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On the creation of a stable and convex static meniscus, appropriate for the growth of a single crystal rod with specified constant radius

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In this paper a theoretical procedure for the creation of a stable static meniscus, appropriate for the growth of a single crystal rod, with a priori specified size (upper radius) and shape (convex), is presented. The method is based on explicit formulas, established as mathematical theorems and combine them for locate the controllable part p of the pressure difference across the free surface in order to create a stable static meniscus with a priori specified size and shape. In fact it consists in a set of calculus which leads to the determination of the melt column height between the shaper top level and the crucible melt level in function of the pressure of the gas flow (introduced in the furnace for release the heat) in order to obtain the desired meniscus. The procedure is presented in general and is numerically illustrated. The numerical illustration reveals situation when convex meniscus can not be created, only convex-concave meniscus exists, which is appropriate for the growth. The novelty is that for the second order axi-symmetric Young-Laplace equation three boundary conditions are specified (instead of two) and that value p of the controllable part of the pressure difference is found for which the boundary conditions are satisfied.

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1. Introduction

Up until 1993-1994 information concerning the dependence of the meniscus free surface shape and size on the controllable part of the pressure difference across the free surface, in the growth of single crystal rod by edge-defined film-fed growth (E.F.G.) technique, is summarized in [1]. According to [1], there is no complete analysis of the solution of the general equation describing the meniscus free surface. For the general equation only numerical integrations were carried out for a number of process parameter values that are of practical interest.

In [2] the authors investigate the influence of the controllable part of the pressure difference across the free surface on the size and shape of the meniscus free surface for rods, in the case of middle-range Bond numbers (i.e. $B_0 = 1$) which most frequently occurs in practice and has been left out of the regular study in [1]. A numerical approach is used in order to solve the meniscus surface equation written in terms of the arc length of the curve. The stability of the static meniscus free surface is analyzed by means of the Jacobi equation. The conclusion is that a large number of static menisci having drop-like shapes are unstable.

The authors of papers [3], [4] consider automated crystal growth processes based on weight sensors and computers. An expression for the weight of the meniscus, contacted with crystal and shaper of arbitrary shape, in which there are terms related to the hydrodynamic pressure (the pressure under the crystallization front), is given

In [5] the author shows that the hydrodynamic pressure is too small to be considered in the automated crystal growth. In [6] theoretical and numerical study of meniscus shape under symmetric and asymmetric configuration is undertaken. A meniscus dynamics model is developed to consider meniscus shape and its dynamics, heat and mass transfer around the die-top and meniscus, interaction of solidification with meniscus and tube thickness variation. The parametric studies are conducted to reveal the correlations among tube thickness, effective height, pull rate, die top temperature and crystal environmental temperature.

Finally, in [7] the general axisymmetric Young-Laplace equation is considered for the boundary conditions: $z(r_0) = 0$; $z'(r_0) = -\tan\alpha_c$ $z'(r_1) = -\tan\left(\pi/2 - \alpha_g\right)$ when $0 < \alpha_c < \pi/2 - \alpha_g < \pi/2$. Explicit formulas are established prescribing the boundaries of the ranges where the parameter p has to be chosen, or can be chosen, in order to obtain a stable, convex solution.

In the present paper it is shown in which kind the inequalities proved in [6] have to be combined and used for the determination of the melt column height between the shaper top level and the horizontal crucible melt level in function of the pressure of the gas flow introduced in the furnace for release the heat, in order to obtain a stable static meniscus, having a convex free surface, appropriate for the growth of a rod with an a priori given constant radius r_1. Appropriate means, that at the left end of the free surface, the angle between the tangent to the free surface and the vertical is equal to the growth angle α_g, i.e. $z'(r_1) = -\tan\left(\pi/2 - \alpha_g\right)$.

The thermal problem concerning the setting of the thermal conditions which assure that for the obtained meniscus at the point $(r_1, z(r_1))$ the solidification conditions are satisfied is not considered in this paper.

