JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS, Vol. 10, No. 11, November 2008, p. 2894 - 2897

Manipulation of cooperative emission using laser pulses

N. CIOBANU*, N. A. ENAKI
Institute of Applied Physics, Academy of Sciences of Moldova, Academiei street 5, Chisinau MD-2028, Moldova

The cooperative spontaneous emission from an ensemble of Λ type three-level atoms dressed by standing wave in resonance with coherent field is discussed. It is analyzed the spatial interference effect of fluorescent field as function of the distance between the radiators and relative position of atoms in the standing wave. The equations that describe the cooperative effect between the atom *a* and *b* are obtained.

(Received September 1, 2008; accepted October 30, 2008)

Keywords: Collective resonance fluorescence, Travel wave, Standing wave resonator, Cooperative emission

1. Introduction

Recently, a great attention in quantum optics is devoted to the study of spontaneous emission from two or more atoms trapped in the standing wave of external coherent field [1-3]. This interest is stimulated by a large number of publications dedicated to the cooperative resonance of atomic ensembles in the travel wave approximation [4, 5].

An interesting problem is connected with the modification of spontaneous emission in a three-level system stimulated by external coherent field [6] that is used successfully for such novel effects as lasing without inversion [7], quantum information processing [8]. Much attention has been focused on the spontaneous emission dynamics from multi-level atoms [9-10].

Following this idea, we discuss the cooperative spontaneous emission from an ensemble of Λ type three-level atoms dressed by standing wave that is in resonance with electromagnetic field. For this situation we studied the behavior of such atoms in the traveling and standing waves. Taking in to account the dependence of fluorescent spectrum, the spontaneous emission rate on the intensity of external field and its quantum statistical proprieties, the new control possibilities of cooperative spontaneous emission phenomena are investigated. So, for large values of laser field intensity, the control of spontaneous emission is possible at two frequencies and atom-atom interaction process. The dependence of cvasi-energeticall levels of the atoms on its position in the standing wave is discussed. In this case, the atoms become undistinguished, they are situated in the equivalent points of standing wave. The kinetic equations that describe the cooperative effect between the atoms *a* and *b* are solved.

2. The model hamiltonian and master equation

Let study the cooperative generation light in a three-level system stimulated by the presence of higher intensity standing wave that is in resonance with one atomic transition. This transition in the strong field limits takes place between the cvasienergetical levels of dressed atomic energetically spectrum which as will be shown below drastically is modified in the standing wave of resonator in comparison with similar transition in the strong traveling wave of free space [4]. The scheme of excitation of a three-level system represented in the *Fig.1* corresponds to the situation in which the atomic system is excited at second level and the low frequency of coherent field is in resonance with second $|2>$ and third $|3>$ states.

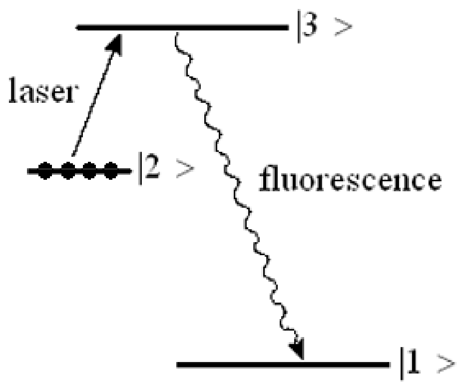

Fig. 1 Scheme of the energy excitations of an ensemble of three-level atoms.

In order to study the cooperative generation of fluorescent field relatively to the frequency $\omega_{31} = (|E_3 - E_1|)/\hbar$, we neglect the spontaneous emission rate at this frequency in comparison with similar spontaneous process at frequency $\omega_{32} = (|E_3 - E_2|)\hbar$, considering that the dipole momentum transition d_{31} are larger than d_{32}. In this approximation the atom-field system can be described by the Hamiltonian

$$H = -\sum_{j=1}^{N}\sum_{\alpha=1}^{3}\hbar\omega_{3\alpha}a_{\alpha j}^{+}a_{\alpha j} + \sum_{k}\hbar\omega_k b_k^{+}b_k + i\hbar\sum_{j=1}^{N}\Omega_j(a_{3j}^{+}a_{2j}e^{i(\vec{k}_0,\vec{r}_j)(1-\lambda)-i\omega_0 t} - h.c.) + \frac{i\omega_{31}d_{31}}{c}\sum_{k}\sum_{j=1}^{N}g_k(b_k a_{3j}^{+}a_{1j}e^{i(\vec{k},\vec{r}_j)} - h.c.). \tag{1}$$

