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Linearity and nonlinearity in the electrooptic modulation with crystals of class $\bar{4}2m$. The Stokes parameters analysis

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An analysis of the polarization dynamics of the light modulated by means of the KDP type crystals is performed in terms of the Stokes parameters of the modulated light. Both the shape of the modulated signal and its spectral structure are presented. The linear and nonlinear ranges of the modulator operation are studied both theoretically and experimentally. Some particular cases of practical interest are presented.

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1. Introduction

The electrooptic effect is largely applied in various devices such as directional couplers, optical switches, bistable devices, integrated optics and also in phase, amplitude, frequency or polarization modulation of light, in Q-switching and mode locking in lasers, etc. [1, 2]. Most of these applications are in dynamical regime, i.e. when the voltage applied to the crystal has a time-varying component. In this case, the state of polarization of the emerging light is also a time-varying one.

From a theoretical viewpoint the time evolution of the polarization state may be analysed in the matrix (Jones and Mueller) [3, 4, 5], or pure operatorial [6, 7] formalisms.

Both the Jones and Stokes –Mueller formalisms were introduced in the analysis of the polarization dynamics of the output of the anisotropic laser [8, 9, 10].

On the other hand, the analysis of the polarization dynamics of the light emerging from various kinds of modulators was performed till now only in Jones formalism [11,12,13] or in the Pauli-algebraic one [14,15].

In the present paper we analyse the linearity and nonlinearity in the electrooptic modulation with crystals of class $\bar{4}2m$ (KDP type), in terms of observable quantities, the Stokes parameters, in the particular case of the longitudinal linear electrooptic effect.

The electrooptic modulation is linear, in related to the Stokes parameters, for a sinusoidal modulating signal (the voltage applied to the crystal), if the temporal evolution of the Stokes parameters has also sinusoidal form and their spectrum contains only frequency of the modulating signal. Contrary, the electrooptic modulation is nonlinear, if the temporal evolution of the Stokes parameters abates from the sinusoidal shape and their spectrum contains upper frequencies or the temporal evolution of the Stokes parameters keep sinusoidal shape, but their spectral structure has the harmonic component with a higher frequency compared to the frequency of the modulating signal.

2. Theoretical analysis of the temporal evolution and spectral structure of the Stokes parameters

The KDP crystal is a uniaxial crystal, it has a fourfold axis of symmetry, the optic axis Z, and the two mutually orthogonal twofold axes of symmetry, the crystallographic axes, the X and Y axes, that lie in the plane normal to Z. By applying an electric field parallel to the optical axis of the crystal, Z, it become biaxial crystal with the new induced principal axes $X^{'}$ and $Y^{'}$ rotated at an angle of 45° with respect to the X and Y crystallographic axes [1].

We consider the modulation arrangement presented in figure 1, where K is a KDP longitudinal electooptic modulator. The incident light on modulator is linearly polarized along X axis at 45° to the electrically induced axes of the crystal. In the following we perform the calculi in the reference system OXY.

The refractive indices for the light polarized along the induced principal axes in a longitudinal electrooptic modulator with KDP crystal are [1]:

$$n_{x'} = n_0 + \frac{n_0^3}{2} r_{63} E_z ,$$
$$n_{y'} = n_0 - \frac{n_0^3}{2} r_{63} E_z , \qquad (1)$$

where n_0 is the unperturbate crystalline index of refraction, r_{63}, the electrooptic coefficient for the electric field applied along the crystallographic Z axis and E_z electric field applied to the crystal. If we apply to the crystal a voltage with a d.c. component U_0, as well as a harmonically-varying one, $U_m \sin\Omega t$, the phase shift between linearly polarized components along the principal axes $X^{'}$ and $Y^{'}$ is:

$$\Phi = 2(\Phi_0 + \Gamma \sin\Omega t) \tag{2}$$

where:

$\Phi_0 = \dfrac{\pi}{\lambda_0} n_0^{\,3} r_{63} U_0$, $\Gamma = \dfrac{\pi}{\lambda_0} n_0^{\,3} r_{63} U_m$, λ_0 is the vacuum wavelength of the incident light.