2. The free surface equation

For the single crystal rod growth by E.F.G. method, in hydrostatic approximation the free surface of the static meniscus is described by the Laplace-Young capillary equation [8]:

$$\gamma \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \rho \cdot g \cdot z - p \qquad (2.1)$$

Here: γ is the melt surface tension; ρ denotes the melt density; g is the gravity acceleration; $1/R_1$, $1/R_2$ denote the main normal curvatures at a point M of the free surface; z is the coordinate of M with respect to the Oz axis, directed vertically upwards; p is the controllable part of the pressure difference across the free surface:

$$p = p_m - p_g - \rho \cdot g \cdot H . \qquad (2.2)$$

In the last formula: p_m denotes the hydrodynamic pressure in the meniscus melt under the free surface and is due to the thermal convection created by the thermal gradients but usually it is accepted to be equal to zero; $p_g \geq 0$ denotes the gas pressure of the gas flow (on the free surface), introduced in the furnace in order to release the heat from the rod wall and free surface; H denotes the melt column height between the horizontal crucible melt level and the shaper top level (see Fig.1). H is positive when the crucible melt level is under the shaper top level and H is negative, when the shaper top level is under the crucible melt level. When H is positive, then the hydrostatic pressure $\rho \cdot g \cdot H$ acts in the same direction as the hydrostatic pressure $\rho \cdot g \cdot z$ and when H is negative , then $\rho \cdot g \cdot H$ acts in the opposite direction as $\rho \cdot g \cdot z$. The pressure difference, which appears in the Young-Laplace equation is: $\Delta p = p_g - [p_m - (\rho \cdot g \cdot z + \rho \cdot g \cdot H)] = \rho \cdot g \cdot z + p_g + \rho \cdot g \cdot H - p_m$ $= \rho \cdot g \cdot z - [p_m - p_g - \rho \cdot g \cdot H] = \rho \cdot g \cdot z - p$

p is called the controllable part of the pressure difference, because when p_m is negligible, then p is controllable by H and by the pressure difference of the gas at the entrance and at the exit of the furnace.

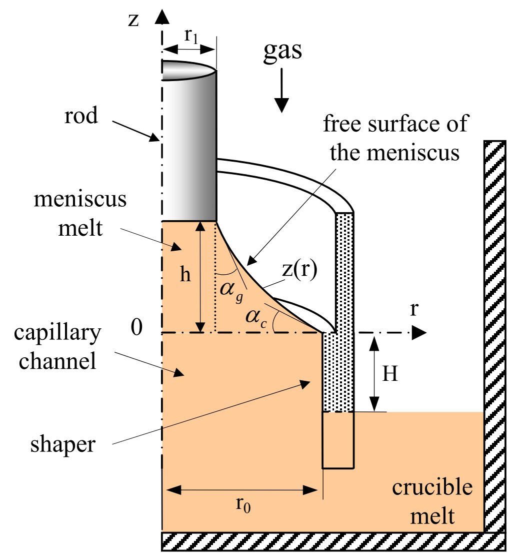

Fig. 1.Axisymmetric meniscus geometry in the rod growth by E.F.G. method.

To calculate the meniscus free surface shape and size is convenient to employ the Laplace-Young equation (2.1) in its differential form [8].

$$\left[1 + (z_y)^2\right] \cdot z_{xx} - 2 \cdot z_x \cdot z_y \cdot z_{xy} + \left[1 + (z_x)^2\right] \cdot z_{yy} = = \frac{g \cdot \rho \cdot z - p}{\gamma} \cdot \left[1 + (z_x)^2 + (z_y)^2\right]^{3/2} \qquad (2.3)$$

For the growth of a single crystal rod of radius r_1, $0 < r_1 < r_0$, the differential equation (2.3) of the axi-symmetric meniscus free surface is given by the formula:

$$z'' = \frac{\rho \cdot g \cdot z - p}{\gamma}\left[1 + (z')^2\right]^{3/2} - \frac{1}{r} \cdot \left[1 + (z')^2\right] \cdot z' \quad \text{for } 0 < r_1 \leq r \leq r_0 \qquad (2.4)$$

which is the Euler equation for the energy functional of the melt column:

$$I(z) = \int_{r_1}^{r_0} \left\{ \gamma \cdot \left[1 + (z')^2\right]^{1/2} + \frac{1}{2} \cdot \rho \cdot g \cdot z^2 - p \cdot z \right\} \cdot r \cdot dr \qquad (2.5)$$

$$z(r_1) = h > 0, \quad z(r_0) = 0$$

.

