Optical soliton perturbation in magneto-optic waveguides with spatio-temporal dispersion

J. VEGA-GUZMAN[a], A. A. ALSHAERY[b], E. M. HILAL[b], A. H. BHRAWY[c,d], M. F. MAHMOOD[a], L. MORARU[e], ANJAN BISWAS[d,f,*]

[a]*Department of Mathematics, Howard University, Washington, DC-20059, USA*

[b]*Department of Mathematics, Faculty of Science for Girls, King Abdulaziz University, Jeddah, Saudi Arabia*

[c]*Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt*

[d]*Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah-21589, Saudi Arabia*

[e]*Department of Chemistry, Physics and Environment University Dunarea de Jos Galati 111 Domneasca Street, 800201 Galati, Romania*

[f]*Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA*

This paper obtains optical soliton solutions in magneto-optic waveguides with several perturbation terms. In addition to group-velocity dispersion, spatio-temporal dispersion term is taken into consideration. There are three types of nonlinear media studied in this paper. They are Kerr law, power law and log-law nonlinearities. Bright, dark and singular soliton solutions are obtained. Numerous constraint conditions naturally emerge for solitons to exist.

(Received July 30, 2014; accepted September 11, 2014)

Keywords: Solitons, Magneto-optic waveguides, Integrability

1. Introduction

Optical solitons is one of the most fascinating areas of research in nonlinear optics. There is lots of research activity that are being conducted in various corners of the world in this filed [1-15]. This lead to several results that have various practical applications. This paper will address the dynamics of optical soliton propagation through magneto-optic waveguides in presence of spatio-temporal dispersion (STD). This dispersion term is considered in addition to the usual group velocity dispersion (GVD) and it provides well-posedness to the problem [8, 13]. Magneto-optic waveguides provides a means for compelling solitons to move from a state of attraction to a state of isolation [2, 3, 15].

The nonlinear Schrödinger's equation (NLSE), with perturbation terms, will dictate the dynamics of the propagation of solitons through these waveguides. There are three types of nonlinear media that will be touched upon, in this paper. These are Kerr law, power law and log law. For the first two cases, bright, dark and singular soliton solutions to the governing equation will be obtained. The integrability conditions, also known as constraint conditions will naturally fall out of the integration scheme. In the past, this problem has been studied with unperturbed NLSE, and the results are reported [15]. This paper is therefore an extension of earlier results.

2. Mathematical model

The mathematical model that describes the dynamics of soliton propagation through optical fibers in presence of magneto-optic field is given by the following coupled system of NLSE [2, 3, 15].

$$\begin{aligned} &iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 F(|q|^2) + \eta_1 F(|r|^2)\right\} q = \\ &= Q_1 r + i\left\{\alpha_1 q_x + \lambda_1 \left(|q|^2 q\right)_x + \nu_1 \left(|q|^2\right)_x q + \theta_1 |q|^2 q_x\right\} \end{aligned} \tag{1}$$

$$\begin{aligned} &ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 F(|r|^2) + \eta_2 F(|q|^2)\right\} r = \\ &= Q_2 q + i\left\{\alpha_2 r_x + \lambda_2 \left(|r|^2 r\right)_x + \nu_2 \left(|r|^2\right)_x r + \theta_2 |r|^2 r_x\right\} \end{aligned} \tag{2}$$

In equations (1) and (2) a_j represents the coefficients of GVD while b_j, for j = 1, 2 are the coefficients of STD. The functional F is the type of nonlinearity that will be considered. On the right hand side Q_j represents the magnetic field effect, while α_j are the coefficients of inter-modal dispersion. Also, λ_j represents the coefficients of self-steepening terms in order to avoid shock-wave formation, ν_j are the coefficients of nonlinear dispersion, while θ_j also gives nonlinear dispersion. Besides Q_j, all terms on the right hand sides, are treated as strong perturbation terms. This paper will carry out the integration of the model given by (1) and (2), in order to extract soliton solutions to the equations. This will be possible provided the type of nonlinearity is given. This will be discussed in the next three sections after an elementary analysis in the remainder of this section.

