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Models for few-cycle optical solitons

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The propagation of few-cycle optical pulses (FCPs) in nonlinear media can be described by means of a model of modified Korteweg-de Vries-sine Gordon (mKdV-sG) type. This model has in some special situations the advantage of being 'integrable', which allows us to study the interactions between FCPs. In addition, it is very general: we show that all other non-slowly varying envelope approximation models of FCP propagation which can be found in the literature, especially the so-called 'short pulse equation', are in fact approximations or special cases of the mKdV-sG model. Finally, an analogous model valid in the case of a quadratic nonlinearity will be discussed.

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1. Introduction

Few-cycle optical pulses, since their first experimental realization in 1999 are the matter of intensive research activity (see the comprehensive review [1]). From the fundamental point of view, many phenomena involving ultra-short optical pulses (with very broad spectra) are studied in detail at present, such as the supercontinuum generation. A new and interesting physics appears in the study of these unique phenomena.

It is worthy to mention that the slowly varying envelope approximation (SVEA) is no longer valid under these conditions. Although generalizations have been proposed and have proven their efficiency [2], a completely different approach to the study of few-cycle pulses, which completely abandons the SVEA is desirable.

We showed, for the first time to our knowledge, that by using the reductive perturbation method (or multiscale expansion) applied to the Maxwell-Bloch equations, universal equations such as the modified Korteweg-de Vries (mKdV) or sine-Gordon (sG) ones could account for the propagation of few cycle solitons in a transparent Kerr medium [3]-[7].

These earlier studies were then generalized to a system consisting of two atomic transitions, one below and one above the range of propagated wavelengths. As a result, a model of mKdV-sG-type equations was put forward [8]-[10]. In certain cases this nonlinear dynamical system is completely integrable by means of the Inverse Scattering Transform (IST) method. It admits stable solutions of 'breather' type, which also give a good account of few cycle soliton propagation. Integrability allowed us to investigate the interaction of solitons and it was found that no phase matching is required [11]. The propagation of few-cycle pulses in a quadratic medium has also been described by either a Korteweg-de Vries (KdV) or a Kadomtsev-Petviashvili (KP) equation, in (1+1) or (2+1) dimensions respectively, which evidenced either the stability of a plane wavefront, for a normal dispersion, or the formation of a localized spatiotemporal half-cycle soliton, for an anomalous dispersion [12]-[13]. The present paper briefly summarizes these new results.

2. The most general SVEA model

The mKdV-sG equation is:

$$E_z + c_1 \sin\left(\int^t E\right) + c_2\left(E^3\right)_t + c_3 E_{ttt} = 0. \qquad (1)$$

As discussed above, it can be derived from Maxwell-Bloch equations, and describes FCP soliton propagation in a Kerr medium. It reduces to mKdV equation if $c_1 = 0$, and to sG equation if $c_2 = c_3 = 0$. The two latter equations are completely integrable by means of the IST, and Eq. (1) is also completely integrable if $c_3 = 2c_2$ [14]. It admits breather solutions, which describe FCP solitons (see Fig. 1.)

Other non-SVEA models have been proposed to describe FCP soliton propagation. Among them is the so-called Short-Pulse Equation (SPE) [15]:

$$E_{zt} = E + \frac{1}{6}\left(E^3\right)_{tt}. \qquad (2)$$

The SPE is completely integrable [16], and accounts for FCP soliton propagation. It is easily shown that the SPE can be derived from mKdV-sG [17]: let us first perform a small amplitude approximation, so that the sine term in Eq. (1) reduces to $c_1 \int^t E$, then the mKdV-type dispersion is neglected: $c_3 = 0$. A linear change of variables allows to fix the values of the remaining coefficients to $c_1 = -1$, $c_2 = -1/6$, and derivation with respect to t yields exactly the SPE equation (2).

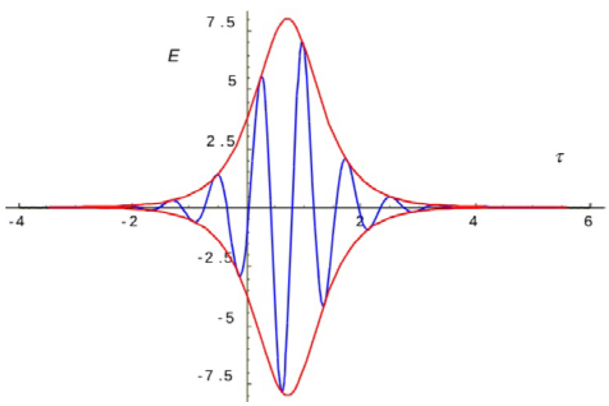

Fig. 1. A FCP soliton described by the breather solution of the mKdV-sG equation (1). Blue: the analytical profile, red: its exact envelope.

