

One-dimensional photonic crystal as an omni-directional reflector

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In this article, the reflection properties in one-dimensional dielectric-dielectric photonic crystals have been studied. We have used SiO₂ as material of low refractive index and Si/Te as a high refractive index material. Two Structures SiO₂/Si and SiO₂/Te have been studied. The effect of refractive index contrast on omni-directional reflection is investigated. Reflectivity of proposed structures is plotted as a function of frequency and omni-directional PBGs are computed theoretically. To obtain reflectance, we used transfer matrix method for solving Maxwell's equations for electromagnetic wave in proposed structures.

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1. Introduction

Photonic band gap (PBG) materials which are photonic crystals with forbidden photonic bands, have received considerable attention for the study of their fundamental physical properties as well as for potential applications in photonic devices since the pioneering works of Yablonovitch and John have been reported [1-3]. Photonic band gap (PBG) materials may be designed in one, two and three dimensions. But multilayered periodic structure with different refractive index materials, which are nothing but one-dimensional PBG materials are attractive because such structures can be fabricated more easily at any wavelength scale and their analytical study and numerical calculations are simpler. Omni-directional dielectric reflector is a mirror having almost cent percent reflectivity at any angle of incidence for both TE and TM polarized electromagnetic waves. Recently, such mirrors are realized and have been manufactured; and the conditions for their realization have been formulated. Thus, periodic structures have become important structures in photonics [4-8].

In 1998, Fink et al. [9,10] reported, for the first time, that one-dimensional dielectric lattice displays total omni-directional reflection for incident light under certain conditions. They constructed a stack of nine alternate polystyrene/tellurium layers having a thickness of a few micrometres and demonstrated omni directional reflection over the wavelength range from 10-15 micrometres. Further works by various researchers found many interesting results. Gallas et al. [11] reported the annealing effect in the Si/SiO₂ PBG based omni-directional reflectors. Chen et al. [12] fabricated six bi-layers of SiO₂ and TiO₂ quarter wave films using the sol gel method and found an omni-directional photonic band gap of about 70 nm in near infrared region. Chigrin et al. [13,14] fabricated a lattice consisting of 19 layers of

Na₃AlF₆/ZnSe and found that omni-directional photonic band gap exists in the spectral range 604.3 to 638.4nm. Ojha et al. [15,16] theoretically studied omni-directional high reflector for infrared wavelengths, large omni-directional reflection using combination of periodic and Fibonacci structures respectively.

In this paper, we shall discuss the design of omni-directional reflectors having a large range of frequency over which omni-directional reflection can be achieved. We can achieve the design of such an omni-directional reflector for by using SiO₂/Si and SiO₂/Te multilayer systems.

2. Theoretical Analysis

Consider the electromagnetic wave propagation in one dimensional (1D) system that consists of alternate layers of dielectric materials with different refractive indices. We consider the system to be isotropic and nonmagnetic. The schematic diagram of the structure considered is shown in the Fig. 1. The refractive index profile of the PBG structure is given by

$$n(x) = \begin{cases} n_L, & 0 < x < d_L \\ n_H, & d_L < x < d_H \end{cases} \quad \text{with } n(x) = n(x + d) \quad (1)$$

where d_L and d_H are the thicknesses of the layers and $d = d_L + d_H$ is the period. Here the x-axis is taken along the direction normal to the layers and assume that the materials are nonmagnetic. To calculate the reflectivity of this PBG structure, we have used transfer matrix method (TMM) [17].

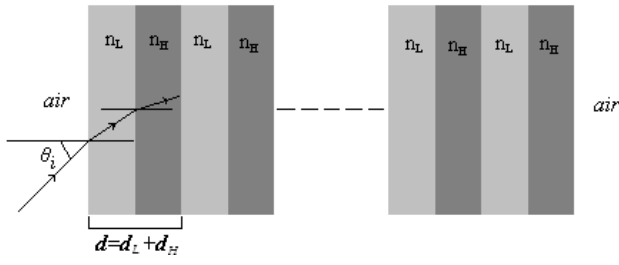


Fig. 1. The schematic diagram of PBG structure.

The electric field for the Maxwell equation can be written as

$$E = E(x) \cdot \exp[i(\omega t - \beta \cdot z)] \quad (2)$$

where, β is the z-component of the wave vector and ω is angular frequency.

