JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 13, No. 8, August 2011, p. 1004 - 1009

Mode transforming properties of taper microlensed fibers

N. LAMBRACHE*, H. TEIMOORI[a]
Alef Photonics, Ottawa, Canada
[a]*Alef Photonics, Ottawa, Canada*

Drowned tapered and microlensed single-mode fibers offer significant manufacturing advantages as waveguide couplers. However, controlled manufacturing requires a better understanding of beam propagation in such photonics devices. The authors investigate the mode transforming properties of drowned tapered and microlensed fiber optics and suggest optimal geometries.

(Received May 4, 2011; accepted August 10, 2011)

Keywords: Waveguide coupling, Tapered and microlensed single-mode fibers

1. Lightwave modes and gaussian approximations

The introduction of the concept of mode is a fundamental requirement for accurate modeling of light propagation in fiber optics. In general terms, a propagation mode is a self-consistent electric field distribution with the specific property that its shape in the trasnsverse direction remains constant during propagation. The simplest type of mode in free space is the plane wave. However, plane waves do not resemble real waves since they have an infinite transverse extent and therefore other modes limited to the transverse spatial dimension are of higher interest.

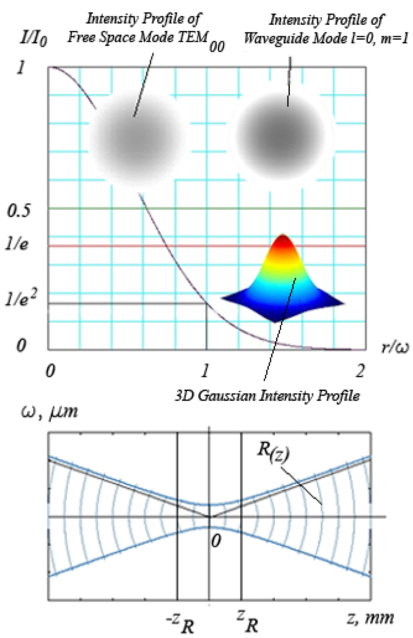

Fig. 1. Gaussian Beams.

The simplest type of mode satisfying the above requirements is the Gaussian mode [1]. Gaussian beams contract or expand during propagation, but the amplitude profile is only transversally scaled and has a constant – Gaussian – shape. Gaussian modes are members of families of modes and their number is infinite. The most frequently used mode families are Hermite-Gauss and Laguerre-Gauss. Within a mode family the Gaussian mode is the fundamental mode and during the propagation the higher order modes transversally change in proportion to that of the fundamental mode.

For light propagation in a waveguide the self-consistency condition for a mode is stricter than for free space modes. Mode rescaling is not permitted in waveguides. An overall phase change per unit length – described by the propagation constant β - and losses or gains of total optical power are acceptable. Waveguides have finite number of guided propagation modes and the intensity distribution of each mode has a finite extent around the core of the waveguide. A single-mode fiber has only a single guided mode per polarization direction. Waveguides also have cladding modes and their intensity distributions fill the whole cladding and core regions and exhibit substantial propagation losses at the outer interface of the cladding [2].

Gaussians are radially symmetric distributions with electric field variations given by:

$$E_s = E_0 \exp\left(-\frac{r^2}{\omega_0^2}\right) \tag{1}$$

The Fourier Transform of a Gaussian is also a Gaussian distribution and Gaussian source distributions remain Gaussian at every point along their propagation path. This property is useful in visualizing the field distributions anywhere in optical systems. The intensity of the field is also a Gaussian:

$$I_s = \eta E_s E_s^* = \eta E_0 E_0^* \exp\left(-\frac{2r^2}{\omega_0^2}\right) \quad (2)$$

The definition of the size of Gaussians is somewhat arbitrary because the Gaussian has no obvious boundaries. Figure 1 shows the Gaussian intensity distribution of a typical He-Ne laser. The parameter ω_0, called the *Gaussian Beam Radius*, is the radius at which the intensity has decreased to $1/e^2$ of its value on its axis. Because of the unique self-Fourier Transform characteristic of Gaussians, the transverse distribution intensity remains Gaussian and only its radius ω and the radius of curvature of the wavefront R change with the position z, as shown in figure 1. The equations describing the *Gaussian Beam Radius* ω and wavefront radius R are:

$$\omega(z) = \omega_0^2\left(1+\left(\frac{\lambda z}{\pi\omega_0^2}\right)^2\right)^{\frac{1}{2}} \quad (3)$$

$$R(z) = z\left(1+\left(\frac{\pi\omega_0^2}{\lambda z}\right)^2\right) \quad (4)$$

