly describe all the aspects of the electromagnetic behavior of the CRLH line. However, some significant features of the CRLH may be easier highlighted by describing the transmission lines by ladder circuits obtained by chaining several unit cell. In this sense, it is useful to modify the circuit in Fig. 2 into the circuit in Fig. 5.

There are two particular extreme situations in which the transmission line with unit cell shown in Fig. 5 can be simplified. On one hand, when $L_L = C_L = G_L = 0$, the propagation of a signal along the transmission line corresponds to the wave propagation through a conventional medium, associated with a right-handed triad between the electric field **E**, magnetic field **H** and the wave vector **k**. This particular case will be called purely right-handed (PRH) case. On the other hand, when only the components L_L , C_L and G_L are included, a new model of transmission line is obtained, which in fact is the dual of the PRH transmission line., and will be referred as the PLH line.

For a general CRLH transmission line, shunt and series resonance frequencies f_{se} and f_{sh} are defined by

$$f_{se} = \frac{1}{2\pi \sqrt{L_R C_L}} \text{ and } f_{sh} = \frac{1}{2\pi \sqrt{L_L C_R}}.$$
 (1)

In the unbalanced case $f_{se} \neq f_{sh}$ and the frequency response of the line presents a transmission gap between the shunt and series resonance. In our case $f_{sh} < f_{se}$ (Fig. 6), but they may change places without significant modification in the CRLH line properties. The maximum gap attenuation in the dependence of the $|S_{21}|$ versus frequency corresponds to a central frequency $f_0 = \sqrt{f_R f_L} = \sqrt{f_{se} f_{sh}}$.

While describing the CRLH line with a ladder circuit, a unit cell corresponds to a portion of finite length of the CRLH line. When analyzing such a ladder circuit, two cutoff frequencies are put in evidence due to this finite length: f_{cL} limits the left handed range at low frequencies and f_{cR} limits the "right –handed region" at high frequencies. When the length of CRLH cell tends to zero, f_{cL} tend to zero and f_{cR} to the infinity. For CRLH ladder networks with different numbers N of identical cells, both these cutoff frequencies are better emphasized for large N as shown in Fig. 6.

In the balanced case $f_{sh}=f_{se}$ which corresponds to the closure of the gap between the left-handed and right handed ranges. In this case, the cut-off frequencies of the CRLH ladder network are given by the expressions.

$$f_{cL}^{bal} = \frac{\left|1 - \sqrt{1 + x}\right|}{2\pi\sqrt{L_R C_R}} \text{ and } f_{cR}^{bal} = \frac{(1 + x)}{2\pi\sqrt{L_R C_R}} \text{ where}$$
$$x = \sqrt{\frac{L_R C_R}{L_L C_L}} \tag{2}$$

In the unbalanced case, the expressions (2) can also be used for the cut-off frequencies f_{cL} and f_{cR} as fairly good approximations.



Fig. 6. |S₂₁| dependence versus frequency for three CRLH networks with N identical cells. a) N =10, b) N =20, c) N =80.



Fig. 7. Dispersion/attenuation diagrams for infinitely small unit cells a) $|\beta_{RL}|$, $\beta_{RL} < 0$ for $f < f_{sh}$ and $\beta_{RL} > 0$ for $f > f_{se}$ ($f_{se} = 4.11$ GHz, $f_{sh} = 3.18$ GHz)b) α_{RL} , c) propagation constant β_R of the PRH transmission line, d) absolute value $|\beta_L|$ propagation constant $\beta_L < 0$, for the PLH transmission line.

The real and imaginary part of the complex propagation constant $\gamma_{RL} = \alpha_{RL} + j\beta_{RL}$ are presented in Fig. 7 for a CRLH line with $L_R = 2$ nH, $C_R = 1$ pF, $R_R = 0$, $L_L = 2.5$ nH, $C_L = 0.75$ pF, $G_L = 0$. For comparison, the propagation constant of a PRH line with $L_R = 2$ nH, $C_R = 1$ pF and of a PLH line with $L_L = 2.5$ nH, $C_L = 0.75$ pF are also presented. In this case, $f_{se} = 4.11$ GHz, $f_{sh} = 3.18$ GHz.

The dispersion/attenuation diagram for CRLH transmission line shows a lower pass-band $(f < f_{sh})$ designated as left-handed range, a band-stop band $(f_{sh} < f < f_{se})$ and an upper pass-band $f > f_{se}$ designated as the right-handed range. In the stop-band, the wave is attenuated despite the loss-less character of the components of the ladder network.

At low frequencies, the propagation constant β_{RL} of the CRLH transmission line approaches asymptotically by the propagation constant β_L of the perfectly left-handed transmission line. On the other hand, at very high frequencies, β_{RL} tends to β_R of the conventional righthanded transmission line (Fig. 7).