The KDP electrooptic modulator behaves as a time varying phase shift plate. The characteristic angles of the wave plate are: θ - the fast axis azimuth (the fast axis of the modulator is the $Y^{'}$ axis) and Φ - the phase shift introduced by the plate between the $E^{'}_x$ and $E^{'}_y$ components of the electric field.

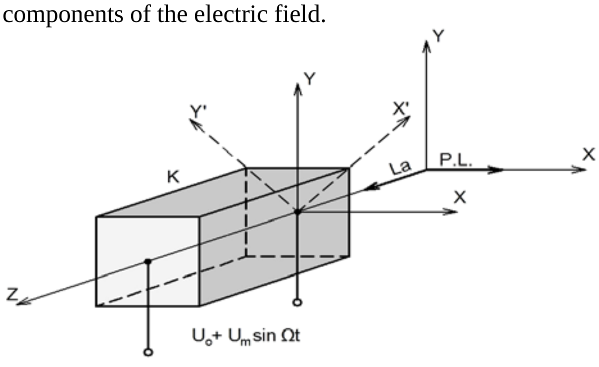

Fig. 1. The longitudinal electrooptic effect in the KDP crystal.

The Mueller matrix, M, of a plate having the characteristic angles, θ and Φ, is given by [16]:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \cos\Phi \sin^2 2\theta & (1-\cos\Phi)\sin 2\theta \cos 2\theta & -\sin\Phi \sin 2\theta \\ 0 & (1-\cos\Phi)\sin 2\theta\cos 2\theta & \sin^2 2\theta + \cos\Phi\cos^2 2\theta & \sin\Phi\cos 2\theta \\ 0 & \sin\Phi\sin 2\theta & -\sin\Phi\cos 2\theta & \cos\Phi \end{pmatrix} \tag{3}$$

In our experiment the incident light is linearly polarized along the X axis, having the Stokes vector (we will take the input Stokes vector normed to 1, i.e. the input intensity equal to 1):

$$S_{in} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

In this case the fast axis azimuth, θ, is 135° (the figure 1). Thus, the Mueller matrix of the modulator is:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Phi & 0 & \sin\Phi \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\Phi & 0 & \cos\Phi \end{pmatrix} \tag{5}$$

The Stokes vector, S_{out}, of the emergent light from the modulator is:

$$S_{out} = MS_{in} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Phi & 0 & \sin\Phi \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\Phi & 0 & \cos\Phi \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ \cos\Phi \\ 0 \\ -\sin\Phi \end{pmatrix} = \begin{pmatrix} 1 \\ \cos 2(\Phi_0 + \Gamma\sin\Omega t) \\ 0 \\ -\sin 2(\Phi_0 + \Gamma \sin\Omega t) \end{pmatrix} \tag{6}$$

The output Stokes vector has two constant components (S_0 and S_2) and two variable components (S_1 and S_3). The KDP crystal being non-absorbant the S_0 parameter which signifies the total output intensity is constant, namely equal to one. Since the incident light on the KDP modulator is linearly polarized along the X axis, at 45° to $X^{'}$ and $Y^{'}$ induced axes, the S_2 parameter is equal to zero.

By the decomposition of sine and cosine functions in equation (6), we get the following Stokes vector:

$$S_{out} = \cos(2\Gamma\sin\Omega t)\begin{pmatrix} 1 \\ \cos 2\Phi_0 \\ 0 \\ -\sin 2\Phi_0 \end{pmatrix} + \sin(2\Gamma \sin\Omega t)\begin{pmatrix} 1 \\ -\sin 2\Phi_0 \\ 0 \\ -\cos 2\Phi_0 \end{pmatrix} \tag{7}$$

By using the decomposition formulas [17]:

$$\cos\left(2\Gamma\sin\Omega t\right) = J_0(2\Gamma) + 2\sum_{k=1}^{\infty} J_{2k}\left(2\Gamma\right)\cos\left(2k\Omega t\right)$$

$$\sin\left(2\Gamma\sin\Omega t\right) = 2\sum_{k=1}^{\infty} J_{2k-1}\left(2\Gamma\right)\sin\left[\left(2k-1\right)\Omega t\right]$$

equation (7) may be developed in Fourier series as follows:

$$S_{out} = J_0(2\Gamma)\begin{pmatrix}1\\ \cos 2\Phi_0\\ 0\\ -\sin 2\Phi_0\end{pmatrix} + 2\sum_{k=1}^{\infty} J_{2k}(2\Gamma)\cos(2k\Omega t)\begin{pmatrix}1\\ \cos 2\Phi_0\\ 0\\ -\sin 2\Phi_0\end{pmatrix} +$$

$$+2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right]\begin{pmatrix}1\\ -\sin 2\Phi_0\\ 0\\ -\cos 2\Phi_0\end{pmatrix} \quad (8)$$

where $J(2\Gamma)$ are Bessel functions of the first kind.