Within each layer, the electric field distribution can be expressed as the sum of an incident plane wave and a reflected plane wave. Incident light can be either transverse electric (TE) (electric field is perpendicular to the plane of incidence) or transverse magnetic (TM) mode (magnetic field is perpendicular to plane of incidence). The amplitudes of the fields in the first layer of N^{th} unit cell and $(N-1)^{\text{th}}$ unit cell are related by a matrix ,

$$\begin{pmatrix} A_{N-1} \\ B_{N-1} \end{pmatrix} = m_N \begin{pmatrix} A_N \\ B_N \end{pmatrix} \quad (3)$$

where $m_N = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ is the transfer matrix, A_N and B_N

respectively are the coefficients of the right and left hand side propagating waves in the first layer of the N^{th} unit cell.

Where, the matrix elements given as

$$m_{11} = e^{ik_1 a} \left[\cos(k_2 b) + \frac{i}{2} \left(\gamma + \frac{1}{\gamma} \right) \sin(k_2 b) \right] \quad (4a)$$

$$m_{12} = e^{-ik_1 a} \left[-\frac{i}{2} \left(\gamma - \frac{1}{\gamma} \right) \sin(k_2 b) \right] \quad (4b)$$

$$m_{21} = e^{ik_1 a} \left[\frac{i}{2} \left(\gamma - \frac{1}{\gamma} \right) \sin(k_2 b) \right] \quad (4c)$$

$$m_{22} = e^{-ik_1 b} \left[\cos(k_2 b) - \frac{i}{2} \left(\gamma + \frac{1}{\gamma} \right) \sin(k_2 b) \right] \quad (4d)$$

The dispersion relation between K and ω for Bloch waves, can be expressed as

$$K(\omega) = \frac{1}{d} \cos^{-1} \left[\frac{1}{2} (m_{11} + m_{22}) \right] \quad (5)$$

The reflection and transmission coefficients can be related easily between the plane wave amplifications

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = M \begin{pmatrix} 1 \\ r \end{pmatrix} \quad (6)$$

and $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ with $M_{11} = m_{11} U_{N-1} - U_{N-2}$,
 $M_{21} = m_{21} U_{N-1}$, $M_{12} = m_{12} U_{N-1} - U_{N-2}$,
 $M_{22} = m_{22} U_{N-1} - U_{N-2}$ and
 $U_N = \frac{\sin[(N+1) \cdot K(\omega) \cdot d]}{\sin[K(\omega) \cdot d]}$ and N is the number of unit cells. The reflection coefficient is given by

$$r = \frac{M_{21}}{M_{22}} \quad (7)$$

The associated reflectance (R) is obtained by taking the absolute square of r [18]

$$R = |r|^2 \quad (8)$$

In the next section, using equation (8), we will compute reflectivity of the structure considered here for both TE and TM modes of polarization.

3. Results and discussion

(i) SiO₂/Si system

For the sake of calculations, we consider a structure by choosing SiO₂ and Si materials as the low and high refractive index materials respectively. The refractive index for SiO₂ is $n_L=1.5$ and for Si is $n_H=3.7$. We consider both the materials amorphous and isotropic. The thickness of the layers are taken as $d_L=83.3\text{nm}$ and $d_H=33.8\text{nm}$ according to the quarter wave stack condition $d_L=\lambda_C/4n_L$ and $d_H=\lambda_C/4n_H$, where $\lambda_C=500\text{nm}$. The structure considered to have 10 unit cells.

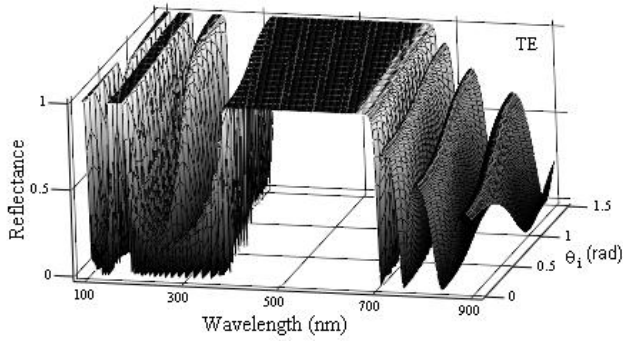


Fig. 2. Reflectance spectra of SiO_2/Si system for TE polarization.

The reflectance spectra of proposed one-dimensional PBG structure, is shown in Fig. 2 and 3 for TE and TM mode of polarizations respectively. These spectra are plotted in terms of wavelength and for incident angle θ_i . Figure 4, represents the photonic band structure which can be obtained by the projection of $R \approx 1$ for figure 2 and 3. Hence, the forbidden bands represent the region with reflectivity greater than 98%. In figure 4, the shaded region gives the total omnidirectional reflection (ODR) band. The data corresponding to nearly 100% reflectance is summarized in Table 1.