The hamiltonian (1) takes into account the interaction with coherent field of standing wave (third term) and fluorescent electromagnetic field (fourth term). Here, $a_{\alpha j}^{+}$, $a_{\alpha j}$, are the generation and annihilation operator for level α and atom *j*; d_{31} and d_{32} represent the dipole momentum transitions between the levels $|3>\leftrightarrow|1>$ and $|3>\leftrightarrow|2>$, respectively. $b_k^{+}(b_k)$ is the generation (annihilation) operator of fluorescent electromagnetic field (EMF); $g_k = \sqrt{2\pi c^2\hbar/V\omega_k}(\vec{e}_\lambda,\vec{\mathbf{d}}_{31}/d_{31})$ represents the interaction constant with vacuum fluctuations; V is the quantification volume; $\vec{e}_\lambda$ and $\vec{\mathbf{d}}_{31}$ represent the photons polarization vector and the dipolar moment of transition, respectively. The atomic energy is measured relatively to the excited state $|3>$ ($E_3 = 0$).

Considering that coherent field is in resonance with transition $|3>\leftrightarrow|2>$, we can define the Rabi frequency, Ω_j, for traveling, $\lambda = 0$ and standing $\lambda = 1$ waves in the following

$$\Omega_j = \begin{cases} \Omega_0 & , for\ \lambda = 0 \\ \Omega_0\sin(k_0, r_j) & , for\ \lambda = 1 \end{cases} \tag{2}$$

In order to eliminate the time dependence from Hamiltonian (1), we can make the following transformation $H^{eff} = UHU^{-1} - i\hbar U\partial U^{-1}/\partial t$, where the operator U is represented through the detuning, $\Delta = \omega_{32} - \omega_0$,

$$U = \exp\{i\frac{\Delta t}{2}\sum_{j}(a_{3j}^{+}a_{3j} - a_{2j}^{+}a_{2j}) - it\sum_{j=1}^{N}\sum_{\alpha=1}^{3}\omega_{3\alpha}a_{\alpha j}^{+}a_{\alpha j} + it(\frac{\Delta}{2} + \omega_{31})\sum_{k}b_k^{+}b_k\}.$$

After this transformation the effective Hamiltonian of the system becomes

$$H^{eff} = \sum_{j}\frac{\hbar\Delta}{2}(a_{3j}^{+}a_{3j} - a_{2j}^{+}a_{2j}) + i\hbar\sum_{j=1}^{N}\Omega_j(a_{3j}^{+}a_{2j}e^{i(\vec{k}_0,\vec{r}_j)(1-\lambda)} - h.c.) + \hbar\sum_{k}(\omega_k - \frac{\Delta}{2} - \omega_{31})b_k^{+}b_k + \frac{i\omega_{31}d_{31}}{c}\sum_{k}\sum_{j=1}^{N}g_k(b_k a_{3j}^{+}a_{1j}e^{i(\vec{k},\vec{r}_j)} - h.c.). \tag{3}$$

As the effective Hamiltonian is time independently, let us consider that classical field is strong so that $\Omega_j\hbar >> \hbar\gamma_{\omega p}$, where $\gamma_{\omega n}$ is the collective spontaneous emission line width at transition $|2>\leftrightarrow|3>$. In order to diagonalize the first and second terms of the Hamiltonian (3) we introduce the Bogoliubov transformation

$$\tilde{a}_{3j} = \alpha C_{3j} + \beta C_{2j}, \qquad \tilde{a}_{2j} = \gamma C_{3j} + \delta C_{2j}$$

for atomic operators

$$\tilde{a}_{3j}^{+} = a_{3j}^{+}e^{\frac{i(\vec{k}_0,\cdot\vec{r}_j)}{2}(1-\lambda)+\frac{i\pi}{4}},\ \tilde{a}_{3j} = a_{3j}e^{-\frac{i(\vec{k}_0,\cdot\vec{r}_j)}{2}(1-\lambda)-\frac{i\pi}{4}},$$
$$\tilde{a}_{2j}^{+} = a_{2j}^{+}e^{-\frac{i(\vec{k}_0,\cdot\vec{r}_j)}{2}(1-\lambda)-\frac{i\pi}{4}},\ \tilde{a}_{2j} = a_{2j}e^{\frac{i(\vec{k}_0,\cdot\vec{r}_j)}{2}(1-\lambda)+\frac{i\pi}{4}}.$$