We note: $S'_{1out} = \begin{pmatrix}1\\ \cos 2\Phi_0\\ 0\\ -\sin 2\Phi_0\end{pmatrix}$,

$$S'_{2out} = \begin{pmatrix}1\\ -\sin 2\Phi_0\\ 0\\ -\cos 2\Phi_0\end{pmatrix} \quad (9)$$

The vectors S'_{1out} and S'_{2out} are time invariant and correspond to a static alteration of the incident light state of polarization. The states of polarization corresponding to these vectors depends on d.c voltage, U_0, applied of the modulator. These vectors, generally, correspond to the elliptical polarization state. For certain values of the d.c. voltage applied to the modulator, the vectors S'_{1out} and S'_{2out} correspond to the linear and circular polarization states.

Each emergent elementary state of polarization oscillates as a whole at an integer multiple of the modulating frequency, ν ($\nu = \frac{\Omega}{2\pi}$, Ω is the angular frequency of the modulating voltage).

In the following we present the spectral structure of the Stokes vector of the light modulated by means of longitudinal linear electrooptic effect in the KDP crystal in two important particular cases in applications: no bias voltage and quarter wave plate bias voltage applied of the KDP modulator.

a) $U_0 = 0$. For no bias voltage applied to the modulator, $2\Phi_0 = 0$, and from (8), it is obtained:

$$S_{out} = J_0(2\Gamma)\begin{pmatrix}1\\1\\0\\0\end{pmatrix} + 2\sum_{k=1}^{\infty} J_{2k}(2\Gamma)\cos(2k\Omega t)\begin{pmatrix}1\\1\\0\\0\end{pmatrix} + 2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right]\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} \quad (10)$$

The time invariant vectors which appear in expression (10) correspond to the linearly polarized light along X axis and left circularly polarized light:

$$S'_{1out} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix},\ S'_{2out} = \begin{pmatrix}1\\0\\0\\-1\end{pmatrix} \quad (11)$$

The even-order spectral vectors correspond to the linearly polarized light along X axis, all even- order components of the modulated light have the same state of polarization as the incident light. The odd-order spectral vectors correspond to the left circularly polarized light, thus, the modulator acts on the odd-order components of the light as a quarter wave plate with the fast axis at -45° to the X axis.

b) $U_0 = U_{\lambda/4}$. For a quarter wave plate bias voltage applied to the modulator
(the modulator is biased to the middle point of the linear portion of the intensity transmission characteristic by applying a d.c. voltage or by inserting a quarter-wave plate in the light path, before or after the modulator), $2\Phi_0 = \frac{\pi}{2}$, and from (8), it is obtained:

$$S_{out} = J_0(2\Gamma)\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} + 2\sum_{k=1}^{\infty} J_{2k}(2\Gamma)\cos(2k\Omega t)\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} + 2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right]\begin{pmatrix}1\\-1\\0\\0\end{pmatrix} \quad (12)$$

In this case, the time invariant vectors which appear in expression (12) correspond to the left circularly polarized light and linearly polarized light along Y axis:

$$S'_{1out} = \begin{pmatrix}1\\0\\0\\-1\end{pmatrix},\ S'_{2out} = \begin{pmatrix}1\\-1\\0\\0\end{pmatrix} \quad (13)$$

The even-order spectral vectors correspond to the left circularly polarized light, the modulator acts in the same way as in the case odd-order components of the modulated light in absence the d.c. voltage. The odd-order spectral vectors correspond to the linearly polarized light along Y axis, the modulator acts as a half wave plate on the odd-order components of the modulated light.

