# Modal SSDI-Max Technique of a Smart Beam Structure: broadband excitation

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Semi-active control is based on modal control strategy that needs very some energy for work but is effective only when the excitation is targeted on a unique mode. To improve the performance of semi-active control in the case of a broadband excitation, a modal approach SSDI-Max is proposed. This present paper presents an analysis of the performance of the SSDI-Max damping technique with a Beam-structure. It relies on simulations, made with the Matlab-Simulink environment, using a realistic model of a beam structure previously identified. The proposed method aims at maximizing the amplitude of the piezoelectric actuator by the definition of an optimal switching time according to the targeted mode chosen. Starting at this time, an algorithm is implemented to wait for the next voltage extremum within a given time window. The performances of the SSDI-Max method for the control of single mode of the structure are described in the case of pulse and noise excitations. Finally the influence of the delay time window used is described again for pulse and noise excitations for various modes.

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## 1. Introduction

Effective vibration control can improve the precision of the machining, reduce the fatigue load and prolong the operational life of industrial structures. Plenty of vibration control methods have been developed and already applied into the commercial arena. Smart materials especially piezoelectric material based vibration control techniques grow rapidly during the past two decades. These approaches can be generally classified as three catalogs which are passive control, active control and semi-active or semi-passive control.

Passive control is the earliest developed vibration control methods. These methods are easy to implement, sensor and power needless and unconditional stable [1]. The main disadvantage of the passive control is that the control bandwidth is too narrow. Once the structure dynamic properties changes, the control system would need to be retuned. In addition, the passive control needs to add non negligible mass to the structure which could be unacceptable in aerospace field.

Active control is developed based on the development of the computer science [2, 3]. The control system needs sensors to monitor the displacement or velocity of the structure. The sensed signals then send to a controller in order to obtain a control signal by employing specific algorithms. The control signal would be amplified by power amplifier or directly exert on actuators to generate feedback force on the structure which usually has the opposite phase with the external excitation. The power consumption would be large and the spill-over could be induced for high frequency control leading to instability.

Semi-active control or semi-passive control possesses partial advantages of both passive and active control. Usually, these control strategies are hysteretic or nonlinear in nature. By using small amount of energy, these methods can change the structural dynamic properties by changing the control state thus achieving the damping. The semipassive and semi-active methods can be distinguished by how we use the small amount external energy. The criterion is that if this energy is only used for power-up the control system but not used for inducing the control force, such method can be ranged with semi-passive method. Otherwise, it is a semi-active control. However, the energy used for inducing the control force is very small comparing with conventional active control. Several control systems have been proposed and investigated in the literatures, include active variable stiffness (AVS) in which the stiffness can be switched between high and low values [4], electro-rheological (ER) dampers and magnetorheological (MR) dampers which damp the structural vibrations by tuning the intensity of electric field or magnetic field with the structural motion [5, 6].

Among kinds of semi-active and semi-passive vibration control methods, Synchronized Switch Damping (SSD) techniques are proved to be an effective treatment. Compared with the passive methods, the system has the immunity against the structure dynamic properties shift due to the environmental change. They are also compact, lightweight which is convenient to apply to specific

structure with weight or size restriction. Compared with the active control, SSD control system is very simple to implement and can easily be self-powered from the vibration itself. In these techniques, the switch in the circuit is intermittently switched leading to a non-linear voltage processing. The piezo-force induced by such voltage always shows an opposite sign with the structure velocity which leading the vibration suppression on the structure.

Synchronized Switch Damping on Short circuit (SSDS) was proposed by Richard et.al. [7]. The circuit of SSDS is very simple which only consists of a wire, a switch and a small resister. In order to maximizing the damping, Synchronized Switch Damping on Inductor (SSDI) is proposed by Richard et.al. In [8]. The voltage amplitude in SSD techniques is significant. Synchronized Switch Damping on Voltage source (SSDV) technique is proposed to artificially enhance such voltage especially for systems with low electromechanical coupling [9, 10]. In the original SSDV, the sign of the continuous voltage source changes with the structural speed direction that increases the piezo-voltage during the inversion process. However, it would lead stability problems. Since the absolute value of the voltage source is constant, which images the force induced by this voltage is also constant. It could excite the structure when the structural vibration level is low instead of suppressing the vibration. Badel et.al developed an enhanced SSDV so called SSDV on adaptive voltage source [11].