The coefficients α, β, γ and δ are chosen in according with the diagonalization requirements

$$\alpha = \frac{1}{\sqrt{2}}[1 + \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}}]^{\frac{1}{2}} = -\delta, \quad \beta = \frac{1}{\sqrt{2}}[1 - \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}}]^{\frac{1}{2}} = \gamma.$$

After this transformation the Hamiltonian (3) can be represented through the new cvasienergetical operators

$$H = H_0 + H_{\text{int}}, \tag{4}$$

where

$$H_0 = \sum_{k}\hbar(\omega_k - \frac{\Delta}{2} - \omega_{31})b_k^{+}b_k + \hbar\sum_{k}\tilde{\Omega}_j(U_{3j}^{3} - U_{2j}^{2}),$$

$$H_{\text{int}} = \frac{i\omega_{31}d_{31}}{c}\sum_{k}\sum_{j=1}^{N}g_k[b_k e^{i\left[(\vec{k},\vec{r}_j) - \left(\frac{(\vec{k}_0,\cdot\vec{r}_j)}{2}(1-\lambda)+\frac{\pi}{4}\right)\right]}(\alpha U_{1j}^{3} + \beta U_{1j}^{2}) - \hbar.c.],$$

and $\tilde{\Omega}_j = \sqrt{\Omega_j^2 + \Delta^2}$. Here the operators $U_{3j}^{1} = C_{3j}a_1$, $U_{2j}^{1} = C_{2j}a_1^{+}$ and $U_{1j}^{3} = C_{3j}^{+}a_1$, $U_{1j}^{2} = C_{2j}^{+}a_1$ represent the transition between the new cvazienergetical levels with populations $U_{3j}^{3} = C_{3j}^{+}C_{3j}$, $U_{2j}^{2} = C_{2j}^{+}C_{2j}$ and ground state $|1>$. The commutation relations for these operators are

$$[U_\beta^\alpha, U_\delta^\gamma] = U_\delta^\alpha\delta_{\gamma\beta} - U_\beta^\gamma\delta_{\alpha\delta}. \tag{5}$$

Let us study the cooperative spontaneous emission from the dressed exited states of an atomic ensemble.

Introducing the generalized, *O-operator* for dressed atomic subsystem and using the method of elimination of EMF operators [4], one can obtain the following master equation for the mean value of operator $< O(t) >$

$$\frac{d < O(t) >}{dt} = i \sum_j \tilde{\Omega}_j < [U_{3j}^3 - U_{2j}^2, O(t)] >$$
$$- \sum_k \sum_{j,l=1}^{N} J_{j,l}(k) \{\alpha^2 < [U_{1j}^3(t), O(t)] U_{3l}^1(t) > \zeta(\omega_k - \frac{\Delta}{2} - \omega_{31} - \tilde{\Omega}_j)$$
$$+ \beta^2 < [U_{1j}^2(t), O(t)] U_{2l}^1(t) > \zeta(\omega_k - \frac{\Delta}{2} - \omega_{31} + \tilde{\Omega}_j) + h.c.\}, \quad (6)$$

where

$$J_{j,l}(k) = (\frac{\omega_{31} d_{31}}{c\hbar})^2 g_k^2 \exp i \left[(\vec{k}, \vec{r}_{jl}) - \left(\frac{(\vec{k}_0, \vec{r}_{jl})}{2} (1 - \lambda) + \frac{\pi}{4} \right) \right] \quad (7)$$

describes the behavior of exchange integral between atoms j and l at the frequency ω_k,

$$\zeta(\omega_k - \frac{\Delta}{2} - \omega_{31} \pm \tilde{\Omega}_j) = i \frac{P}{\omega_k - \frac{\Delta}{2} - \omega_{31} \pm \tilde{\Omega}_j} + \pi\delta(\omega_k - \frac{\Delta}{2} - \omega_{31} \pm \tilde{\Omega}_j) \quad (7)$$

The first and second terms from expresion (7) represent the integration in the sense of principal value and Dirack delta function respectively. Using this equation one can study the behavior of atomic subsystem dressed by external standing wave in the process of spontaneous emission. In order to find the mutual influence of neighbor atoms we will analyze the correlation between two atoms situated in the equivalent positions of standing wave of resonator.