From the equation (8) we can determine the spectral structure of the Stokes parameters of the light modulated by longitudinal linear electrooptic effect in KDP crystal:

$$S_0 = 1$$
$$S_1 = J_0(2\Gamma)\cos 2\Phi_0 - 2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right]\sin 2\Phi_0 + 2\sum_{k=1}^{\infty} J_{2k}(2\Gamma)\cos(2k\Omega t)\cos 2\Phi_0$$
$$S_2 = 0 \tag{14}$$
$$S_3 = -J_0(2\Gamma)\sin 2\Phi_0 - 2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right]\cos 2\Phi_0 - 2\sum_{k=1}^{\infty} J_{2k}(2\Gamma)\cos(2k\Omega t)\sin 2\Phi_0$$

The Stokes parameters S_0 and S_2 have been characterized previous. The other Stokes parameters, S_1 and S_3, have a complex spectral structure.

The Stokes parameters S_1 and S_3 have a steady term as well as harmonics terms at, generally, all multiples of the modulating frequency. The phase difference between the different harmonic components of the Stokes parameters is not constant in time, because their angular frequencies are different. The amplitudes of various harmonics are dependent upon the corresponding order Bessel functions of 2Γ, implicitly of the alternative voltage amplitude, U_m. Thus, the linear and nonlinear ranges of the modulator operation are given by the amplitude of the alternative voltage applied to the KDP modulator. By increasing the modulation index, higher and higher order harmonics appear in the spectrum of the S_1 and S_3 the Stokes parameters. The harmonic structure of the S_1 and S_3 Stokes parameters depends on the d.c. bias , for appropriate values of the U_0, all the odd order or all the even order harmonics may be suppressed in block.

The electrooptic modulation is linear, in related to the Stokes parameters S_1 and S_3, for a arbitrary d.c. voltage applied to the modulator, if their spectral structure contains only first harmonic component, whose frequency is equal to the frequency of the alternative voltage applied to the modulator. The amplitudes of various harmonics are dependent upon the corresponding order Bessel functions of 2Γ, thus the Bessel functions of upper order ($k > 1$) should be negligible related to Bessel function of one order: $\frac{J_k}{J_1} \le 0,01$. From (14), the Stokes parameters S_1 and S_3 have the expressions:

$$S_1 = J_0(2\Gamma)\cos 2\Phi_0 - J_1 \sin 2\Phi_0 \sin \Omega t$$
$$S_3 = -J_0(2\Gamma)\sin 2\Phi_0 - J_1 \cos 2\Phi_0 \sin \Omega t \tag{15}$$

Previous, we noted that the harmonic structure of the Stokes parameters S_1 and S_3 depends on the bias voltage applied to the KDP crystal. We will analyse the spectral structure of the Stokes parameters S_1 and S_3 in two cases important in applications: no d.c. bias voltage and quarter wave bias voltage applied to the modulator.

a) $U_0 = 0$. For no bias voltage applied to the modulator, $2\Phi_0 = 0$, and from (14), it is obtained:

$$S_1 = J_0(2\Gamma) + 2\sum_{k=1}^{\infty} J_{2k}(2\Gamma)\cos(2k\Omega t)$$
$$S_3 = -2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right] \tag{16}$$

The spectrum of the Stokes parameter S_1 contain a constant termen and only the even order harmonics of the modulation frequency , but the spectrum of the parameter S_3 contain odd order harmonics of the modulation frequency.

The spectrum of the Stokes parameter S_1 contains only the second harmonic component if $\frac{J_{2k}}{J_2} \le 0,01$, for $k > 1$. Also, the spectrum of the Stokes parameter S_3 contains only the first harmonic component if $\frac{J_{2k+1}}{J_1} \le 0,01$, for $k > 1$. Thus, the parameter S_1 has a double frequency with respect to frequency of the parameter S_3 and, implicitely, with respect to the modulation frequency. The electrooptic modulation is linear in related to the Stokes parameters S_3, and nonlinear in related to the Stokes parameters S_1. The Stokes parameters S_1 and S_3 have the expressions:

$$\begin{aligned} S_1 &= J_0(2\Gamma) + 2J_2(2\Gamma)\cos(2\Omega t) \\ S_3 &= -2J_1(2\Gamma)\sin(\Omega t) \end{aligned} \tag{17}$$