To integrate the coupled NLSE (1) - (2) we assume a solution structure of the form

$$q(x,t) = P_1(x,t)\, e^{i\phi(x,t)} \tag{3}$$

$$r(x,t) = P_2(x,t)\, e^{2i\phi(x,t)} \tag{4}$$

where $P_l(x, t)$, for l = 1, 2 represents the amplitude component of the soliton, and phase factor is given by with

$$\phi(x,t) = -\kappa x + \omega t + \theta \tag{5}$$

Here κ is the frequency of the solitons while ω represents the wave number and θ is the phase constant. Substituting (3) and (4) into (1) and (2) and then decomposing into real and imaginary parts gives

$$\begin{aligned} &a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + (b_l \kappa\omega - \omega - a_l \kappa^2 - \alpha_l \kappa) P_l - \\ &- \kappa(\lambda_l + \theta_l) P_l^3 + \left\{\xi_l F(P_l^2) + \eta_l F(P_{\bar{l}}^2)\right\} P_l - Q_l P_{\bar{l}} = 0 \end{aligned} \tag{6}$$

with $\bar{l} = 3 - l$, and imaginary parts yields

$$\begin{aligned} &(1 - \kappa b_l) \frac{\partial P_l}{\partial t} + (\omega b_l - 2 a_l \kappa - \alpha_l) \frac{\partial P_l}{\partial x} = \\ &= (3\lambda_l + 2\nu_l + \theta_l) P_l^2 \frac{\partial P_l}{\partial x} \end{aligned} \tag{7}$$

From equation (7) is possible to retrieve the speed of the soliton

$$v = \frac{b_l \omega - 2 a_l \kappa - \alpha_l}{1 - b_l \kappa} \tag{8}$$

as long as the constraints

$$\kappa b_l \neq 1 \tag{9}$$

$$3\lambda_l + 2\nu_l + \theta_l = 0 \tag{10}$$

remain valid. Notice that $P(x, t)$ can be represented as $g(x - vt)$ where the function g is the soliton wave profile depending on the type of nonlninearity, and v is the speed of the soliton. Now, equating the two values of the solitons speed (8) leads to

$$a_1 = a_2,\; b_1 = b_2,\; \alpha_1 = \alpha_2 \tag{11}$$

Consequently the equation (8) reduces to

$$v = \frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa} \tag{12}$$

subject to

$$\kappa b \neq 1 \tag{13}$$

Equation (12) shows that the speed of the soliton is a rational quantity and so this can lead to soliton speed control, which can be applied to address Internet bottleneck, which is a growing problem in Internet data transfer. It is worth to mention that the speed of the soliton given by (12) stays valid as long as constraints (9) and (10) hold. Therefore the coupled NLSE for the perturbed magneto-optic waveguide is re-casted as

$$\begin{aligned} &iq_t + aq_{xx} + bq_{xt} + \left\{\xi_1 F(|q|^2) + \eta_1 F(|r|^2)\right\} q = \\ &= Q_1 r + i\left\{\alpha q_x + \lambda_1 \left(|q|^2 q\right)_x + \nu_1 \left(|q|^2\right)_x q + \theta_1 |q|^2 q_x\right\} \end{aligned} \tag{14}$$

$$\begin{aligned} &ir_t + ar_{xx} + br_{xt} + \left\{\xi_2 F(|r|^2) + \eta_2 F(|q|^2)\right\} r = \\ &= Q_2 q + i\left\{\alpha r_x + \lambda_2 \left(|r|^2 r\right)_x + \nu_2 \left(|r|^2\right)_x r + \theta_2 |r|^2 r_x\right\} \end{aligned} \tag{15}$$

and as a consequence the real part equation (6) modifies to

$$\begin{aligned} &a \frac{\partial^2 P_l}{\partial x^2} + b \frac{\partial^2 P_l}{\partial x \partial t} + (b\kappa\omega - \omega - a\kappa^2 - \alpha\kappa) P_l - \\ &- \kappa(\lambda_l + \theta_l) P_l^3 + \left\{\xi_l F(P_l^2) + \eta_l F(P_{\bar{l}}^2)\right\} P_l - Q_l P_{\bar{l}} = 0 \end{aligned} \tag{16}$$

For the next three sections the study of the integrability of (16) will be stressed for three different types of nonlinearity.

3. Kerr law

For Kerr law nonlinearity, $F(s) = s$. As a consequence, the equations (14)-(15) modify to

$$\begin{aligned} &iq_t + aq_{xx} + bq_{xt} + \left\{\xi_1 |q|^2 + \eta_1 |r|^2\right\} q = \\ &= Q_1 r + i\left\{\alpha q_x + \lambda_1 \left(|q|^2 q\right)_x + \nu_1 \left(|q|^2\right)_x q + \theta_1 |q|^2 q_x\right\} \end{aligned} \tag{17}$$

$$\begin{aligned} &ir_t + ar_{xx} + br_{xt} + \left\{\xi_2 |r|^2 + \eta_2 |q|^2\right\} r = \\ &= Q_2 q + i\left\{\alpha r_x + \lambda_2 \left(|r|^2 r\right)_x + \nu_2 \left(|r|^2\right)_x r + \theta_2 |r|^2 r_x\right\} \end{aligned} \tag{18}$$