During the early research stage, SSD techniques usually deal with the monomodal vibration case. However, researchers' put more and more attention to the multimode vibration control in the recent years. Several methods listed below are developed to refine SSD application for lower mode control in the multimode vibration. They could be classified as two catalogues: mode distinguishes based approaches and time window based approaches.

For multimode vibration, the sampled signal usually consists of the information from different modes. Mode distinguish based approaches are used to separate this signal into several components in which contains only single mode information. Harari et.al proposed to use modal observer to select the desired modes [12]. The propose of modal observer is similar with numerical filter, that is to distinguish each mode from the global motion. The further researches found that even the switching occurs at the extrema of the distinguish mode, it may still not be the optimal control for the target mode [13]. A SSDI-Max is then developed with an enhanced switching law for a bimodal vibration control [14, 12, 15, 16] aiming at dissipated more energy of the structure.

This present paper presents an analysis of the performance of the SSDI-Max damping technique with a Beam-structure. It relies on simulations, made with the Matlab-Simulink environment, using a realistic model of a beam structure previously identified. The proposed method aims at maximizing the amplitude of the piezoelectric actuator by the definition of an optimal switching time according to the targeted mode chosen. Starting at this time, an algorithm is implemented to wait for the next voltage extremum within a given time window.

The smart structure modeling along with the beam structure investigated is first described. Then the SSDI-Max strategy is exposed. The performances of the SSDI-Max method for the control of single mode of the structure are described in the case of pulse and noise excitations. Finally the influence of the delay time window used is described again for pulse and noise excitations for various modes.

## 2. Smart Structure

#### 2.1 Smart Structure Modeling

The aim of the method presented here is to control a structure submitted to a broadband excitation with a minimum of actuators and sensors. Its principle is based on modal control and the type of control chosen to eliminate operative power energy and amplifier is SSDI control. The control of energy quantity due to only the kinetic energy of the structure is often limited, particularly in the field of transportation structures. Modal control makes it possible to concentrate this energy on targeted mode in order to increase modal damping. So, a modal modeling of the smart structure is thus necessary.

The electromechanical behavior equations of a smart structure using usual assumption are [17]:

$$m\ddot{\delta} + c\dot{\delta} + k^E\delta = -\alpha V + \beta F \tag{1}$$

$$I = \alpha \delta - C_0 \dot{V} \tag{2}$$

with  $\delta$  the nodal displacement vector, *m*, *c* and  $k^E$  are respectively the mass, damping and stiffness matrices when the piezoelectric patches are in short circuit,  $\alpha$  is the electromechanical coupling matrix, *V* is voltage vector of the *i* piezoelectric patches, *I* is the electric current vector, and  $C_0$  is the diagonal capacitance matrix. *F* is the force applied to the system.

By using the following variable change where  $\delta$  is the mode shape matrix limited to *n* modes and *q* the modal displacement vector:

$$\delta = \phi q \tag{3}$$

The equations (1) and (2) can be well represented by the projection in the modal basis by:

$$M\ddot{q} + C\dot{q} + K^{E} q = -\theta V + \beta F \tag{4}$$

$$I = \theta^t \dot{q} - C_0 \dot{V} \tag{5}$$

with *M*, *C*,  $K^E$  are respectively the mass, damping and stiffness modal matrices, and  $\theta$  is the modal electromechanical coupling matrix with *[n, i]* matrix size, This later is defined as follows:

$$\theta = \phi^t \alpha \tag{6}$$

The structure being assumed lightly damped, with proportional damping and with modes sufficiently decoupled, and by using a norm such as modal matrix is the identity matrix, (4) can be written as a function of  $\xi$  the modal damping vector,  $\omega^E$  is the frequency vector when the actuator is in short circuit, and  $\omega^D$  is the frequency vector when it is in open circuit. Thus the modal mechanical matrices of (4) become:

$$M = I_d; \ C = 2 \operatorname{diag}(\xi) \operatorname{diag}(\omega^D); \ K^E = \operatorname{diag}\left(\left(\omega^E\right)^2\right)$$
(7)