The meniscus is appropriate for the growth of a rod of constant radius r_1 if the solution $z = z(r)$ of the eq.(2.4), describing the meniscus free surface, satisfy the following conditions:

$$\text{a. } z'(r_1) = -\tan\left(\pi/2 - \alpha_g\right)$$

(2.6)

$$\text{b. } z'(r_0) = -\tan\alpha_c$$

c. $z(r_0) = 0$ and $z(r)$ is strictly decreasing on $[r_1, r_0]$, where $r_0 > 0$ is the shaper radius, α_g is the growth angle, α_c is the contact angle between the meniscus free surface and the edge of the shaper top and $0 < \alpha_c < \pi/2 - \alpha_g$ (Fig.1).

Comments: Condition a. expresses that at the point $(r_1, z(r_1))$ (left end of the free surface where the thermal conditions for solidification have to be assured) the angle between the tangent line to the free surface and the vertical is equal to the growth angle α_g, i.e. the tangent to the crystal wall is vertical.

Condition b. expresses that at the point $(r_0, 0)$ (the right end of the free surface) the angle between the tangent line to the free surface and the shaper top, i.e. contact angle, is equal to α_c.

Condition c. expresses that the right end of the free surface is fixed to the outer edge of the shaper.
Usually, eq. (2.4) is transformed into the system:

$$\begin{cases} \dfrac{dz}{dr} = -\tan\alpha \\ \dfrac{d\alpha}{dr} = \dfrac{p - \rho \cdot g \cdot z}{\gamma} \cdot \dfrac{1}{\cos\alpha} - \dfrac{1}{r} \cdot \tan\alpha \end{cases} \tag{2.7}$$

for which the conditions a. – c. become:

$$z(r_0) = 0;\ \alpha(r_0) = \alpha_c;\ \alpha(r_1) = \pi/2 - \alpha_g;\ z(r) \text{ is strictly decreasing on } [r_1, r_0]. \tag{2.8}$$

3. The dependence of the meniscus shape and size on the controllable part of the pressure difference across the free surface

If for a certain value of the controllable part of the pressure difference p, the solution $z(r)$ of eq.(2.4), which satisfies $z(r_0) = 0$ and $z'(r_0) = -\tan\alpha_c$ is globally concave ($z''(r) < 0$), then $z(r)$ can not satisfy a condition of the type $z'(r_1) = -\tan\left(\pi/2 - \alpha_g\right)$ when $r_1 \in (0, r_0)$. Hence a static meniscus having such a free surface is not appropriate for the growth of a rod with constant radius r_1. Therefore, the interest is to find the values of the pressure p for which there exists a solution $z(r)$ of eq.*(2.4)* which satisfies $z(r_0) = 0;\ z'(r_0) = -\tan\alpha_c$ and $z(r)$ is not globally concave.

Based on the mathematical theorems rigorously established in [6], the following statements can be formulated regarding the creation of an appropriate meniscus.

Statement 1. If there exists a solution $z(r)$ of the eq.(2.3) which satisfies $z(r_0) = 0;\ z'(r_0) = -\tan\alpha_c$ and $z(r)$ is globally convex ($z''(r) > 0$), then for the pressure p the following inequality holds:

$$\begin{aligned} &-\frac{n}{n-1} \cdot \gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \cos\alpha_c + \frac{\gamma}{r_0} \cdot \sin\alpha_c \le p \\ &\le -\frac{n}{n-1} \cdot \gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \sin\alpha_g + \frac{n-1}{n} \cdot \rho \cdot g \cdot r_0 \cdot \tan\left(\pi/2 - \alpha_g\right) \\ &+ n \cdot \frac{\gamma}{r_0} \cdot \cos\alpha_g \end{aligned} \tag{3.1}$$

here $n = \dfrac{r_0}{r_1} > 1$.