3. Discussion and results

To understand the generalized expression (6) for $O(t)$, let us for simplicity study the behavior of two atoms, *a* and *b* in the standing wave. This affirmation can be similar to the case of traveling wave studied in [4]. Considering that atoms are situated in the volume less than half of wave length let us study the situation for which $k_0 r_A \approx k_0 r_B = (n + 1/2)\pi$. In this case the Rabi frequency from the expression (2) takes the extreme values for standing wave. As follows, the exchange integral, $J_{j,l}(k)$, does not depend on the atomic positions. Indeed, if the operator $O(t)$ coincides with $U_{a2}^2(t)$ or $U_{b2}^2(t)$ from expression (6) one can obtain the following equation for the population operators in the dressed state scheme

$$\frac{d < U_{a2}^2(t) >}{dt} = 2B\{< U_{a2}^2(t) > + < U_{a1}^2(t) U_{b2}^1(t) >\}. \quad (8a)$$

In order to close this system of equations in analogical way we can found the other equations for atomic correlators $< U_{a1}^2(t) U_{b2}^1(t) >$, $< U_{a2}^2(t) U_{b1}^1(t) >$, $< U_{a2}^2(t) U_{b2}^2(t) >$, and $< U_{a3}^2(t) U_{b2}^3(t) >$

$$\frac{d < U_{a1}^2(t) U_{b2}^1(t) >}{dt} = -2A < U_{b2}^3(t) U_{a3}^2(t) > -2B\{< U_{a2}^2(t) U_{b2}^2(t) > - < U_{a2}^2(t) U_{b1}^1(t) > - < U_{a1}^2(t) U_{b2}^1(t) >\},$$

$$\frac{d < U_{a2}^2(t) U_{b1}^1(t) >}{dt} = -2A < U_{a2}^2(t) U_{b3}^3(t) > -2B\{< U_{a2}^2(t) U_{b2}^2(t) > - < U_{a1}^2(t) U_{b2}^1(t) > - < U_{a2}^2(t) U_{b1}^1(t) >\},$$

$$\frac{d < U_{a2}^2(t) U_{b2}^2(t) >}{dt} = 4B < U_{a2}^2(t) U_{b2}^2(t) >, \quad (8b)$$
$$\frac{d < U_{a3}^2(t) U_{b2}^3(t) >}{dt} = 2(A + B) < U_{a3}^2(t) U_{b2}^3(t) >,$$

where $A = \frac{4}{3} \frac{\omega_{31}^2 d_{31}^2}{c^3 \hbar} \omega_k \alpha^2$ and $B = \frac{4}{3} \frac{\omega_{31}^2 d_{31}^2}{c^3 \hbar} \omega_k \beta^2$.

From system (8) we can write the motion equations that describe the cooperative effect between the atom *a* and *b*

$$< U_{a2}^2(t) > = < U_{b2}^2(t) > = -\frac{1}{4} \frac{\Omega_j^2}{\Omega_j^2 + \Delta^2} \frac{1}{A - B} \exp(2Bt) \{B \exp(2At) + A[1 - \exp(2Bt)] - B\} - 2(1 + \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}})(2Bt - 1)\exp(4Bt),$$

$$< U_{a1}^2(t) U_{b2}^1(t) > = < U_{a2}^2(t) U_{b1}^1(t) > = -B(1 + \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}}) \exp(4Bt)t - \frac{1}{4} \frac{\Omega_j^2}{\Omega_j^2 + \Delta^2} \frac{A}{A - B} \exp(2Bt) \{\exp(2At) - \exp(2Bt)\},$$

$$< U_{3a}^2(t) U_{b2}^3(t) > = < U_{2a}^2(t) U_{b3}^3(t) > = \frac{1}{4} \frac{\Omega_j^2}{\Omega_j^2 + \Delta^2} \exp[2(A + B)t],$$

$$< U_{a2}^2(t) U_{b2}^2(t) > = \frac{1}{2} \left(1 + \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}} \right) \exp(4Bt). \quad (9)$$

In the case when the distance between the atoms is much larger than wave length $k_0 r_{AB} >> \lambda$ and the behavior of exchange integral between atoms from expresion (7) takes the form

$$J_{j,l}(k) = \frac{\omega_{31}^2 d_{31}^2}{c^3 \hbar} \omega_k \left[1 - (\vec{n}_d, \vec{n}_{jl})^2\right] \frac{\sin(\frac{\omega_k}{c}, r_{jl})}{(\frac{\omega_k}{c}, r_{jl})} + \left[1 - 3(\vec{n}_d \cdot \vec{n}_{jl})^2\right] \left[\frac{\cos(\frac{\omega_k}{c}, r_{jl})}{(\frac{\omega_k}{c}, r_{jl})^2} - \frac{\sin(\frac{\omega_k}{c}, r_{jl})}{(\frac{\omega_k}{c}, r_{jl})^3} \right]. \quad (10)$$

Here, we neglect the correction in the slow part of exchange integral connected with Rabi frequency. The frequency ω_k represents the Stokes $\omega_s = \omega_{31} - \Omega$ or anti Stokes $\omega_a = \omega_{31} + \Omega$.

