# Matrix analysis of two Mueller imaging polarimeters in transmission

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In this work we performed, in the frame of the Stokes - Mueller matrix formalism, an analysis of two Mueller matrix imaging polarimeters that function in transmission. We theoretically calculated the Mueller matrices of both the polarization state generator and the polarization state analyzer of the Mueller matrix imaging polarimeters. It also presents the calculation, in a particular case, of the Mueller matrix of the system and of the Stokes vector of the emergent polarized light from polarization state analyzer for one of Mueller matrix imaging polarimeters.

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## 1. Introduction

The Mueller matrix imaging polarimeter is a dual rotating retarder Mueller matrix polarimeter [1 - 4], with the addition of a CCD detector to measure an array of Mueller matrices associated with multiple ray paths through an optical system [2]. Mueller matrix imaging polarimetry is a powerful imaging technique used to provide high precision measurements for the Mueller matrices at every pixel of a scene [5, 6]. The Mueller matrix of the sample can be determined by measuring a series of different generated and analyzed state [7]. The Mueller matrix imaging polarimeters could work, in respect to sample, in transmission, reflection, retroreflection, and variable-angle scattering [7-11].

In the Stokes-Mueller matrix formalism, the Mueller matrix is a 4x4 real matrix that characterizes the properties of polarization of an optical element, and the Stokes vector describes the state of polarization of the light. If $S$ is the Stokes vector of the incident light on optical element, $M$ is the Mueller matrix of the optical element, then the state of polarization of the emergent light from optical element is given by $S'$ Stokes vector, defined by relation: $S' = MS$, or in explicit form [12,13]:

$$S' = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \tag{1}$$

,where $m_{ij}$ are the $M$ Mueller matrix elements and $S_i$ are $S$ Stokes vector elements.

The Mueller matrix contains all the information needed for determine the influence of the optical element on the state of polarization of the incident light on this, e.g.: diattenuation, retardance, depolarization and polarizance [2, 3, 10].

The Mueller matrix imaging polarimeters have the following five component parts: first one is the source, a laser system that emits polarized/ unpolarized light, second one is for prepare the incident optical field on sample, the polarization state generator (PSG), third one meet the interaction of the laser radiation with different samples, interaction followed by the change of polarizing properties of the laser radiation, fourth one is used for analysis of the state of polarization of the emergent light from sample, the polarization state analyzer (PSA), and last represented by the CCD camera, for the opto-electronic detection, and a computer.

This article presents the matrix analysis of two types of polarimeters of Mueller matrix imaging, functioning in transmission: the first one has PSG consisting of a linear polarizer followed by a quarter-wave plate and PSA represented by a quarter-wave plate and a linear polarizer [1, 2], the second one has PSG consisting only of a quarter-wave plate, because the light from the laser system is polarized [14], PSA is represented also by a quarter-wave plate and a linear polarizer. For both Mueller matrix imaging polarimeters we determine the Mueller matrices of the PSG and PSA. In addition, for the second Mueller matrix imaging polarimeter we have calculated the Mueller matrix of the system and the Stokes vector of the emergent polarized light from PSA, in an important case, without the sample. The data collection process, for Mueller matrix imaging polarimeters in transmission, is allowed to proceed with no sample in place as a calibration [10].

## 2. Matrix analysis of two configurations of Mueller imaging polarimeters

First, we determine the Mueller matrices of the polarization state generator and the polarization state analyzer of the Mueller matrix imaging polarimeter configured by two linear polarizers and two rotating quarter-wave plates.

In our study the linear polarizers have the parallel transmission axes, the same Mueller matrix, this depend on the azimuth $\theta$ of the polarizers transmission axis. Quarter-wave plates are rotated, fast axes azimuths, $\rho_1$ and $\rho_2$, are variable in time. The diagram of this polarimeter it is shown in figure 1.