If we use the same small amplitude approximation, but do not neglect the mKdV-type term, we obtain after rescaling so that $c_3 = -\mu$ and $c_1 = c_2 = 1$ the alternative model equation

$$E_{zt} + E - \mu E_{tttt} + \left(E^3\right)_{tt} = 0 . \tag{3}$$

Eq. (3) was first prosposed to model FCP soliton propagation in Ref. [18] and it has shown FCP pulse compression [19-20]. Hence we see that all non SVEA models which have been already proposed to model FCP propagation are approximations of the generic mKdV-sG equation (1).

3. Cubic FCP solitons

Let us first consider the defocusing mKdV equation. There are indeed two different mKdV equations:

$$u_z + \sigma u^2 u_t + u_{ttt} = 0 \tag{4}$$

with $\sigma = \pm 1$. For $\sigma = +1$ Eq. (4) is of focusing type, while for $\sigma = -1$ it is of defocusing type. An example of evolution of a FCP according to the defocusing mKdV equation is shown in Fig. 2. It is seen that the dispersion is accentuated by the nonlinear effect.

The mKdV-sG equation with a defocusing mKdV part supports solitons in spite of this. This can be demonstrated as follows: For high frequencies (closer to SVEA), the mKdV-sG equation can be approximately mapped to sG equation [17]. Then the sG breather allows us to construct an approximate soliton, which can be used as input in a numerical resolution of mKdV-sG equation. It evolves with little deformation, as is shown in Fig. 3 in the case of vanishing mKdV dispersion.

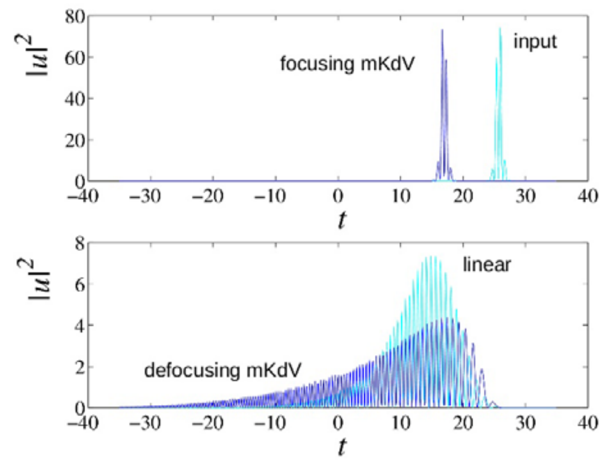

Fig. 2. The evolution of a FCP according to focusing (top) or defocusing (bottom) mKdV equation. The linear evolution is shown, in order to put in evidence the nonlinear dispersion.

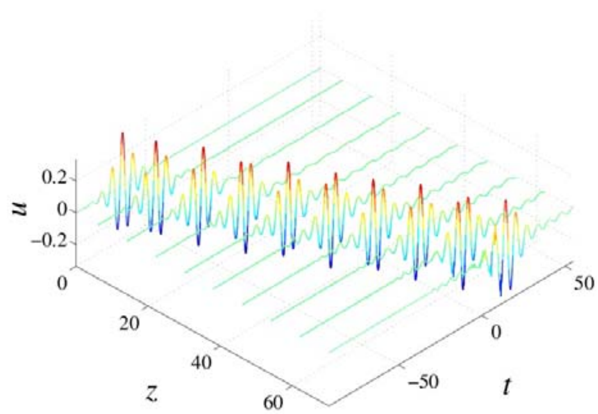

Fig. 3. Evolution of a FCP soliton according to the mKdV-sG equation with defocusing mKdV nonlinearity and vanishing mKdV dispersion. The input is built from the sG breather, according to an approximate mapping of sG equation to mKdV-sG equation.

For really defocusing mKdV dispersion, pulse compression may occur, as shown in Fig. 4. The input used is this computation is the soliton of the nonlinear Schrödinger equation which corresponds to the SVEA limit of mKdV-sG equation. It is clearly seen that SVEA is not valid.