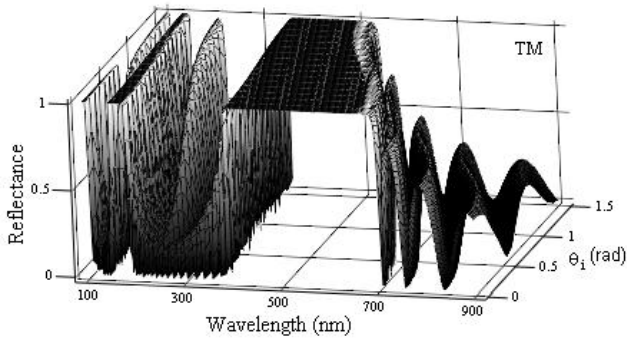


Fig. 3. Reflectance spectra of SiO_2/Si system for TM polarization.

We observe from the data that the TE polarization has its omnidirectional reflection range from 390nm to 658nm and the omni-directional reflection range for the TM polarization is from 319nm to 525nm. The omnidirectional PBGs for both TE and TM polarizations is defined by the edges of the upper photonic band gap at the incident angle 85° and the lower photonic band at incident angle 0° . It is noticeable that the omni-directional PBG for the TM polarization is completely within the omnidirectional PBG corresponding to that of the TE polarization. Therefore, the omnidirectional PBG for the TM polarization is not outside the overall omnidirectional PBG for any polarization. From Table 1 and Figure 4, it is clear that the total ODR (for both TE and TM polarizations), has the bandwidth $\Delta\lambda = (\lambda_H - \lambda_L) = 135\text{nm}$. The upper wavelength edge of ODR band is $\lambda_H = 588\text{nm}$

and the lower wavelength $\lambda_L = 372\text{nm}$. Normalized omnidirectional bandwidth is $\Delta\lambda/\lambda_C = 27\%$ at $\lambda_C = 500\text{nm}$.

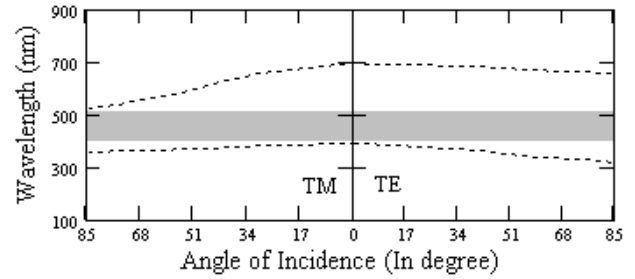


Fig. 4. Photonic band structure of SiO_2/Si for TE and TM modes of polarizations.

Table 1. 100% reflection ranges for TE and TM polarizations at different incident angles for SiO_2/Si system.

Incident angle (in degree)	TE polarization		TM polarization	
	100% Reflection range (nm)	Band width (nm)	100% Reflection range (nm)	Band width (nm)
0	390-695	305	390-695	305
30	373-686	313	382-658	276
60	337-667	330	365-572	207
85	319-658	339	357-525	168

(ii) SiO_2/Te system

In another structure, we consider a structure by choosing SiO_2 and Te materials as the low and high refractive index materials respectively. The refractive index for SiO_2 is $n_L = 1.5$ and for Te is $n_H = 4.6$. We choose Te as a high refractive index material to increase the refractive index contrast (n_H/n_L). We consider both the materials amorphous and isotropic. We consider the thickness of the layers are taken as $d_L = 83.3\text{nm}$ and $d_H = 27.2\text{nm}$ to satisfy the quarter wave stack condition $d_L = \lambda_c/4n_L$ and $d_H = \lambda_c/4n_H$, where $\lambda_c = 500\text{nm}$ for omni-directional reflection. We consider the structure to have 10 unit cells.

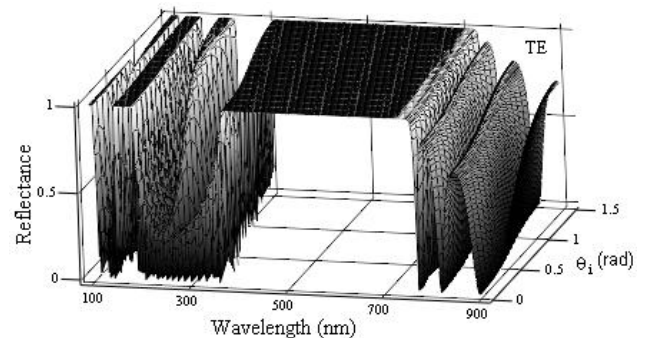


Fig. 5. Reflectance spectra of SiO_2/Te PBG for TE polarization.