By separating the actuators and sensors voltages, named, respectively,  $V_a$  and  $V_s$ , Equation (4) becomes:

$$M\ddot{q} + C\dot{q} + K^{E} q = -\theta_{a} V_{a} - \theta_{s} V_{s} + \beta F$$
(8)

In an open circuit or when the sensor voltage is monitored with a voltage amplifier, sensor intensity is null, therefore:

$$\theta_s^t q - C_{0s} V_s = 0 \tag{9}$$

And by reintroducing the Equation (9) in Equation (8):

$$M\ddot{q} + C\dot{q} + \left(K^{E} + \theta_{s}\left(C_{0s}\right)^{-1}\theta_{s}^{t}\right)q = -\theta_{a}V_{a} + \beta F$$
(10)

Linear systems (10) and (9) can be written in modal state under form:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad \qquad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$
(11)

With x being the state vector,  $u = [F, V_a]$  is the control vector,  $y = [q, \dot{q}, V_s]$  is the output vector, A, B, C are the state matrices:

$$A = \begin{bmatrix} 0 & I_{d} \\ -M^{-1} \left( K^{E} + \theta_{s} C_{0s}^{-1} \theta_{s}^{t} \right) & -M^{-1} C \end{bmatrix},$$
  
$$B = \begin{bmatrix} 0 \\ +M^{-1} \beta & -M^{-1} \theta_{a} \end{bmatrix}, C = \begin{bmatrix} I_{d} & 0 \\ 0 & I_{d} \\ C_{0s}^{-1} \theta_{s}^{t} & 0 \end{bmatrix}$$
(12)

 $V_a$  Calculated by the following relation:

$$V_a = C_{0a}^{-1} \theta_a^t q \tag{13}$$

 $C_{0a}$  is the capacity of actuator and the matrix  $C_{0s}$  is capacity sensors.

#### 2.2. Smart Structure definition

The structure that will be used in the following simulations and analyses is a clamped-free smart beam composed of a beam in dur aluminium and four P188 piezoelectric patches bonded on the beam. One is used as actuator, two others as sensors, and the last for the excitation. The characteristics of the smart structure are given in table 1. Figure 1 illustrates this beam. This structure has been identified according to the previously described model. The measurement process and parameter identification is described in [18]. Table 2 summarizes the frequencies of the three modes considered in the model.



Table.1 Materials properties [18].

Aluminum beam:		
density : $\rho = 2800$	Young	Poisson ratio
Kg.m <sup>-3</sup>	modulus E =	v = 0.3
-	$7 \times 10^{11}$ Pa	
Piezoelectric ceramic : P188 Ceramic		
density $\rho c = 7700$		
Kg.m <sup>-3</sup>		
Dielectric	$\epsilon^{s}_{11} = 12.75 \times$	$\epsilon^{S}_{33} = 7.411$
Permittivity:	10 <sup>-9</sup> F/m	$\times 10^{-9} \text{ F/m}$
Elastic	$s_{11}^{E} = 15.44 \times$	$s_{33}^{E} = 20.09$
compliances :	$10^{12} \text{ m}^2/\text{N}$	$\times 10^{12} \text{ m}^2/\text{N}$
Charge	$d_{31} = -186$	$d_{33} = 425$
coefficients :	pC/N	pC/N

The resonance frequencies of the lower modes are given in Table 2.