In equations (3) and (4) ω_0 is the *Gaussian Beam Radius* and λ is the wavelength. The above mentioned parameters occur in the same combination in both equations and they are often merged into a single parameter known as the *Rayleigh Range*, z_R [1] and at $z = z_R$, R is minimum:

$$z_R = \frac{\pi\omega_0^2}{\lambda} \quad (5)$$

Optical fibers have two essential parts: the core and the cladding. The core acts as a cylindrical waveguide, while the cladding prevents the guided mode from interacting with anything outside of the fiber. For a constant refractive index difference Δn between the core and the cladding, the mode profile within the core is described by the equation [2]:

$$f(r) = AJ_0\left(n_c^2 k_0^2 - \beta^2\right)^{\frac{1}{2}} \cdot r\ , \quad r < a \quad (6)$$

In equation (6), J_0 is the Bessel function of first kind, k_0 is the free space wavenumber, and β represents the propagation constant. For $r = a$ the profile is represented by:

$$f(r) = A\exp\left(-\frac{r}{\omega_G}\right)^2 \quad (7)$$

The function $f(r)$ from equation (7) strongly resembles a Gaussian for all r values, provided the mode width parameter ω_G is properly chosen. The Gaussian approximation described by equation (7) provides a convenient single-parameter characterization of the fundamental mode for weakly guiding, step index, single-mode fibers.

A model of beam to guided-mode coupling requires the summation of plane waves to describe the incoming beam and the guided mode and cannot be entirely based on ray optics since beams of finite sizes are made up of combinations of plane waves with different propagation directions. Efficient coupling requires overlapping of the incident beam profile $f_b(r)$ and the outgoing mode profile $f_m(r)$. Henry and Verbeek [03] define the coupling efficiency as the ratio between the power in the guided mode and the power in the incoming beam. If $f_b(r)$ and $f_m(r)$ have the same polarization and interface reflection is negligible the coupling efficiency is described by the equation:

$$\eta_c = \int_0^H f_b^*(r) f_m^*(r) dr = \sum_q^N DFT^*\left(f_b\right)_q DFT\left(f_m\right)_q \quad (8)$$

In equation (08), $f_b^*(r)$, $f_m^*(r)$ and DFT^* are the complex conjugates of their respective functions and DFT is the Discrete Fourier Transform. Henry and Verbeek equation helps identify the main factors affecting coupling efficiency.

2. Mode transforming properties in tapered core-cladding single-mode fibers

Several methods are used to manufacture tapered and microlensed single-mode fibers. For typical coupling applications the taper is shaped on a controlled process by drowning the partly melted tip of the fiber [4]. Such approaches generate fibers with tapered core and cladding. Microlenses with spherical surfaces are also generated in the process. The characteristic geometry of a core-cladding taper is shown in Fig. 2.

Manufacturing processes must be able to control the geometry of the taper and microlens as a way to control the spot size of the coupler.

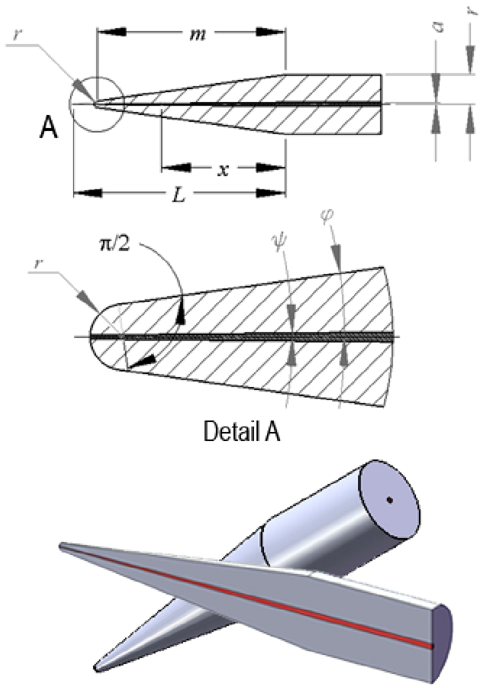

Fig. 2. Tapered fiber geometry.

Analytical modelling of the dependence of the spot size on the geometry of the taper requires the following approximations:

- The mode in the fiber taper at any point along the taper is equal to the equivalent mode of the straight fiber with the same core diameter. This approximation is in accordance with the *Local Mode Theory* [3], [4], and requires a gradual, adiabatic taper.
- The analysis accounts for the coupling from the core-guided mode to the cladding-guided mode
- The fundamental fiber mode is represented with a Gaussian beam approximation.