Comments: According to this statement, in order to create an appropriate static meniscus having a convex free surface on $\left[r_0/n, r_0\right]$, the pressure p has to be chosen in the range given by the inequality (3.1). Formula (3.1) can be used for a rough evaluation of the pressure p which has to be realized when the rod radius r_1 which has to be grown is given by $r_1 = r_0/n$.

A consequence of the Statement 1 is: if for an appropriate static meniscus having a convex free surface, r_1 is close to zero, then p verifies:

$$p \ge -\gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \cos\alpha_c + \frac{\gamma}{r_0} \cdot \sin\alpha_c \tag{3.2}$$

Formula (3.2) can be used for a rough evaluation of the pressure p, which has to be realized, when the rod radius r_1 (which has to be grown) is close to zero.

Statement 2. If the pressure p satisfies:

$$p < -\frac{n}{n-1} \cdot \gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \cos\alpha_c + \frac{\gamma}{r_0} \cdot \sin\alpha_c \tag{3.3}$$

then there exists $r_1 \in \left[r_0/n , r_0 \right]$ and a solution $z(r)$ of eq.(2.4), which satisfies (2.6) and $z''(r) > 0$.

Comments: According to the above statement, if the pressure p is chosen such that inequality (3.3) holds, then an appropriate static meniscus, having a convex free surface on a certain interval $[r_1, r_0]$, $r_0/n < r_1 < r_0$, is obtained. This meniscus is appropriate for the growth of a rod of radius r_1.

Formula (3.3) can be used for the evaluation of the pressure p which has to be realized when the rod radius r_1 (which has to be grown) has to be in the range $\left[r_0/n , r_0 \right]$.

A consequence of the Statement 2 is that if for p the following inequality holds:

$$p < -\gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \cos\alpha_c + \frac{\gamma}{r_0} \cdot \sin\alpha_c \, , \quad (3.4)$$

then there exists r_1 in the range $(0, r_0)$ and a static meniscus having globally convex free surface on $[r_1, r_0]$ which is appropriate for the growth of a rod of constant radius r_1.

Statement 3. If for $n > n' > 1$ and p the following inequalities hold:

$$-\frac{n'}{n'-1} \cdot \gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \sin\alpha_g + \frac{n'-1}{n'} \cdot \rho \cdot g \cdot r_0 \cdot \tan\left(\pi/2 - \alpha_g\right) + n' \cdot \frac{\gamma}{r_0} \cdot \cos\alpha_g < \\ < p < -\frac{n}{n-1} \cdot \gamma \cdot \frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0} \cdot \cos\alpha_c + \frac{\gamma}{r_0} \cdot \sin\alpha_c \, , \quad (3.5)$$

then there exists r_1 in the interval $\left[\frac{r_0}{n}, \frac{r_0}{n'} \right]$ and a solution $z(r)$ of the eq. (2.4) which satisfies (2.6) and $z''(r) > 0$ on $[r_1, r_0]$.

Comment: Formula (3.5) can be used for the evaluation of the pressure p, which has to be realized, when the rod radius r_1 (which has to be grown) has to be in the range $\left[\frac{r_0}{n}, \frac{r_0}{n'} \right]$.

Statement 4. If p satisfies:

$$p < \frac{\gamma}{r_0} \cdot \sin\alpha_c \quad (3.6)$$

then a solution $z(r)$ of the eq.(2.4) which satisfies $z(r_0) = 0;\ z'(r_0) = -\tan\alpha_c$ is globally convex, i.e. $z''(r) > 0$, and vice versa.

Comment: Formula (3.6) can be used for the evaluation of the pressure p for which the meniscus free surface is globally convex and potentially the condition $z'(r_1) = -\tan\left(\pi/2 - \alpha_g\right)$ can be realized (i.e. the meniscus can be appropriate).