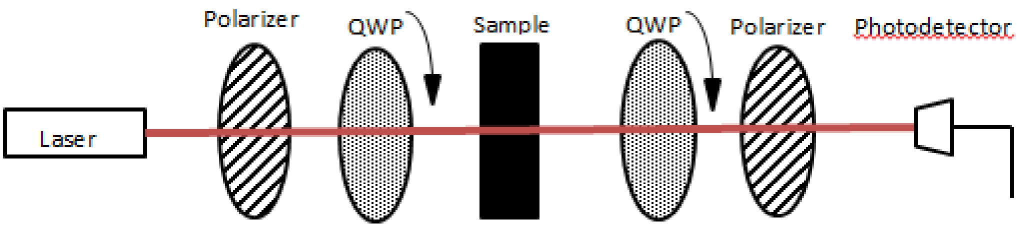


*Fig. 1 The Mueller matrix imaging polarimeter with two linear polarizers and two rotated quarter-waves plates.*

The Mueller matrices corresponding to optical elements are:

$M_1$–matrix of the first ideal homogenous linear polarizer

$M_2$ –matrix of the first ideal homogenous qurter-wave plate

$M_3$–matrix of the second ideal homogenous quarter-wave plate

$M_4$–matrix of the second ideal homogenous linear polarizer

Polarization state generator and polarization state analyzer are composed from a linear polarizer and a quarter-wave plate.

Thus, according to [12] those matrices have the following forms:

$$M_1 = M_4 = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2)$$

$$M_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_1 & \cos 2\rho_1 \sin 2\rho_1 & -\sin 2\rho_1 \\ 0 & \cos 2\rho_1 \sin 2\rho_1 & \sin^2 2\rho_1 & \cos 2\rho_1 \\ 0 & \sin 2\rho_1 & -\cos 2\rho_1 & 0 \end{pmatrix} \quad (3)$$

$$M_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 0 & \cos 2\rho_2 \sin 2\rho_2 & \sin^2 2\rho_2 & \cos 2\rho_2 \\ 0 & \sin 2\rho_2 & -\cos 2\rho_2 & 0 \end{pmatrix} \quad (4)$$

For both types of polarimeters, polarizers and quarter-wave plates matrices will be express according to those matrices, given in (2 – 4).

Mueller matrix of PSG it is $M_G = M_2\, M_1$:

$$M_G = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_1 & \cos 2\rho_1 \sin 2\rho_1 & -\sin 2\rho_1 \\ 0 & \cos 2\rho_1 \sin 2\rho_1 & \sin^2 2\rho_1 & \cos 2\rho_1 \\ 0 & \sin 2\rho_1 & -\cos 2\rho_1 & 0 \end{pmatrix}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

thus Mueller matrix of the PSG is:

$$M_G = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\rho_1 \cos 2(\theta-\rho_1) & \cos 2\theta \cos 2\rho_1 \cos 2(\theta-\rho_1) & \sin 2\theta \cos 2\rho_1 \cos 2(\theta-\rho_1) & 0 \\ \sin 2\rho_1 \cos 2(\theta-\rho_1) & \sin 2\rho_1 \cos 2\theta \cos 2(\theta-\rho_1) & \sin 2\theta \sin 2\rho_1 \cos 2(\theta-\rho_1) & 0 \\ \sin 2(\rho_1-\theta) & \cos 2\theta \sin 2(\rho_1-\theta) & \sin 2\theta \sin 2(\rho_1-\theta) & 0 \end{pmatrix} \tag{6}$$

Mueller matrix of PSA it is $M_A = M_4 M_3$:

$$M_A = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 0 & \cos 2\rho_2 \sin 2\rho_2 & \sin^2 2\rho_2 & \cos 2\rho_2 \\ 0 & \sin 2\rho_2 & -\cos 2\rho_2 & 0 \end{pmatrix}$$

$$M_A = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2(\theta-\rho_2) \\ \cos 2\theta & \cos 2\theta \cos 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\rho_2 \cos 2\theta \cos 2(\theta-\rho_2) & \cos 2\theta \sin 2(\theta-\rho_2) \\ \sin 2\theta & \sin 2\theta \cos 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\theta \sin 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\theta \sin 2(\theta-\rho_2) \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{7}$$

Both matrices, $M_G$ and $M_A$, depend on the transmission axes azimuths of the polarizers and on fast axes azimuths of the quarte-wave plates. Since the angles $\rho_1$ and $\rho_2$ are variable in time, the Mueller matrices of PSG and PSA describe the modulation of polarization state of the incident light and emergent light from sample.