By approximating the fundamental fiber mode with a Gaussian radial distribution, the radius of the waist of a microlens with a focal distance f and radius r in image space is given by the equation:

$$\omega_f = \frac{\omega_l}{\sqrt{1+\left(\frac{\pi\omega_l^2}{\lambda}\right)^2 \cdot \left(\frac{n_c-1}{r} - \frac{n_c}{R_l}\right)^2}} \qquad (09)$$

In equation (09) λ is the free space wavelength, ω_l is the Gaussian beam radius at the microlens, and R_l is the radius of curvature of the wavefront at the lens. The focal length f of the melted fiber lens is given by the equation:

$$f = \frac{r}{n-1} \qquad (10)$$

In equation (10) r represents the radius and n represents the refractive index of the microlens.

For single-mode fibers the mode radius ω_l - defined as the radial distance at which the field amplitude is $1/e^2$ of its maximum - can be approximated by Marcuse's equation [03]:

$$\omega_l = \left(0.65 + \frac{1.619}{V^{\frac{3}{2}}} + \frac{2.879}{V^6}\right) \cdot a \qquad (11)$$

The V *Number* or normalized frequency is defined by the equation:

$$V = \frac{2\pi a}{\lambda} NA = \frac{2\pi a}{\lambda}\sqrt{n_c^2 - n_{cl}^2} \qquad (12)$$

In equation (12) a is the core radius, n_c the refractive index of the core, and n_{cl} the refractive index of the cladding. Marcuse's equation is valid for V *Number*s between 0.8 and 2.5.

For a drawn taper it is assumed that the cladding and the core radii maintain their initial ratio [04]. To calculate the mode radius at a given point on the taper, it is necessary to calculate the fundamental cladding-guided mode radius, to compare it with the core-guided mode radius, and to choose the smaller of the two as the mode radius. This is equivalent to assuming that the core-guided mode couples completely to the fundamental cladding-guided mode at the point at which the two mode radii are equal.

Using the notations from Fig. 2 and based on the assumption that the cladding and the core radii maintain their initial ratio:

$$L = \frac{d_{cl}}{2\tan\varphi} \qquad (13)$$

$$\tan\psi = \frac{a}{L} = \frac{a_x}{L-x} \qquad (14)$$

The core radius at the distance x from the beginning of the tapered region is:

$$a_x = \frac{a(L-x)}{L} \qquad (15)$$

Other geometrical parameters of interest are:

$$m = L - \frac{r}{\sin\varphi} \tag{16}$$

$$\psi = \arctan\left(\frac{d_c}{d_{cl}}\tan\varphi\right) \tag{17}$$

The core-guided mode radius is calculated with the equation:

$$\omega_{l\,c} = \left(0.65 + \frac{1.619}{\left(\frac{2\pi a(L-x)\sqrt{n_c^2 - n_{cl}^2}}{\lambda L}\right)^{1.5}} + \frac{2.879}{\left(\frac{2\pi a(L-x)\sqrt{n_c^2 - n_{cl}^2}}{\lambda L}\right)^{6}}\right)\cdot\frac{a(L-x)}{L} \tag{18}$$

The cladding-guided mode radius is calculated with the equation:

$$\omega_{l\,cl} = \left(0.65 + \frac{1.619}{\left(\frac{2\pi r_{cl}(L-x)\sqrt{n_c^2 - 1}}{\lambda L}\right)^{\frac{3}{2}}} + \frac{2.879}{\left(\frac{2\pi r_{cl}(L-x)\sqrt{n_c^2 - 1}}{\lambda L}\right)^{6}}\right)\cdot\frac{(L-x)r_{cl}}{L} \tag{19}$$

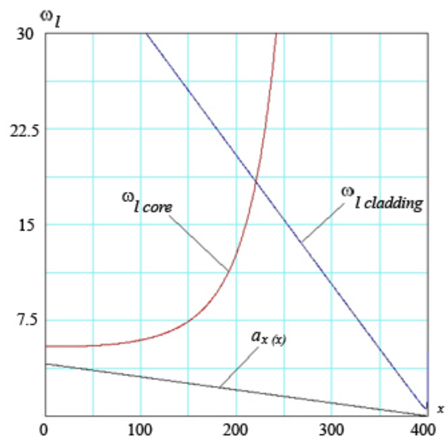

Fig. 3. Mode radius variation in core-cladding tapers.