Statement 5. If for $p > \frac{\gamma}{r_0} \cdot \sin\alpha_c$ there exists r_1, $0 < r_1 < r_0$ $\left(r_1 = r_0/n ;\ n > 1 \right)$ and a solution $z(r)$ of the eq.(2.4) which satisfies (2.6), then for p the following inequalities hold:

$$\frac{\gamma}{r_0} \cdot \sin\alpha_c < p \le \frac{n-1}{n} \cdot \rho \cdot g \cdot r_0 \cdot \tan\left(\frac{\pi}{2} - \alpha_g \right) + n \cdot \frac{\gamma}{r_0} \cdot \cos\alpha_g \quad (3.7)$$

Comment: Formula (3.7) is useful for the evaluation of the pressure p, which has to be realized, for create an appropriate meniscus having non globally convex (convex-concave) free surface.

Statement 6. If for $n > 1$ and p the following inequality holds:

$$p > \frac{n-1}{n} \cdot \rho \cdot g \cdot r_0 \cdot \tan\alpha_c + n \cdot \frac{\gamma}{r_0} \quad (3.8)$$

then the solution $z(r)$ of the eq.(2.4) which satisfies $z(r_0) = 0;\ z'(r_0) = -\tan\alpha_c$ is concave $(z''(r) < 0)$ on $\left[r_0/n , r_0 \right]$ and the conditions $z'(r_1) = -\tan\left(\pi/2 - \alpha_g\right)$, $z(r)$ is strictly decreasing on $\left[r_0/n , r_0 \right]$ can not be realized on it.

Comment: Formula (3.8) is useful for a rough evaluation of the p values for which the meniscus is not appropriate for the growth of a rod of radius r_0/n.

Statement 7. If a solution $z(r)$ of eq.(2.4) which satisfies (2.6), is globally convex ($z''(r) > 0$), then $z(r)$ is minimum of the energy functional of the melt column.

Comment: The above statement establishes that if the appropriate meniscus free surface is convex, then the static meniscus is stable.

4. Creation of an appropriate static meniscus by the choice of the melt column height

In this sequence first it will be shown theoretically in which kind the explicit formulas, presented in the previous sequence, can be combined and used for the determination of the pressure p, (which has to be used) for the creation of a stable and convex static meniscus, appropriate for the growth of a single crystal rod when $\alpha_c, \alpha_g, \rho, \gamma, r_0, r_1$ are given a priori. After that, it will be shown in which kind the melt column height (which has to be used) has to be found, when the pressure of the gas flow, introduced in the furnace to release the heat, is given.

In order to find the pressure p, which has to be used for the creation of a convex meniscus, having the bottom radius equal to r_0 and the top radius equal to r_1, the following limits, presented in the above section, have be considered:

$$l_1(n) = \frac{n-1}{n}\cdot\rho\cdot g\cdot r_0\cdot\tan\left(\pi/2 - \alpha_g\right) + \frac{n\cdot\gamma}{r_0}\cdot\cos\alpha_g$$

$$\overline{L}(n) = -\frac{n}{n-1}\cdot\gamma\cdot\frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0}\cdot\sin\alpha_g + \frac{n-1}{n}\cdot\rho\cdot g\cdot r_0\cdot\tan\left(\pi/2 - \alpha_g\right) + n\cdot\frac{\gamma}{r_0}\cdot\cos\alpha_g$$

$$l_2(n) = \frac{n-1}{n}\cdot\rho\cdot g\cdot r_0\cdot\tan\alpha_c + \frac{n\cdot\gamma}{r_0}$$

$$l_3 = \frac{\gamma}{r_0}\cdot\sin\alpha_c$$

$$l_4 = -\gamma\cdot\frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0}\cdot\cos\alpha_c + \frac{\gamma}{r_0}\cdot\sin\alpha_c$$

$$\underline{L}(n) = -\frac{n}{n-1}\cdot\gamma\cdot\frac{\pi/2 - (\alpha_c + \alpha_g)}{r_0}\cdot\cos\alpha_c + \frac{\gamma}{r_0}\cdot\sin\alpha_c .$$