Table.2. Frequencies of the three simulated modes of the beam [18]

Modes	Frequency
Mode 1	31.86 Hz
Mode 2	171.52 Hz
Mode 3	433.96 Hz

#### 3. Modal SSDI Control

#### **3.1 SSDI control**

The semi-active control implemented is the SSDI, Synchronized Switch Damping on Inductor, which is efficient without actuation energy supply [8]. The SSDI consists in connecting the piezoelectric element to a specific electrical circuit composed of a switch and an inductance L connected in series (Figure 2 (a)). In the case of sinusoidal excitation, the switch is almost always open, and in this case the voltage and strain vary proportionally. When the voltage is extremum, the switch is closed until the voltage on the piezoelectric element has been reversed. The inversion is possible thanks to the capacitance  $C_0$  of the piezoelectric element and the inductance L which constitute an electric oscillator, as shown in the following equation:

$$\frac{d^2 V_a}{d t^2} + \frac{1}{C_0 L} V_a = 0$$
(14)

By choosing inductance L correctly, the electric period is very short compared to the mechanical vibration period and the switch is kept closed for exactly half a period, allowing the complete inversion of the piezoelectric actuator voltage (Fig. 2 (c)).

The repetitive voltage inversion process induces a self generating voltage crenel function nearly in quadrature with the displacement. The voltage amplitude is magnified by the repetitive switch process. As, this voltage generates stress, the resulting forces are out of phase with the speed thus creating energy dissipation.

The voltage inversion imperfection is due to losses in the inversion network, the reversed voltage  $V_{after}$  is lower than the voltage prior the inversion  $V_{prior}$ . An inversion factor  $\gamma$  is defined as:

$$V_{after} = -\gamma . V_{prior}$$
(15)

This  $\gamma$  coefficient is related to the electrical quality factor of the oscillating network.



Fig.2. (a) The SSDI circuit, (b) The typical voltage waveforms, where  $V_a$  is the piezoelectric actuator voltage and  $q_1$  is the corresponding first modal displacement and (c) the voltage inversion.

## 3.2 Modal SSDI control

In the case of a wide bandwidth excitation, many local extrema appear on the actuator voltage  $V_a$  as shown in Figure 2 (b). In this case, the SSDI strategy, which consists of switching at each voltage actuator extremum, is not optimal. Given that the structure vibrates mainly on the eigen frequencies, the proposed control focuses on the modes and uses one SSDI controller for each controlled mode driven by the targeted modal displacement. Even in the case of highly structure complex motion, the actuator voltage is reversed when the modal displacement of the targeted mode is maximum. This inversion on the maximum of the modal displacement is possible thanks to its numerical reconstruction by the observer. Figure 2 (b) shows the waveform of the modal displacement qi, which

determines the moment of the inversion and the waveform of the actuator voltage  $V_a$  of the piezoelectric element.

The inversion generated by the control SSDI generates a crenel function in quadrature with the targeted modal displacement (Fig. 2 (c)) that can be expressed by:

$$V_a(t) = \frac{\theta'}{C_{0a}} q(t) + h(t)$$
(16)

The first term of the Equation (16) corresponds to the voltage that should appear on the open-circuited actuator. The inversion involves the second term is a self generated crenel function that is always in phase with the sign of  $\dot{q}_i(t)$ , i.e., the targeted modal speed. The frequency of the crenel function is the same as the frequency of the mode controlled. The self generated crenel function could re-inject harmonics. Moreover, its amplitude can be modulated by the uncontrolled modes. Consequently, the control can interact with uncontrolled modes, resulting in spillover. The modulation frequency depends on the frequency ratio of the structure over the frequency of the controlled mode.

# 3.3 Energetic analysis

The energy E at time t is composed of kinetic energy, potential energy, mechanical losses and energy coupling. The latter type of energy for each mode corresponds to the share of mechanical energy is converted into electrical energy [19].

$$E = \left[\frac{1}{2}M\dot{q}\right]_{0}^{t} + \left[\frac{1}{2}K^{E}q^{2}\right]_{0}^{t} + \int_{0}^{t}C\dot{q}^{2}dt + \int_{0}^{t}\theta V\dot{q}dt \quad (7)$$

Table.3. Terms of energy [19]

Kinetic energy	$\frac{1}{2}M\dot{q}^2$
Elastic potential energy	$\frac{1}{2}K^Eq^2$
Viscous losses	$\int C \frac{d^2}{q^2} dt$
Energy transferred	∫ <i>θV</i> ġ dt

Compared to the potential and kinetic energy, mechanical losses and modal coupling energy is weak and can be neglected. Therefore the modal energy at time t can be calculated as follows:

$$E = \left[\frac{1}{2}M\,\dot{q}\right]_{0}^{t} + \left[\frac{1}{2}K^{E}q^{2}\right]_{0}^{t}$$
(18)

The calculation of the modal energy is real-time matrices from modal mass and mechanical stiffness and the modal displacements and modal velocities.