The mode radius variation into the taper is represented in Fig. 3. As the core-guided mode propagates along the taper it first contracts slightly, then spreads into the cladding, and gradually couples to the cladding-guided mode. It is assumed that the coupling occurs only from the core-guided mode to the fundamental cladding-guided mode.

Equation (09) will estimate the spot size accurately, unless:

$$|R_l| >> \frac{n_c r}{n_c - 1} \tag{20}$$

The radius of curvature of the Gaussian beam can be expressed as:

$$R_l \approx \frac{r}{\frac{dr}{dz}} = \frac{r}{\tan\varphi} \tag{21}$$

In equation (21) z is the direction of propagation. Introducing (21) into (20):

$$\omega_f = \frac{\omega_l}{\sqrt{1 + \left(\frac{\pi\omega_l^2}{\lambda}\right)^2 \cdot \left(\frac{n_c - 1 - n_c\tan\varphi}{r}\right)^2}} \tag{22}$$

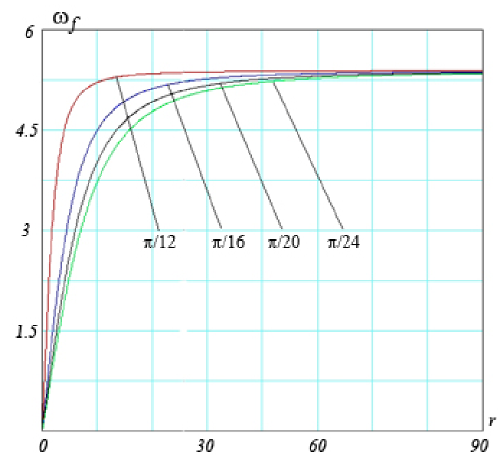

Fig. 4. Spot size variation with taper angle and lens radius.

3. Taper finite element modelling

Limitations and ambiguities of analytical approaches require a more accurate modelling of light wave propagation in tapered and microlensed single-mode fibers. Numerical solutions to the paraxial approximation of the Helmholtz equation for monochromatic waves are based on finite difference beam propagation method. By restricting the wave propagation to narrow angles and neglecting polarization effects, the scalar field assumption is valid and the wave equation can be written in the known Helmholtz form:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} + k^2(x,y,z)\phi = 0 \tag{23}$$

For most guided-wave problems, the fastest variation of the field ϕ is the phase variation due to propagation along the guiding axis. Assuming the guiding axis is z, this fast variation can be factored out by with the slowly varying field $u(x,y,z)=\phi(x,y,z)/e^{i\vec{k}z}$. $\vec{k}$ is the reference wavenumber and represents the average variation of the field ϕ. It is common practice to express the wavenumber in terms of a reference refractive index $\vec{n}$ as in $\vec{k}=k_0\vec{n}$. Under these assumptions one can write the three dimensional Beam Propagation Method (BPM) equation:

$$\frac{\partial u}{\partial z}=\frac{i}{2\vec{k}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\left(k^2-\vec{k}^2\right)u\right) \tag{24}$$

For a given input field $u(x,y,z=0)$, the BPM equation (24) describes the evolution of the field in the space $z>0$. Furthermore, the 3D equation (24) can be simplified for 2D by omitting all dependencies on y.

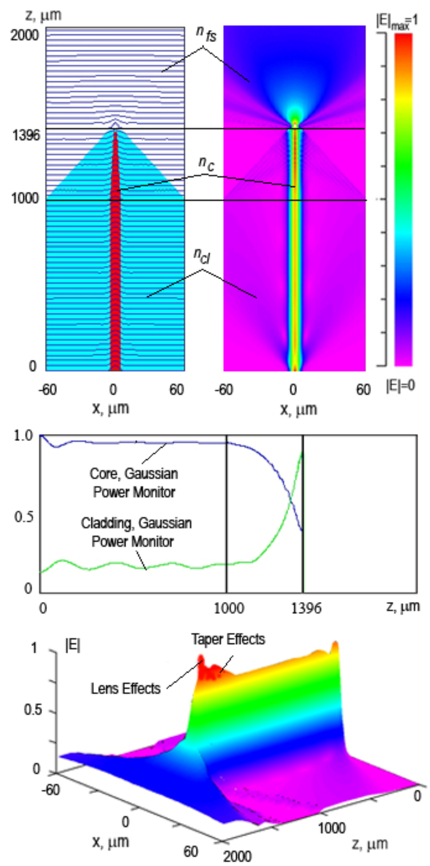

Fig. 5. Taper beam propagation simulation.