- According to Statement 1, for an appropriate static and convex meniscus, p has to be searched in the range: $[\underline{L}(n_1), \overline{L}(n_1)]$, where $n_1 = r_0/r_1$.
- From this range the following p values have to be excluded:
  - the values of p higher than $l_2(n_1)$. (That is because according to Statement 6 for such values of p the free surface is concave on $[r_1, r_0]$ and is not appropriate for the growth.)
  - the values of p higher than l_3. (That is because according to Satement 4 for these values of p the free surface is concave at r_0 and it is not anymore globally-convex.)
  - the values of p less than l_4 when the obtained upper radius r of the meniscus for $p_1 = l_4$ is higher than the desired radius r_1, and the values of p higher than l_4 when the obtained upper radius r of the meniscus for $p_1 = l_4$ is less than the desired radius r_1, respectively.(That is according to the Statement 3 and comments following Statement 1.)
- Excluding the above mentioned values of p, a part P of the range $[\underline{L}(n_1), \overline{L}(n_1)]$ is obtained, where the values of p have to be searched.
- In order to obtain the right value of p, the initial value problem (2.6), (2.7) has to be integrated numerically for different values of p in P and the obtained upper radii of the menisci have to be represented versus p. The right value of p can be seen on this graphic. Using this right value of p, the melt column "height", which has to be used is given by: $H = -\frac{1}{\rho\cdot g}\cdot[p + p_g]$, where $p_g \geq 0$ is the pressure of the gas flow.

In the following the above described general procedure will be illustrated for a melt having thermo-physical properties similar to NdYAG melt:

$\alpha_c = 0.523\ rad = 30^\circ$; $\alpha_g = 0.2967\ rad = 17^\circ$;

$\rho = 3.6\times10^3\ kg/m^3$; $\gamma = 7.81\times10^{-1}\ N/m$; $g = 9.81\ m/s^2$.

The calculus was performed in MathCAD V.13. using $r_0 = 5 \times 10^{-3} m$; for :
$r_1 = 4 \times 10^{-3} m$; $r_1 = 2.5 \times 10^{-3} m$ and $r_1 = 0.55 \times 10^{-3} m$, respectively.
In Fig.2 the considered limits are represented for $n \in [1, 10]$.

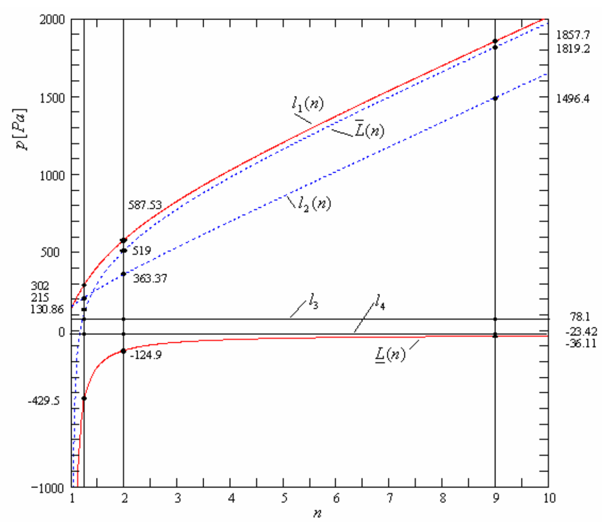

Fig. 2. The possible p ranges which have to be investigated for $n = 1.25;\ 2;\ 9$.

When $r_1 = 2.5 \times 10^{-3} m$, then $n_1 = 2$ and for an appropriate static and stable meniscus, p has to be searched in the range:

$$\left[\underline{L}(2), \overline{L}(2)\right] = \left[-124.942,\ 518.985\right] Pa.$$

From this range the values p greater than $l_2(2) = 363.374\, Pa$ have to be excluded. Hence, the range where p has to be searched is: [-124.942, 363.374] Pa.
From this range the values of p higher than $l_3(2) = 78.1\, Pa$ have to be excluded too. Hence, the range where p has to be searched becomes: [-124.942, 78.1] Pa.
We have to search now for $p = l_4 = -23.432\, Pa$ the point $r \in (0, r_0]$ at which $z'(r) = -\tan\left(\pi/2 - \alpha_g\right)$.
This can be made integrating numerically the system (2.5) for $p = -23.432\, Pa$ and $z(r_0) = 0, \alpha(r_0) = \alpha_c$ The result of this integration is represented on Fig. 3.