#### 3.4 SSDI Max modal control

#### 3.4.1 Principle

The technical implementation SSDI-Max is an alternate modal-SSDI control which increases the growth of the piezoelectric actuator voltage. Damping performance is strongly dependent on this voltage amplitude. The SSDI allows a natural growth of this voltage using a cumulative effect [17]. However, if this cumulative effect is affected by local maximum, the voltage magnification is not optimal. It is the objective of the modal-SSDI-Max technique to correct this point.

SSDI-Max strategy is to wait for the next extreme point of tension after an extremum of displacement modal fashion targeted to reverse the voltage across the piezoelectric element. Thus, the inversion is done from a higher voltage but still approximately synchronized with the targeted modal shift mode which implies a significant increase in depreciation. This method allows using energy methods to increase uncontrolled depreciation of a targeted fashion.



Fig..3. Shape of the modal displacement and the voltage across an actuator controlled by the control law SSDI-Max.

#### 3.4.2 Algorithm SSDI-Max

As soon as a maximum modal shift is achieved, the signs of tension and its derivative are recorded.

• If the voltage is positive and the derivative is negative, the reverse voltage is immediate.

• If the voltage is positive and the derivative is positive, the system waits for the next maximum voltage to reverse voltage.

• If the voltage is negative, the system waits until the voltage is positive and its derivative negative before reversing the voltage.

The algorithm is symmetric if a minimum modal displacement is reached.

### 4. Simulation

The simulations are performed using the Matlab/Simulink<sup>TM</sup> software environment. The inversion

coefficient  $\gamma$  is set to 0.7 which is a realistic value [8]. The first step in SSDI-Max validation is the control of a single mode of the structure. This single mode control is considered with multi-sinusoidal excitation, white noise excitation and pulse excitation (the first, second and third mode) of structure. The effect of the time window on SSDI-Max performance is established in the later section.

#### 4.1 White noise excitation

Like a pulse this type of broadband excitation results in vibration energy shared on all the three considered modes of the model.



Fig.4. Modal displacement and the voltage across the actuator when the single mode is targeted by the control. (a) mode 1 control, (b) mode 2 control, (c) mode 3 control.

Fig.4 (a, b and c) shows a comparison of the modal SSDI and modal SSDI-Max damping performances. It shows a partial time window representative of the 3s long

simulation time. The Fig.4 (a) lower plot presents the first modal coordinate which is targeted by the control. Fig.4 (b) lower plot presents the second modal coordinate which is targeted by the control and Fig.4 (c) lower plot presents the third modal coordinate which is targeted by the control The upper Fig. (a, b and c) illustrates the control voltage actuator. Again it is clearly visible that a slight shift in the switching time definition leads to a much improved actuator voltage amplitude, thus resulting in more efficient damping.

In the frequency regime this reduction of targeted mode 1, mode 2 and mode 3 are illustrated in Fig. 5.



Fig.5. Modal displacement and the voltage across the actuator when the single mode is targeted by the control. (a) mode 1 control, (b) mode 2 control, (c) mode 3 control.

#### 4.2 Pulse excitation

The excitation is a wide frequency square force pulse 50µs long, and with normalized amplitude.

Figure 6 illustrates the simulation results in the time domain in the case of Modal displacement and the voltage build-up improvement, Fig.6. (a) for mode 1 targeting, Fig.6. (b) for mode 2 targeting. Fig.6. (c) for mode 3 targeting.