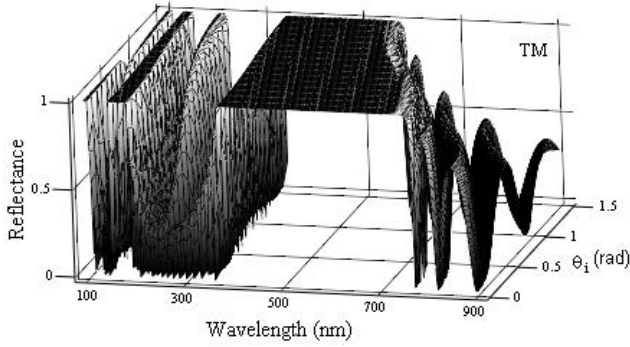


Fig. 6. Reflectance spectra of SiO₂/Te PBG for TM polarization.

The reflectance spectra of proposed one dimensional PBG structure, is shown in Fig. 5 and 6 for TE and TM mode of polarizations respectively. These spectra are plotted in terms of wavelength and for incident angle θ_i . Figure 7, represents the photonic band structure which can be obtained by the projection of figures 5 and 6 at $R \approx 1$. Hence, the forbidden bands represent the regions with reflectivity greater than 98%. In figure 7, the shaded region gives the total omnidirectional reflection band. The data corresponding to nearly 100% reflectance is summarized in table 2.

Table 2. 100% reflection ranges for TE and TM polarizations at different incident angles.

Incident angle (in degree)	TE polarization		TM polarization	
	100% Reflection range (nm)	Band width (nm)	100% Reflection range (nm)	Band width (nm)
0	372-764	392	372-764	392
30	357-756	399	364-724	360
60	325-741	416	350-635	285
85	309-734	425	343-588	245

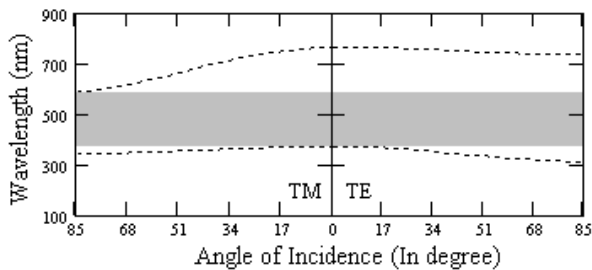


Fig. 7. Photonic band structure of SiO₂/Te for TE and TM modes of polarizations.

We observe from the data that the TE polarization has its omni-directional reflection range from 372nm to 734nm and the omni-directional reflection range for TM polarization from 372nm to 588nm. The omnidirectional PBGs for both the TE and the TM polarizations is defined by the edges of the upper photonic band gap at the incident angle 85° and the lower photonic band at incident angle 0°. It is noticeable that the omni-directional PBG for the TM polarization is completely within the omni-directional PBG corresponding to that of the TE polarization. Therefore, the omni-directional PBG bandwidth for the TM polarization is not more than the overall omni-directional PBG bandwidth for any polarization. From Table 2 and Figure 7, it is clear that the total ODR bandwidth (for both TE and TM polarizations), has the bandwidth $\Delta\lambda=(\lambda_H-\lambda_L)=216\text{nm}$. The upper wavelength edge of ODR band is $\lambda_H=588\text{nm}$ and the lower wavelength $\lambda_L=372\text{nm}$. Normalized omnidirectional bandwidth is $\Delta\lambda/\lambda_C=43.2\%$ at $\lambda_C=500\text{nm}$. It is observed that if we increase the refractive index contrast, i.e. the ratio of the high refractive index to the low refractive index, the ODR bandwidth also increases.

4. Conclusions

We have shown theoretically that we can design a simple one-dimensional dielectric structure that can exhibit total omni-directional reflection of incident wave for a wide wavelength range. By choosing high refractive index contrast materials, we can enhance the ODR ranges of frequency where there is omni-directional reflection from such a PBG structure. Such structures may be employed in optical devices like optical resonator, micro-reflector, etc.

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