The imaginary part (7) is preserved and the corresponding real part equation (16) is given by

$$\begin{aligned} &a \frac{\partial^2 P_l}{\partial x^2} + b \frac{\partial^2 P_l}{\partial x \partial t} + (b\kappa\omega - \omega - a\kappa^2 - \alpha\kappa) P_l + \\ &+ \left\{\xi_l + 2\kappa(\lambda_l + \theta_l)\right\} P_l^3 + \eta_l P_{\bar{l}}^2 P_l = Q_l P_{\bar{l}} \end{aligned} \tag{19}$$

after considering the constraint (10). The ansatz approach will be applied to this equation to retrieve the corresponding bright, dark and singular solitons solution.

3.1 Bright solitons

For bright solitons the assumption for the wave profile is

$$P_l = A_l \mathrm{sech}^{p_l} \tau \tag{20}$$

$$\tau = B(x - vt) \tag{21}$$

Where A_l and B represent the amplitude and inverse width of the soliton, respectively. Substituting (20) into (18) and simplifying leads to

$$\begin{aligned} & A_l \left\{ (b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) + p_l^2 (a - bv) B^2 \right\} \mathrm{sech}^{p_l} \tau + \\ & + \left(\xi_l + 2\kappa(\lambda_l + \nu_l) \right) A_l^3 \mathrm{sech}^{3p_l} \tau + \\ & + p_l (1 + p_l)(a\text{-}bv) A_l B^2 \mathrm{sech}^{p_l + 2} \tau + \\ & + \eta_l A_l A_{\bar{l}}^2 \mathrm{sech}^{p_l + 2p_{\bar{l}}} \tau - Q_l A_{\bar{l}} \mathrm{sech}^{p_{\bar{l}}} \tau = 0 \end{aligned} \tag{22}$$

Balancing principle yields

$$p_l + 2 = 3p_{\bar{l}} \tag{23}$$

so that

$$p_l = 1 \tag{24}$$

for l = 1, 2. Next, from (22), setting the coefficients of the linearly independent functions

$$v = \frac{2aB^2 - \left(\xi_l + 2\kappa(\lambda_l + \nu_l) \right) A_l^2 - \eta_l A_{\bar{l}}^2}{2bB^2} \tag{25}$$

whenever $bB \neq 0$, and for the wave number one obtain

$$\omega = \frac{1}{2(b\kappa - 1)A_l} \begin{bmatrix} 2\kappa(\alpha + a\kappa) A_l + 2Q_l A_{\bar{l}} - \\ - \left(\xi_l + 2\kappa(\lambda_l + \nu_l) \right) A_l^3 - \eta_l A_l A_{\bar{l}}^2 \end{bmatrix} \tag{26}$$

Equating the two expressions for the soliton speed v that arise for l = 1, 2 in (25) implies

$$\frac{A_1}{A_2} = \sqrt{\frac{\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1}{\xi_1 + 2\kappa(\lambda_1 + \nu_1) - \eta_2}} \tag{27}$$

which immediately prompts the constraint

$$[\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1][\xi_1 + 2\kappa(\lambda_1 + \nu_1) - \eta_2] > 0 \tag{28}$$

Similarly, equating the two expressions for the soliton wave number from (26) for l = 1, 2 gives

$$\begin{aligned} & [\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1] A_2^3 A_1 - \\ & - [\xi_1 + 2\kappa(\lambda_1 + \nu_1) - \eta_2] A_1^3 A_2 + 2(A_2^2 Q_1 - A_1^2 Q_2) = 0 \end{aligned} \tag{29}$$

which connects the amplitudes of bright solitons in the two components. Next, by equating (12) and (25) for either l = 1 or l = 2 and considering (27), one get

$$A_1 = B\sqrt{\frac{2[\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1][(1 - b\kappa) - b(b\omega - 2a\kappa - \alpha)]}{(1 - b\kappa)[\xi_1 + 2\kappa(\lambda_1 + \nu_1)][\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2]}} \tag{30}$$

subject to

$$\begin{aligned} & [\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1][(1 - b\kappa) - b(b\omega - 2a\kappa - \alpha)] \times \\ & \times (1 - b\kappa)[\xi_1 + 2\kappa(\lambda_1 + \nu_1)][\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2] > 0 \end{aligned} \tag{31}$$

and

$$A_2 = B\sqrt{\frac{2[\xi_1 + 2\kappa(\lambda_1 + \nu_1) - \eta_2][(1 - b\kappa) - b(b\omega - 2a\kappa - \alpha)]}{(1 - b\kappa)[\xi_1 + 2\kappa(\lambda_1 + \nu_1)][\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2]}} \tag{32}$$

provided

$$\begin{aligned} & [\xi_1 + 2\kappa(\lambda_1 + \nu_1) - \eta_2][(1 - b\kappa) - b(b\omega - 2a\kappa - \alpha)] \times \\ & \times (1 - b\kappa)[\xi_1 + 2\kappa(\lambda_1 + \nu_1)][\xi_2 + 2\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2] > 0 \end{aligned} \tag{33}$$