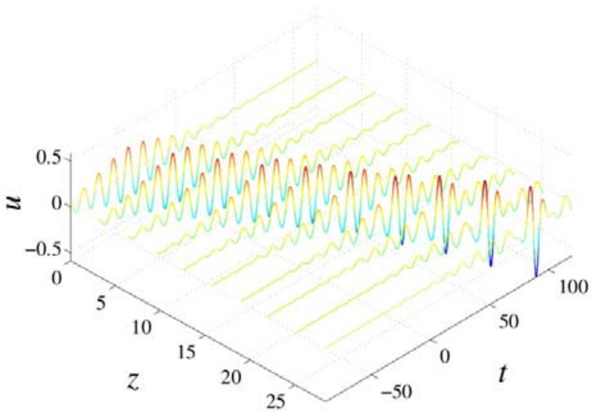

Fig. 4. Pulse compression according to the mKdV-sG model with defocusing mKdV part.

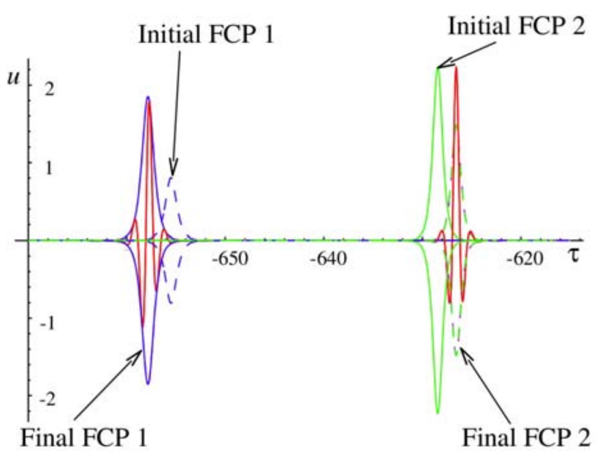

Fig. 5. Interaction of two FCPs. See the text for explanation of the figure.

Let us now turn to FCP soliton interactions. In the integrable case, the 4-soliton solution to mKdV-sG equation [21] gives the 2-breather solution. We can compute the envelope of the FCPs during the interaction [5]. Fig. 5 presents the profile of the two FCPs long after interaction. The envelopes at this time are plotted, and also the envelopes 'before interaction', or more exactly, the envelope of each pulse at the time where the figure is drawn, assuming that they were propagating alone. A shift in location can be seen in Fig. 5; it has been computed explicitly. The rightmost envelopes have not the right amplitudes: this is the consequence of a phase shift which arises during the interaction, and allows to compute it.

4. Quadratic FCP soliton

Starting from either a classical model of elastically bound electron or a quantum two level model, in which a quadratic nonlinearity has been introduced, the reductive perturbation method allows us to derive a KdV model:

$$\partial_\zeta E = A\partial_\tau^3 E + B\partial_\tau (E)^2 \, , \tag{5}$$

in which the dispersion coefficient is

$$A = \frac{1}{6}\frac{d^3k}{d\omega^3}\bigg|_{\omega=0} = \frac{1}{2\mathrm{c}}\frac{d^2n}{d\omega^2}\bigg|_{\omega=0} \, . \tag{6}$$

The nonlinear coefficient is

$$B = \frac{-2\pi}{nc}\chi^{(2)}(2\omega,\omega,\omega)\Big|_{\omega=0} \, . \tag{7}$$

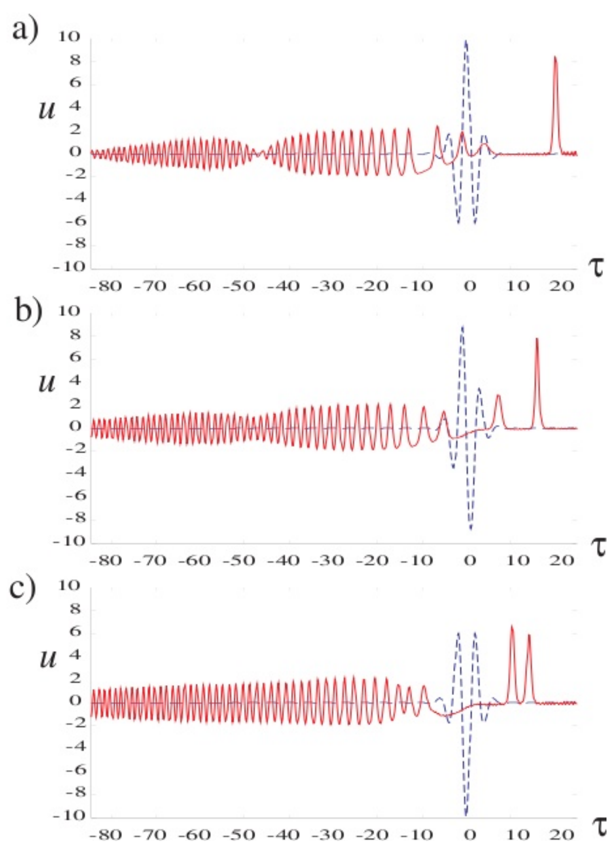

Fig. 6. Formation of a KdV soliton from a FCP. a) Initial carrier-envelope phase $\Phi = 0$: a single soliton is formed. b) $\Phi = \pi/2$: two solitons with different amplitudes. c) $\Phi = \pi$: two identical solitons.