The S parameters of a 7-cell CRLH line calculated by using a full-wave method [7] and corresponding to a ladder circuit [8] are in a good agreement up to 7 GHz. The left-handed range, stop-band and right-handed range are put in evidence (Fig. 8). However, due to the some box resonances and additional electromagnetic influences at high frequencies, there are some differences in the responses for frequencies near the cut off frequency f_{cL} .



Fig. 8. S parameters provided by full-wave analysis for CRLH line with 5.029 mm digit length and 4.815 stub length and by ladder circuit simulation with $L_R=3.1$ nH, $C_R=0.41$ pF, $L_L=2.2$ nH, $C_L=0.46$ pF; a) $|S_{11}|$ full wave, b) $|S_{21}|$ full wave, c) $|S_{11}|$ ladder circuit, d) $|S_{11}|$ ladder circuit.

2. Symmetric CRLH couplers

When CRLH cells are positioned in a symmetrical way on each side of the AB symmetry plane shown in Fig. 1, an edge-coupling will appear. In this case, two different fundamental modes of propagation can exist: the even and odd modes. The even mode is associated with a magnetic wall on the AB symmetry plane along the lines, while the odd mode is associated with an electric wall along the AB plane.

From the ladder circuit model point of view, the odd mode corresponds to a capacitive coupling through a capacitance C_m . The propagation properties of a single CRLH coupled in the odd mode can be obtained by replacing C_R by C_R+2 C_m and leaving unchanged all the other parameters. On the other hand, the even mode corresponds to an inductive coupling between the cells, through a mutual coupling inductance L_m . In order to derive the electromagnetic parameters of a CRLH line in the even mode, one may replace L_R by L_R+2 L_m and keep all the other parameters in Fig. 5 unchanged.



Fig. 9. Symmetric CRLH coupler with 7 cells.

First, couplers with 7-cell CRLH lines were investigated. The ports were designated as in Fig. 9. A very interesting coupling phenomenon occurs in the stopband: the magnitude of the S_{31} is very close to 0 dB. In fact, such a strong coupling is achieved for CRLH lines with relative large number of cells, even for fairly large gap between the lines. Since the coupling signal is directed to port 3 and not to port 4, the coupler can be designated as a backward wave coupler. A very good directivity better than 20 dB, is obtained in a working band between 3.6 and 5.4 GHz. This coupling effect is very different from what is happening in conventional edgecoupled lines couplers: an edge -coupler with conventional microstrip lines can achieve a maximum coupling factor about 10 dB, while the CRLH device in Fig. 9 can exhibit very strong coupling, up to 0 dB. Consequently, it can be used for DC blocking in order to isolate the bias voltages applied to various circuits as well as to block DC and low-frequency voltages while allowing the RF signal to pass through with minimal loss.

While strong coupling (0 dB) requires CRLH lines of at least 7 cells, lower coupling levels can be achieved even with a reduced number of cells. The responses of three-cell and two-cell 3 dB couplers are presented in Figs. 11 and 12, respectively. The 2-cell only exhibits the advantage of being more compact, while its directivity performances are lower than for the 3-cell coupler.



Fig. 10. S parameters for the symmetric coupler in Fig. 9; a) $|S_{I1}|$, b) $|S_{21}|$, c) $|S_{31}|$, d) $|S_{I1}|$.



Fig. 11. S parameters for a 3-cell symmetric coupler. a) $|S_{11}|$, b) $|S_{21}|$, c) $|S_{31}|$, d) $|S_{11}|$.



Fig. 12. S parameters for a 2-cell only symmetric coupler; a) $|S_{11}|$, b) $|S_{21}|$, c) $|S_{31}|$, d) $|S_{11}|$.

3. Asymmetric coupler

An asymmetric coupler can be built from a CRLH line, which is edge-coupled with a conventional microstrip line. The "full-signal" coupling is again achieved to the port 3 in the stop-band of the CRLH line. This asymmetric structure has a reduced complexity, but both the bandwidth and directivity performances are not as good as for the symmetric coupler.



Fig. 13. S parameters for a 7 cell asymmetric coupler; a) $|S_{II}|$, b) $|S_{2I}|$, c) $|S_{3I}|$, d) $|S_{II}|$.

4. Conclusions

In this work, the investigations on different microstrip CRLH lines pointed out the left-handed range, the stopband (for the unbalanced lines) and the right-handed range. For low frequencies above the f_{cL} cut-off frequency, the calculated dispersion characteristic of the CRLH line approaches the dispersion of a purely left-handed line.

By edge-coupling such CRLH lines, devices with new properties, different for the conventional microstrip devices were obtained. Symmetric and asymmetric 0 dB

backward couplers were proposed for DC blocking. Moreover, very short 3 dB couplers with only 2 or 3 unit cells were presented for a wide range of applications in different microwave circuits.

Acknowledgement

The work was partially supported by the Romanian Ministry of Education and Research, project 71-005 / 2007.

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