The authors investigate core-cladding tapered and microlensed devices drown from Corning SMF-28 fiber with the following specifications:

- Mode-Field Diameter MDF at λ =1550 nm: 10.4±0.8 μm
- Core Diameter: 8.2 μm
- Cladding Diameter: 125.0±0.7 μm
- Effective Group Index of Refraction at nominal MFD: 1.4682 nm
- Refractive Index Difference: 0.005
- Numerical Aperture: 0.14

Simulation settings:

- Free Space Wavelength: λ =1550 nm
- Background Refractive Index: 1
- Core Refractive Index: 1.455
- Core/Cladding Refractive Index Difference: 0.005
- Profile: Step Index
- 3D Structure: Fiber
- Crank-Nicholson Implicit Scheme
- Simple Transparent Boundary Condition
- Padé Order: (1,0)
- Mode Calculation Method: Correlation
- Cladding Taper Angle: $\pi/20$
- Core Taper Angle: $\pi/360$
- Taper Length: 395 μm
- Lens Diameter: 13.7 μm
- Lens Radius: 13.85 μm

Fig. 5 shows the simulated beam propagation on a tapered core-cladding single-mode fiber terminated with a microlens. Tapering effects – covering the region 1000 μm to 1395 μm – are visible on the Contour Map (upper right) and Height Coded (bottom) images from Figure 5. The evolution of the Gaussian Power Monitors alongside the propagation confirms the validity of the analytical method proposed by Barnard and Lit and obtained by employing the Local Mode Theory.

The authors performed the evaluation of the Mode Field Radius ω_l of a tapered fiber with the above specified geometry with an LD 8900R Far Field Profiler from Photon Inc. The measurement device is a real time scanning pinhole goniometric radiometer. The instrument is based on a scanning method where neither the detector nor the light source moves, yet the system provides a hemispherical irradiance measurement with 0.05 degree resolution in the angular direction and sub-degree resolution in the azimuthal direction. The measured values are:

- $\omega_{f\,\min}=3.07\mu m$
- $\omega_{f\,\max}=3.49\mu m$
- Standard Deviation:

$$\sqrt{\frac{n\sum_{i=1}^{n}\omega_{fi}^{2}-\left(\sum_{i=1}^{n}\omega_{fi}\right)^{2}}{n(n-1)}}=0.135\mu m,\ n=10$$

- $Mean = 3.28\mu m$

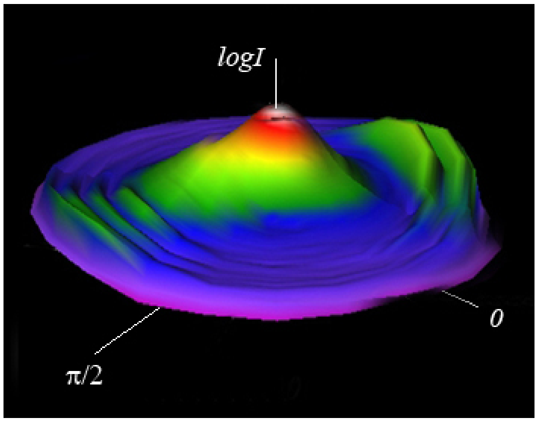

Fig. 6. Measured 3D Mode Field profile intensity.

The predicted value $\omega_f = 3.23\mu m$ falls between the minimum and maximum evaluations and close to the mean.

5. Concluding remarks

The finite element approach and the experimental results confirm the validity of the analytical methods proposed by Barnard and Lit and allow sound recommendations regarding optimal geometry for tapered microlensed single-mode fibers.

The spot radius is independent of the radius of the lens for a well defined range of taper angles and the result is important for manufacturing.

References

[1] B. Saleh, M. Teich, Fundamentals of Photonics, John Wiley & Sons, New York, USA (1991).
[2] B. Buckman, Guided-Wave Photonics, Saunders College Publishing, San Diego, California, USA (1992).
[3] R. Paschotta, Field Guide to Optical Fiber Technology, SPIE Press, Bellingham, Washington, USA (2010).
[4] C. Barnard, J. Lit, Single-Mode Fiber Microlens with Controllable Spot Size, Applied Optics, **30**(15), 1958 (1991).

____
*Corresponding author: Nicholas.Lambrache@alefphotonics.com