From both the resolution of KdV equation by the IST and numerical analysis, it is found that a quadratic FCP soliton can be formed from a FCP input. The soliton is exactly half cycle, with no oscillating tail. In addition, it has a determined polarity. However, a large part of energy is dispersed, and soliton formation strongly depends on initial carrier-envelope phase Φ , see Fig. 6.

In (2+1) dimensions, KdV equation becomes the Kadomtsev-Petviashvili (KP) equation: KP I or KP II. For a normal dispersion, it is the so-called KP II equation, which admits stable *line solitons*. This corresponds to a nonlinear recovery of the initial wavefront, hence the spatial coherence of the wave is improved by the nonlinear effect (see Fig. 7).

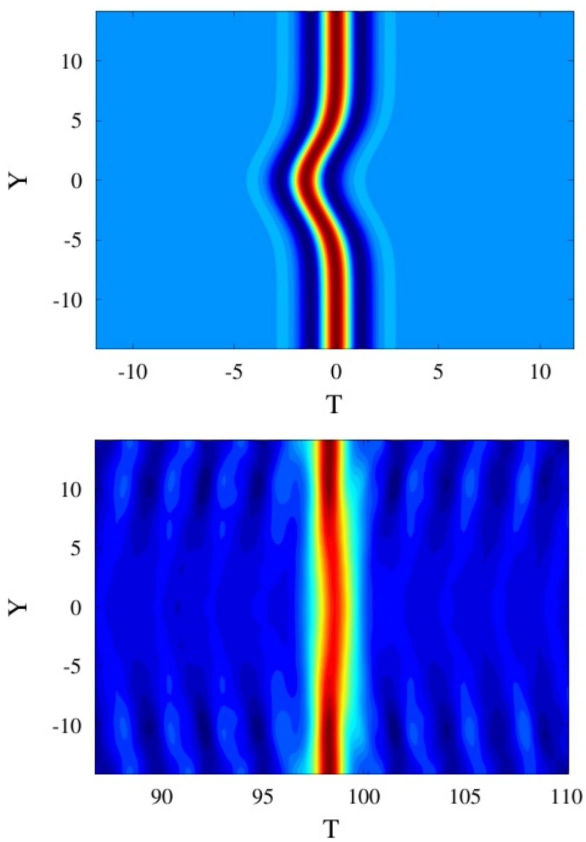

Fig. 7. FCP in quadratic media with normal dispersion: Recovery of a perturbed wavefront according to the KP II equation. Left: input, right: after propagation.

For an anomalous dispersion, the equation is the so-called KP I one, which admits stable localized *lump solutions* [22-23]. The analytical profile of the lumps is shown on Fig. 8. Numerical computation shows that lumps form spontaneously from transverse irregularities.

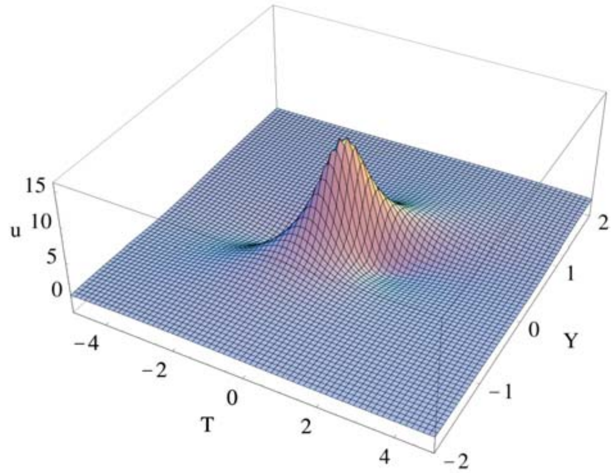

Fig. 8. The two-dimensional FCP soliton in quadratic media with anomalous dispersion, as given by the analytical theory.

5. Conclusions

We developed a theory of optical FCP soliton propagation beyond the commonly used SVEA for both Kerr and quadratic nonlinear media. In this work we have summarized recent results of this theory, emphasizing the generality and interest of the generic mKdV-sG model in the cubic (Kerr) case.

Interactions of Kerr FCP solitons have been described, and quadratic FCP soliton propagation has been also discussed. In both cases, no phase matching is required, which makes a strong contrast with the longer pulses described by the common SVEA theory.

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