The system Mueller matrix, $M_S$, it is the product of the components matrices performed in reverse order of the light propagation direction through system components. If we note $M_p$ – the sample's matrix, then the system matrix will be given by [9, 12]:

$$M_S = M_4 M_3 M_p M_2 M_1 \rightarrow M_S = M_A M_p M_G \tag{8}$$

Farther, we determine the Mueller matrices of both the polarization state generator and the polarization state analyzer of the Mueller matrix imaging polarimeter configured by a linear polarizer and two rotating quarter-wave plates. Also, we calculate the Mueller matrix of the system and the Stokes vector of the emergent polarized light from polarization state analyzer, in the absence of the sample. The empty space is characterized by identity Mueller matrix [10]. If the source light it is a laser which emit polarized light, then the presence of a linear polarizer in the generating system of the state of polarization wouldn't be necessary [14] (fig. 2), thus the PSG is composed only by a quarter-wave plate. PSA it is composed by a linear polarizer and a quarter-wave plate.

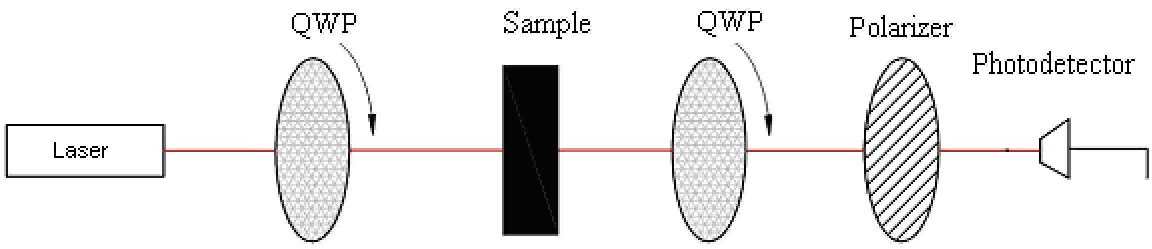


*Fig. 2 The Mueller matrix imaging polarimeter with a linear polarizer and two rotative quarter-wave plates*

The PSG matrix will be the matrix of the first quarter-wave plate:

$$M_G = M_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_1 & \cos 2\rho_1 \sin 2\rho_1 & -\sin 2\rho_1 \\ 0 & \cos 2\rho_1 \sin 2\rho_1 & \sin^2 2\rho_1 & \cos 2\rho_1 \\ 0 & \sin 2\rho_1 & -\cos 2\rho_1 & 0 \end{pmatrix} \quad (9)$$

PSA matrix will be the product of the polarizer and quarter-wave plate matrices:

$$M_A = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2(\theta-\rho_2) \\ \cos 2\theta & \cos 2\theta \cos 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\rho_2 \cos 2\theta \cos 2(\theta-\rho_2) & \cos 2\theta \sin 2(\theta-\rho_2) \\ \sin 2\theta & \sin 2\theta \cos 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\theta \sin 2\rho_2 \cos 2(\theta-\rho_2) & \sin 2\theta \sin 2(\theta-\rho_2) \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

If $S$ is the Stokes vector of the incident light on sample, then $S'$, the Stokes vector of the emergent light from PSA, will be [9, 12]:

$$S' = M_S S = M_4 M_3 M_p M_2 \, S \quad (11)$$

Below it is presented the way to determinate Stokes vector on a simple case, without sample. The empty space is characterized by identity Mueller matrix. Incident light on PSG it is horizontally linear polarized. Polarizers from the PSA structure have the transmission axis at $\theta = 0^\circ$.