b) $U_0 = U_{\lambda/4}$. For a quarter wave plate bias voltage applied to the modulator, $2\Phi_0 = \frac{\pi}{2}$, and from (12), it is obtained:

$$S_1 = -2\sum_{k=1}^{\infty} J_{2k-1}(2\Gamma)\sin\left[(2k-1)\Omega t\right]$$

$$S_3 = -J_0(2\Gamma) - 2\sum_{k=1}^{\infty} J_{2k}\left(2\Gamma\right)\cos\left(2k\Omega t\right) \qquad (18)$$

In this case the spectrum of the parameter S_1 is the same as the spectrum of the parameter S_3 for no bias voltage applied to the crystal. The spectrum of the parameter S_3 contains harmonics that are in antiphase with the harmonics of the parameter S_1 for the case a).

Similarly to the previous case, we obtain for the spectrum of the Stokes parameter S_1 only the first harmonic component if $\frac{J_{2k+1}}{J_1} \le 0,01$, for $k > 1$, and for the spectrum of the Stokes parameter S_3 only the second harmonic component if $\frac{J_{2k}}{J_2} \le 0,01$, for $k > 1$. Here, the parameter S_3 has a double frequency with respect to frequency of the parameter S_1 and, implicitely, with respect to the modulating frequency. The electrooptic modulation is linear in related to the Stokes parameters S_1, and nonlinear in related to the Stokes parameters S_3. The Stokes parameters S_1 and S_3 have the expressions:

$$\begin{aligned} S_1 &= -2J_1(2\Gamma)\sin(\Omega t) \\ S_3 &= -J_0(2\Gamma) - 2J_2(2\Gamma)\cos\left(2\Omega t\right) \end{aligned} \qquad (19)$$

3. Experimental results

The optical scheme of the experimental setup used in the acquisition of the Stokes parameters of the light modulated by longitudinal linear electrooptic effect in KDP crystal is presented in Fig. 2:

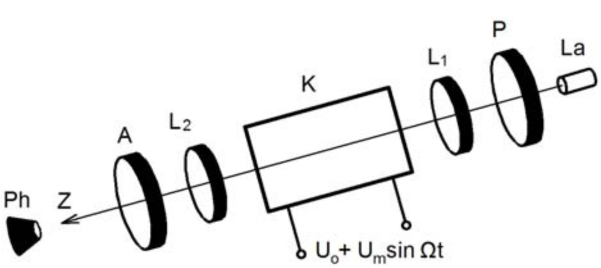

Fig. 2. Experimental setup: La - laser light source P, A - linear polarizers; L_1, L_2 – wave plates, K – KDP modulator; Ph – photodetector.

The experimental setup is formed of the He-Ne laser source ($\lambda = 632,8nm$) La, the linear polarizer P, the KDP modulator, the polarization state analyzer, the photodetector Ph and a computer. The linear polarizer light P, polarize light given by the He-Ne laser along the crystallographic axis X of the modulator. The KDP crystal, modulates the light by means of longitudinal electrooptic effect, the modulating frequency is $\nu = 2245Hz$. To obtain quarter wave plate bias voltage, is inserted a quarter-wave plate L_1 in the light path, before or after the KDP modulator. The quarter-wave plate has neutral lines parallel to the induced principal axes of the KDP modulator. The polarization state analyzer contains the linear polarizer A and the quarter-wave plate L_2.

The photodetector Ph provides an electrical signal proportional to the intensity of the emergent light to the polarization state analyzer. The computer is used for the acquisition and processing experimental data taken from the photodetector. The spectral structure of the parameters S_1 and S_3 is obtained by means of the Fourier transform.

The Stokes parameter S_1 is measured with the linear polarizer A set at azimuth 0° and 90° with respect to X axis, in the absence the quarter-wave plate L_2. In order to measure the Stokes parameter S_3 the quarter-wave plate L_2 must be inserted into optical path with the linear polarizer A set at 45° with respect to X axis.