By performing a substitution of (30) and (32) into identity (29) one get two possible expressions for the soliton frequency,

$$\kappa = \frac{b(b\omega - \alpha) - a}{ab} \tag{34}$$

subject to

$$ab \neq 0 \tag{35}$$

and

$$\kappa = \frac{Q_1(\eta_2 - \xi_1) - Q_2(\eta_1 - \xi_2)}{2[Q_1(\lambda_1 + \nu_1) - Q_2(\lambda_2 + \nu_2)]} \tag{36}$$

conditioned to

$$Q_1(\lambda_1 + \nu_1) \neq Q_2(\lambda_2 + \nu_2) \tag{37}$$

Finally, the bright 1-soliton solution for perturbed magneto-optic waveguides with STD is

$$q(x,t) = A_1 \mathrm{sech}[B(x - vt)] e^{i(-\kappa x + \omega t + \theta)} \tag{38}$$

$$r(x,t) = A_2 \mathrm{sech}[B(x - vt)] e^{i(-\kappa x + \omega t + \theta)} \tag{39}$$

3.2 Dark solitons

For dark solitons the assumption for the wave profile is

$$P_l = A_l \tanh^{p_l} \tau \tag{40}$$

where τ is being defined as in (21) . However for dark solitons the parameters A_l and B are considered herein as free parameters. Substitution of (40) into (17) and (18) leads to

$$\begin{aligned}&A_l\left\{(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) - 2p_l^2(a-bv)B^2\right\}\tanh^{p_l}\tau + \\&+ p_l(1+p_l)(a\text{-}bv)A_lB^2\tanh^{p_l+2}\tau + \\&+ p_l(p_l-1)(a\text{-}bv)A_lB^2\tanh^{p_l-2}\tau + \\&+\left(\xi_l + 2\kappa(\lambda_l+\nu_l)\right)A_l^3\tanh^{3p_l}\tau + \\&+\eta_lA_lA_{\bar l}^2\tanh^{p_l+2p_{\bar l}}\tau - Q_lA_{\bar l}\tanh^{p_{\bar l}}\tau = 0\end{aligned} \quad (41)$$

Balancing principle leads to the same value of p_l as given in (24). Moreover, the standalone linearly independent functions $\tanh^{p_l\pm 2}\tau$ also yields the same value of p_l. Next, setting the coefficients of other linearly independent functions $\tanh^{p_l+j}\tau$ to zero for j = 0, 2 gives

$$v = \frac{2aB^2 + \left(\xi_l + 2\kappa(\lambda_l+\nu_l)\right)A_l^2 + \eta_lA_{\bar l}^2}{2bB^2} \quad (42)$$

and

$$\omega = \frac{1}{(b\kappa-1)A_l}\begin{bmatrix}\kappa(\alpha+a\kappa)A_l + Q_lA_{\bar l} - \\ -\left(\xi_l+2\kappa(\lambda_l+\nu_l)\right)A_l^3 - \eta_lA_lA_{\bar l}^2\end{bmatrix} \quad (43)$$

Now equating the two expressions for the soliton speed v that arise for l = 1, 2 in (42) implies the same relation as given in (27). Consequently, equations (30) - (33) are also valid for dark solitons. Similarly, equating from the soliton wave numbers (43) yield

$$\begin{aligned}&[\xi_2 + 2\kappa(\lambda_2+\nu_2) - \eta_1]A_2^3A_1 - \\&-[\xi_1 + 2\kappa(\lambda_1+\nu_1) - \eta_2]A_1^3A_2 + (A_2^2Q_1 - A_1^2Q_2) = 0\end{aligned} \quad (44)$$

that relates the free parameters A_l. Then, by equating (12) and (42) for either l = 1 or l = 2 and considering (27), one recovers