Fig.6. (a, b and c) (upper) illustrates the voltage buildup improvement showing that small shifts in the switch instants result in amplification of the self generated control voltage. It should be noted that this change led to a much greater reduction of the targeted mode Fig.6. (a, b and c) (lower plot). This result is remarkable since the reference which corresponds to the structure controlled using modal SSDI is already a well damped.



Fig.6. Modal displacement and the voltage across the actuator when the single mode is targeted by the control. (a) Mode 1 control, (b) Mode 2 control, (c) Mode 3 control.

## 4.3 Influence of the time window on the performance of SSDI-Max

The time window that is defined in the strategy SSDI-Max is used to limit the possible time shift prior to switching, is a very important and critic parameter. If it is too small, the voltage will not have the possibility to increase and no significant enhancement will be observed. If it is too long, there is a risk of de-synchronization of the actuator voltage with the targeted modal speed, thus resulting altered damping. In order to define an optimal time window, simulations were made while varying the value of this window from zero (pure modal SSDI) to 2/5th of the targeted mode period for white noise excitation and from zero to 1/5th for pulse excitation.

## 4.4 White noise excitation

Fig.7. illustrates the variation of the targeted mode damping as a function of the time window for white excitation for mode. Fig.7. (a) when mode 1 control, Fig.7. (b) mode 2 control, Fig.7. (c) mode 3 control.



Fig.7. Influence of the time window on the single mode modal SSDI-Max damping for white noise excitation (a) mode 1 is targeted by the control (b) mode 2 is targeted by the control (c) mode 3 is targeted by the control.

Fig.8. shows the corresponding time domain simulation of the targeted modal coordinate for different points of Fig.7.

According to Fig.7. (a), a time window 1/30th of the period is nearly optimal for SSDI-Max control of mode 1, for Fig.7. (b and c) a time window 1/5th of the period is optimal for SSDI-Max control of mode 2 and 3.



Fig.8. The modal displacement of single mode in white noise excitation – (a) Mode 1 control. (b) Mode 2 control. (c) Mode 3 control.

## 4.5 Pulse excitation

Fig. 9. illustrates the variation of the targeted mode damping as a function of the time window for pulse excitation for mode. Fig.9. (a) when mode 1 control, Fig.9. (b) mode 2 control, Fig.9. (c) mode 3 control.

Fig.10. shows the corresponding time domain simulation of the targeted modal coordinate for different points of Fig.9.

According to Fig.9. (a, b and c), a time window 1/10th of the period is nearly optimal for SSDI-Max control of the three mode of structure.





Fig.10. The modal displacement of single mode in pulse excitation– (a) Mode 1 control. (b) Mode 2 control. (c) Mode 3 control.

## 5. Conclusion

The semi-active modal techniques have many advantages. Contrast to passive techniques, it is insensitive to changes in external conditions. On the other hand, the semi-active modal control has the advantage of not consuming any energy action. From the control laws semiactive modal, modal SSDI technique is to reverse the voltage across the piezoelectric element at each extremum of the modal shift target mode. However, in the case of complex or when the excitation amplitude modes targeted

Fig.9. Influence of the time window on the single mode modal SSDI-Max damping for pulse excitation. (a) Mode 1 damping. (b) Mode 2 damping. (c) Mode 3 damping control is lower than that of non-target modes, the damping obtained may be insufficient. To increase the control voltage, the SSDI-Max method is to wait for an extremum of tension instead of reverse voltage immediately after an extremum of displacement modal.

Validation of this concept was done using numeric simulation in the case study of a smart beam structure.

Modal SSDI-Max Damping simulations results showed neatly improved damping performances compared to modal SSDI for the control of a single mode of the structure, in the case of white noise excitation on the three modes of the beam and pulse excitation. Remarkable gains in attenuation were obtained. Finally the influence of the maximum time delay between the targeted modal coordinate extremum and the switch instant was evaluated and results show that a maximum delay of 1/10th of the targeted mode period is nearly optimal for pulse excitation and 1/5th for white noise excitation. Further works aims at implanting and validating experimentally the proposed concept.

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