The analysis of the temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3 is presented both in the case no bias voltage and in the case quarter wave plate bias voltage applied to the KDP modulator in the figures 3, 4, 5 and 6. The figures 3 and 4 present the temporal evolution and the spectral structure of the parameters S_1 and S_3 for the low amplitudes $U_m (U_m \le 0,1U_{\lambda/2}$, $U_{\lambda/2}$ is the half-wave voltage of the modulator) of the alternative voltage applied to the KDP modulator, so are valid the relations (17) and (19). In this case, both Stokes parameters have only the harmonic component of the lowest order. The figures 5 and 6 present the temporal evolution and the spectral structure of the parameters S_1 and S_3 for the high amplitudes $U_m (U_m \cong U_{\lambda/2})$ of the alternative voltage applied to the KDP modulator, in accordance with relations (16) and (18). Because, the upper harmonic components ($k > 2$) have the very low amplitudes compared with the amplitude of the first harmonic component, in the figures 5 and 6, the spectral structure of the Stokes parameters S_1 and S_3 contains only the first two harmonic components.

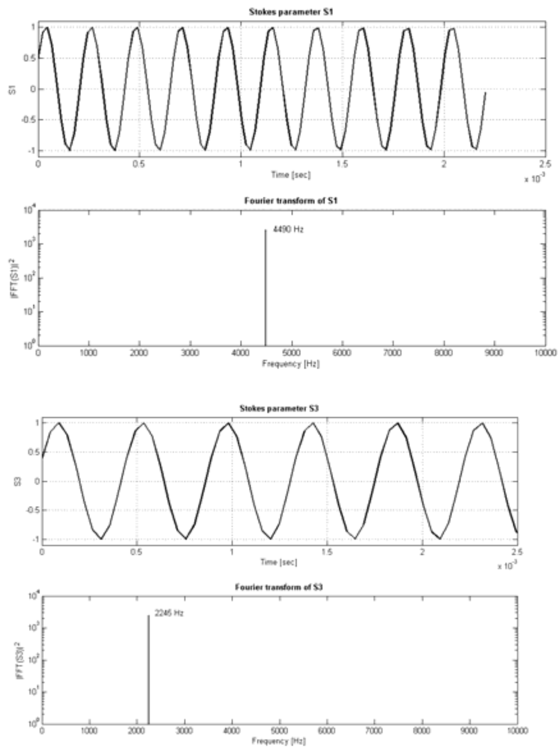

Fig. 3. $U_0 = 0$, $U_m \leq 0{,}1 U_{\lambda/2}$. *The temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3.*

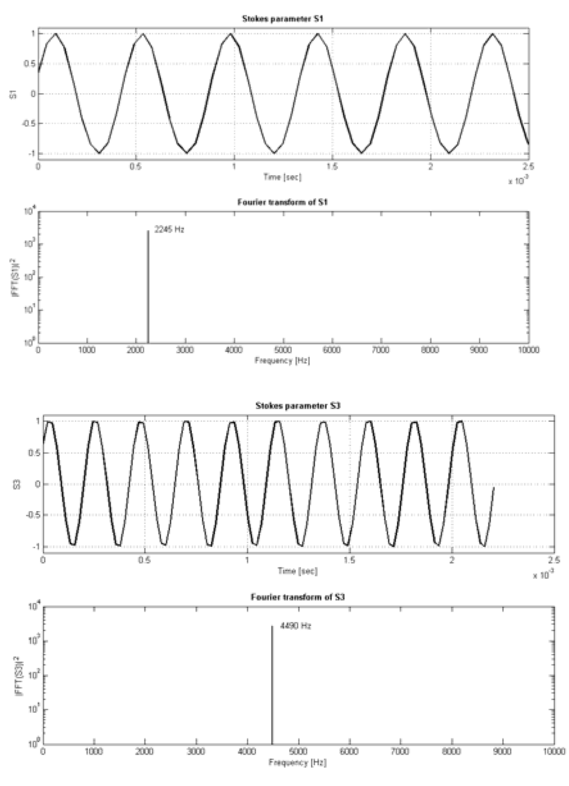

Fig. 4. $U_0 = U_{\lambda/4}$, $U_m \leq 0{,}1 U_{\lambda/2}$. *The temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3.*