Hence

$$S = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, M_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; M_4 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (12)$$

PSA matrix it is given by formula:

$$M_A = M_4 M_3 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 0 & \cos 2\rho_2 \sin 2\rho_2 & \sin^2 2\rho_2 & \cos 2\rho_2 \\ 0 & \sin 2\rho_2 & -\cos 2\rho_2 & 0 \end{pmatrix} \quad (13)$$

after calculating we obtain:

$$M_A = \frac{1}{2}\begin{pmatrix} 1 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 1 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (14)$$

The system Mueller matrix, $M_S = M_A \cdot M_G$, it is :

$$M_S = \frac{1}{2}\begin{pmatrix} 1 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 1 & \cos^2 2\rho_2 & \cos 2\rho_2 \sin 2\rho_2 & -\sin 2\rho_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho_1 & \cos 2\rho_1 \sin 2\rho_1 & -\sin 2\rho_1 \\ 0 & \cos 2\rho_1 \sin 2\rho_1 & \sin^2 2\rho_1 & \cos 2\rho_1 \\ 0 & \sin 2\rho_1 & -\cos 2\rho_1 & 0 \end{pmatrix}$$

without taking into account the factor $\frac{1}{2}$, its components are:

$$m_{11} = 1$$
$$m_{12} = \cos 2\rho_1 \cos 2\rho_2 \cos 2(\rho_2 - \rho_1) - \sin 2\rho_1 \sin 2\rho_2$$
$$m_{13} = \sin 2\rho_1 \cos 2\rho_2 \cos 2(\rho_2 - \rho_1) + \cos 2\rho_1 \sin 2\rho_2$$
$$m_{14} = \cos 2\rho_2 \sin 2(\rho_2 - \rho_1)$$
$$m_{21} = 1$$
$$m_{22} = \cos 2\rho_1 \cos 2\rho_2 \cos 2(\rho_2 - \rho_1) - \sin 2\rho_1 \sin 2\rho_2$$
$$m_{23} = \sin 2\rho_1 \cos 2\rho_2 \cos 2(\rho_2 - \rho_1) + \cos 2\rho_1 \sin 2\rho_2$$
$$m_{24} = \cos 2\rho_2 \sin 2(\rho_2 - \rho_1)$$
$$m_{31} = m_{32} = m_{33} = m_{34} = 0$$
$$m_{41} = m_{42} = m_{43} = m_{44} = 0 \tag{16}$$

Stokes vector of the emergent light from PSA, according to (11) will be:

$$S' = \frac{1}{2}\begin{pmatrix} 1+\cos 2\rho_1 \cos 2\rho_2 \cos 2(\rho_2 - \rho_1) - \sin 2\rho_1 \sin 2\rho_2 \\ 1+\cos 2\rho_1 \cos 2\rho_2 \cos 2(\rho_2 - \rho_1) - \sin 2\rho_1 \sin 2\rho_2 \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

Only Stokes parameters $S_1$ and $S_2$ are different from zero. The first Stokes parameter $S_1$ describes the total intensity of the optical beam, the second parameter $S_2$ represents the preponderance, in intensity, of horizontally polarized light over vertically polarized light [15]. Both parameters depend on typical angles of the quarter-wave plates, $\rho_1$ and $\rho_2$. If the rotation of the quarter-wave plates is at harmonic rates (e.g.: $\rho_2 = 5\rho_1$ ) [1, 2, 9] then results a intensity modulation of the detected signal [2, 9, 16].

## 3. Conclusions

In this paper we determined the Mueller matrices of the PSG and PSA for both Mueller matrix imaging polarimeters. In addition, for the second Mueller matrix imaging polarimeter we have calculated the Mueller matrix of the system and the Stokes vector of the emergent polarized light from PSA, in an important case, without the sample.

The Mueller matrices elements of the polarization state generator and polarization state analyzer of both Mueller matrix imaging polarimeters depend on the transmission axis azimuth $\theta$ of the polarizers and, also, depend on the fast axes azimuths of the quarte-wave plates $\rho_1$ and $\rho_2$. The Mueller matrix of the sample will be determinate by choosing a minimum of sixteen pairs of angles ( $\rho_1$, $\rho_2$ ). In practice, the Mueller matrix imaging polarimeters determine the Mueller matrix of the sample by choosing a different number of angle pairs ( $\rho_1$, $\rho_2$ ), e.g. 16 [17, 18], 34 [18], 48 [14], 60 [2].

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