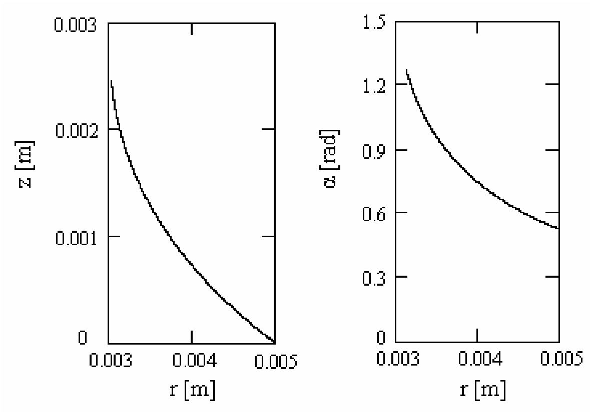

Fig. 3. The result of the integration of system (2.5) for $p = -23.432\, Pa$ and $z(r_0) = 0, \alpha(r_0) = \alpha_c$.

Since the obtained radius is $r = 3.1043 \times 10^{-3}\, m$ and is higher than the desired value $r_1 = 2.5 \times 10^{-3} m$, we have to search the value of p in the range $\left[-23.432,\ 78.1\right] Pa$. The crystal radii obtained by integration of (2.6), (2.7) for different p in the range [-23.432, 78.1] Pa are represented in Fig.4.

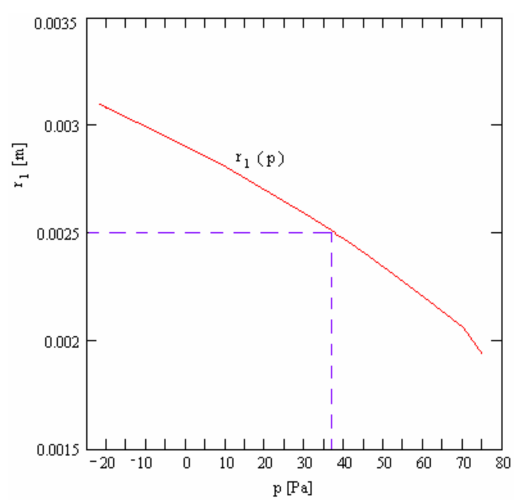

Fig. 4. The crystal radius versus p for $p \in \left[-23.432,\ 78.1\right] Pa$.

This figure shows that the desired rod radius $r_1 = 2.5 \times 10^{-3}\, m$ is obtainable for $p = 37.7\, Pa$. Hence, the right value of p is equal to $37.7\, Pa$.

Consider now the right value of p equal to $p_1 = 37.7\, Pa$ for which $r_1 = 2.5 \times 10^{-3}\, m$. Since $p_1 = p_m - p_g - \rho \cdot g \cdot H$ and according to [5] $p_m \approx 0$, it follows that $37.7\, Pa = -p_g - \rho \cdot g \cdot H$.

Assuming now that the pressure of the gas flow introduced in the furnace for the heat extraction is $p_g \geq 0$, it is found that H is equal to $H = \frac{1}{\rho \cdot g} \cdot \left[-37.7\,Pa - p_g\right]$. Hence the melt column height between the top of the shaper and the crucible melt level has to be : $-H = \frac{1}{\rho \cdot g} \cdot \left[37.7\,Pa + p_g\right] > 0$ with $p_g \geq 0$. It follows : $p_g = 0 \Rightarrow H = 0.106\,cm; \quad p_g = 800\,Pa \Rightarrow H = 2.26$.