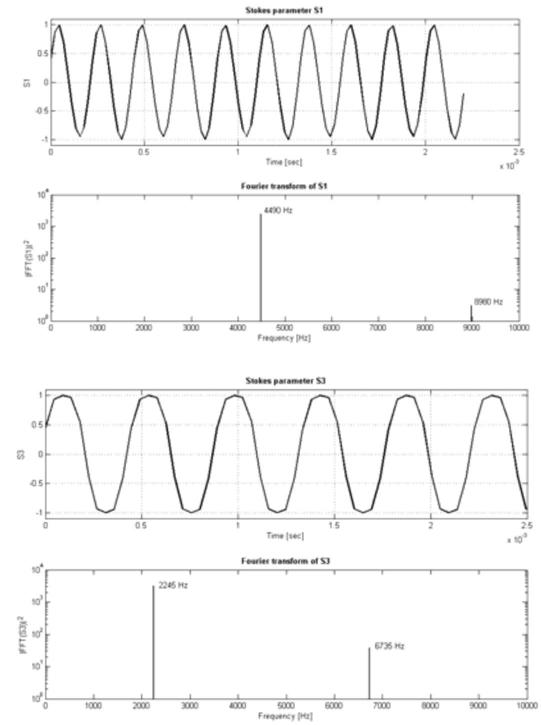

Fig. 5. $U_0 = 0$, $U_m \cong U_{\lambda/2}$. *The temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3.*

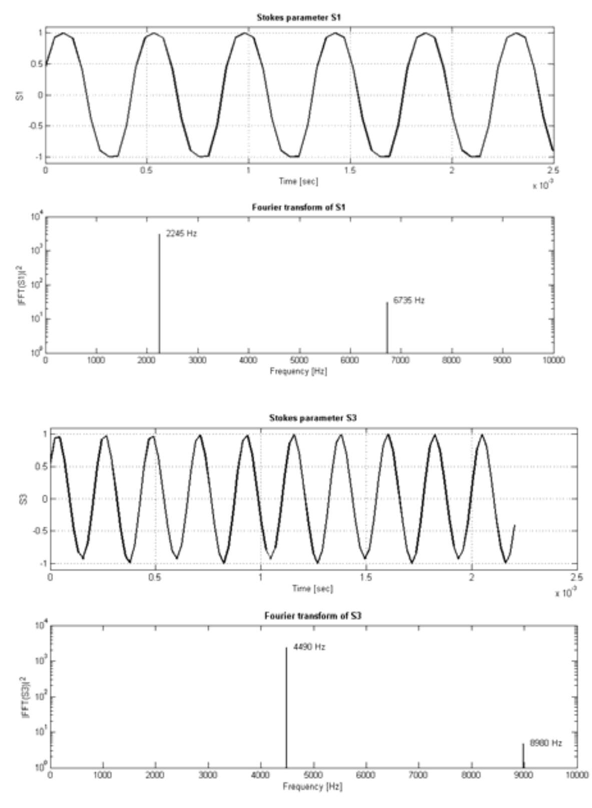

Fig. 6. $U_0 = U_{\lambda/4}$, $U_m \cong U_{\lambda/2}$. *The temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3.*

4. Conclusions

In this work we studied the linearity and nonlinearity in the electrooptic modulation with crystals of class $\bar{4}2m$ (KDP type), in terms of observable quantities, the Stokes parameters, in the particular case of the longitudinal linear electrooptic effect. The Stokes parameters of the modulated light have the following properties:

The first parameter S_0 is unity, the crystal is nonabsorbant, the third term S_2 is nul, since the incident light on the KDP modulator is linearly polarized along the X axis, at 45° to X' and Y' induced axes. The Stokes parameters S_1 and S_3 have a steady term as well as harmonics terms at, generally, all multiples of the modulation frequency. The spectral structure of the Stokes parameters S_1 and S_3 depends on the bias voltage applied to the modulator, for appropriate values of the U_0, all the odd order or all the even order harmonics may be suppressed in block. The amplitudes of the various harmonics are dependent upon the corresponding order Bessel functions of 2Γ, implicitly of the alternative voltage amplitude, U_m. The linear and nonlinear ranges of the modulator operation, in related to the Stokes parameters, are given by the amplitude of the alternative voltage applied to the KDP modulator.

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