Let us consider that the atoms are situated in different loops or nodes of the standing wave. If the moment dipoles of atoms are situated perpendicular $\vec{n}_d \perp \vec{n}_{jl}$ the exchange integral (10) will take the form

$$J^{\perp}_{A,B}(k)=\frac{\omega_{31}^2 d_{31}^2}{c^2\hbar}\frac{1}{r_{A,B}}\{\sin(\omega_\kappa r_{A,B}/c)[1-\frac{1}{(\omega_\kappa r_{A,B}/c)^2}] + \frac{\cos(\omega_\kappa r_{A,B}/c)}{\omega_\kappa r_{A,B}/c}\}. \quad (11)$$

In opposite case, when $\vec{n}_d$ is parallel to the vector $\vec{n}_{jl}$, the exchange integral between the atom a and b becomes

$$J^{\parallel}_{A,B}(\kappa)=\frac{\omega_{31}^2 d_{31}^2}{c\hbar}\frac{2}{\omega_\kappa r_{A,B}^2}\left[\frac{\sin(\omega_\kappa r_{A,B}/c)}{(\omega_\kappa r_{A,B}/c)}-\cos(\omega_\kappa r_{A,B}/c)\right]. \quad (12)$$

As follows from expressions (11) and (12) the exchange integral between the atom a and b depends on its positions and orientation. If the atoms are situated in the loops the Rabi frequency is maximal. Neglecting the influence of Rabi frequency of exchange integral $\omega_\kappa = \omega_{31}$, we observe that $J^{\perp}_{A,B}(\kappa)$ is larger than in the case $r_{AB} \succ \lambda$. In opposite case, when the atoms are located in nodes and the detuning $\Delta = 0$, the Rabi frequency is minimal $\Omega = 0$. In this situation $\sin(\omega_\kappa r_{A,B}/c)=0$ and the exchange integral takes negative values.

4. Conclusions

In this paper we discuss the cooperative spontaneous emission from an ensemble of Λ type three-level atoms dressed by standing wave that is in resonance with coherent field. The behavior of such atoms in the traveling and standing waves was studied. The spatial interference effect of fluorescent field as function of the distance between the radiators and relative position of atoms in the standing wave is analyzed. Taking in to account the dependence of fluorescent spectrum and spontaneous emission rate on the intensity of external field and its quantum statistical proprieties, the new control possibilities of cooperative spontaneous emission phenomena is investigated.

Acknowledgements

Support for this work has been provided by the National Grant No. 06.408.013F and Young Scientists Grant No. 07.408.31 of Academy of Sciences of Moldova.

References

[1] M. Afzelius, M. U. Staudt, H. de Riedmatten, C. Simon, S. R. Hastings-Simon, R. Ricken, H. Suche, W. Sohler and N. Gisin, New J. Phys. **9** 413 (2007)
[2] M. O. Scully, E. S. Fry, Ooi C. H. Raymond, K. Wodkiewicz, Phys. Rev. Lett. **96**, 010501 (2006)
[3] A. Kreuter, C. Becher, G. P.T. Lancaster, A. B. Mundt, C. Russo, H. HaË ffner, C. Roos, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **92**, 203002 (2004)
[4] N. A. Enaki, Yu. M. Shvera, Optics and Spectroscopy, **65**, Issue 2, 272 (1988),
[5] M. R. Ferguson, Z. Ficek, and B. J. Dalton, Phys. Rev. A **54**, 2379 (1996)
[6] Szu-Cheng Cheng, Jing-Nuo Wu, Tzong-Jer Yang, Wen-Feng Hsieh, **arXiv:0805.1107v1**
[7] Li-Feng Cai, **arXiv:quant-ph/0512009v1**
[8] Julien Laurat, Chin-wen Chou, Hui Deng, Kyung Soo Choi, Daniel Felinto, Hugues de Riedmatten, H J Kimble, New Journal of Physics **9**, 207 (2007)
[9] Wenge Yang and Min Xiao, Physics Letters A, **325**, Issue 1, 30 (2004)
[10] Jianming Wen, Shengwang Du and Morton H. Rubin, Phys. Rev. A **76**, 013825 2007
[11] J.H. Li, Eur. Phys. J. D **42**, 467-473 (2007).

*Corresponding author: nellu_ciobanu@yahoo.com