When $r_1 = 4 \times 10^{-3}\,m$ (i.e. $n = 1.25$), reasoning analogously as in the case $r_1 = 2.5 \times 10^{-3}\,m$, it is found that the right p value in the range $[-429.5, -23.432]\,Pa$ has to be searched and it is $p = -215\,Pa$.

It follows:

$p_g = 0 \Rightarrow -H = 0.608\,cm; \quad p_g = 800\,Pa \Rightarrow -H = 2.87\,cm.$

When $r_1 = 0.55 \times 10^{-3}\,m$ (i.e. $n = 9$) reasoning analogously as in the case $r_1 = 2.5 \times 10^{-3}\,m$, it is found that the right p value has to be searched in the range $[-23.432, 78.1]\,Pa$. But we have already seen that for p in this range there is not possible to create a convex meniscus with $r_1 = 0.55 \times 10^{-3}\,m$ (see Fig.4). Only by creating a meniscus with a convex-concave free surface is possible to obtain $r_1 = 0.55 \times 10^{-3}\,m$ (see Fig.5). More exactly, using $p = 174\,Pa$ we obtain a convex-concave free surface with $r_1 = 0.55 \times 10^{-3}\,m$.

It follows:

$p_g = 0 \Rightarrow -H = 0.492\,cm; \quad p_g = 800\,Pa \Rightarrow -H = 2.75\,cm.$

We remark that for p in the range $(78.1,\ 363.374)\,Pa$ the results of the integration of the system (2.5) reveal the existence of non globally convex (convex-concave) free surfaces which are appropriate for the growth of a rod of radius $r_1 < 2.5 \times 10^{-3}\,m$ and the existence of globally concave free surfaces (for $p > 175\,Pa$), which are not appropriate for the growth (see Fig.5).

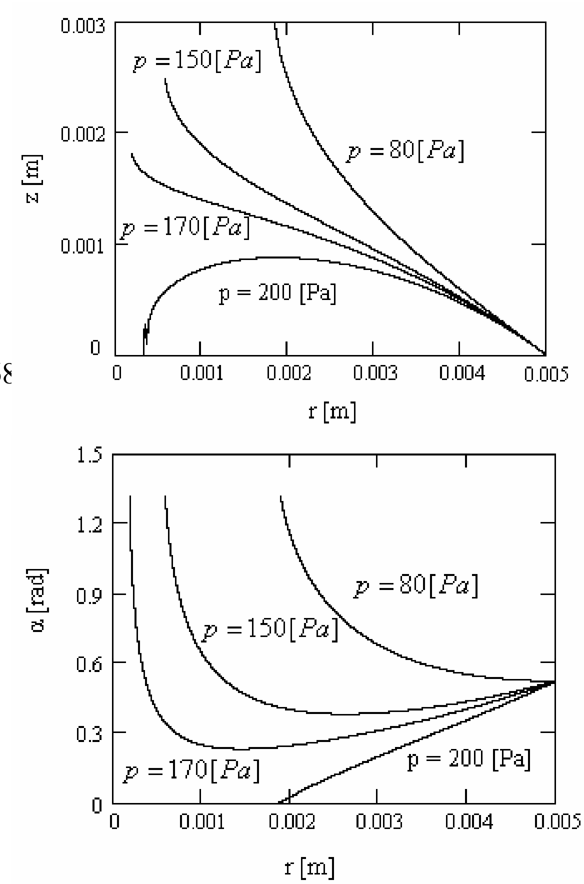

Fig. 5. z and α in function of r for different values of p in the range $(78.1,\ 363.374)\,Pa$.

5. Conclusions

The statements presented in sequence 4 for materials for which $\alpha_c + \alpha_g < \pi/2$, can be used to determine the values of the melt column height, in function of the pressure of the gas flow, which have to be used in order to obtain:

- stable convex static menisci, appropriate for the growth of a rod with an a priori given constant radius
- static menisci which are not appropriate for the growth of a rod with an a priori given constant radius.

The setting of the thermal conditions which assure the solidification at the right places for the obtained convex static menisci, is not discussed in this paper.

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