$$\begin{aligned}&A_1 = \\&= B\sqrt{\frac{2[\xi_2+2\kappa(\lambda_2+\nu_2)-\eta_1][b(b\omega-2a\kappa-\alpha)-a(1-b\kappa)]}{(1-b\kappa)[\xi_1+2\kappa(\lambda_1+\nu_1)][\xi_2+2\kappa(\lambda_2+\nu_2)-\eta_1\eta_2]}}\end{aligned} \quad (45)$$

subject to

$$\begin{aligned}&[\xi_2+2\kappa(\lambda_2+\nu_2)-\eta_1][b(b\omega-2a\kappa-\alpha)-a(1-b\kappa)]\times \\&\times(1-b\kappa)[\xi_1+2\kappa(\lambda_1+\nu_1)][\xi_2+2\kappa(\lambda_2+\nu_2)-\eta_1\eta_2] > 0\end{aligned} \quad (46)$$

and

$$\begin{aligned}&A_2 = \\&= B\sqrt{\frac{2[\xi_1+2\kappa(\lambda_1+\nu_1)-\eta_2][b(b\omega-2a\kappa-\alpha)-a(1-b\kappa)]}{(1-b\kappa)[\xi_1+2\kappa(\lambda_1+\nu_1)][\xi_2+2\kappa(\lambda_2+\nu_2)-\eta_1\eta_2]}}\end{aligned} \quad (47)$$

provided

$$\begin{aligned}&[\xi_1+2\kappa(\lambda_1+\nu_1)-\eta_2][b(b\omega-2a\kappa-\alpha)-a(1-b\kappa)]\times \\&\times(1-b\kappa)[\xi_1+2\kappa(\lambda_1+\nu_1)][\xi_2+2\kappa(\lambda_2+\nu_2)-\eta_1\eta_2] > 0\end{aligned} \quad (48)$$

Then, substituting (45) and (47) into (44) one retrieve the same expressions for κ and corresponding constraints as in (34)- (37). This leads to the dark 1-soliton solution for perturbed magneto-optic waveguides with STD as

$$q(x,t) = A_1\tanh[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \quad (49)$$

$$r(x,t) = A_2\tanh[B(x-vt)]e^{2i(-\kappa x+\omega t+\theta)} \quad (50)$$

3.3 Singular solitons

For singular solitons the assumption for the starting hypothesis is

$$P_l = A_l\operatorname{csch}^{p_l}\tau \quad (51)$$

Where i is being defined as in (21), while the parameters A_l and B are considered herein as free parameters again. Upon substituting (51) into (18) yield

$$\begin{aligned}&A_l\left\{(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) + p_l^2(a-bv)B^2\right\}\operatorname{csch}^{p_l}\tau + \\&+\left(\xi_l - \kappa(\lambda_l+\nu_l)\right)A_l^3\operatorname{csch}^{3p_l}\tau + \\&+ p_l(1+p_l)(a\text{-}bv)A_lB^2\operatorname{csch}^{p_l+2}\tau + \\&+\eta_lA_lA_{\bar l}^2\operatorname{csch}^{p_l+2p_{\bar l}}\tau - Q_lA_{\bar l}\operatorname{csch}^{p_{\bar l}}\tau = 0\end{aligned} \quad (52)$$

Balancing principle gives the same value of p_l as given in (24). Next, from (52), setting the coefficients of the linearly independent functions $\operatorname{csnh}^{p_l+j}\tau$ to zero for j = 0, 2gives for the soliton speed the same expression as in (42), which naturally implies the same relation as given by (27). Therefore, the amplitudes for singular solitons are the same as for dark solitons with corresponding constraints (45) -(48). For the wave number one have

$$\omega = \frac{1}{2(b\kappa-1)A_l}\begin{bmatrix}2\kappa(\alpha+a\kappa)A_l + 2Q_lA_{\bar l} + \\ +\left(\xi_l+2\kappa(\lambda_l+\nu_l)\right)A_l^3 + \eta_lA_lA_{\bar l}^2\end{bmatrix} \quad (53)$$

Equating the two expressions for the soliton wave number from (53) for l = 1, 2 gives

$$\begin{aligned}&[\xi_2 + 2\kappa(\lambda_2+\nu_2) - \eta_1]A_2^3A_1 - \\&-[\xi_1 + 2\kappa(\lambda_1+\nu_1) - \eta_2]A_1^3A_2 - 2(A_2^2Q_1 - A_1^2Q_2) = 0\end{aligned} \quad (54)$$

By substituting (30) and (32) into (54) one get the same expressions for κ and corresponding constraints as in (34)-(37). Hence, the 1-soliton solution for perturbed magneto-optic waveguides with STD is

$$q(x,t) = A_1\operatorname{csch}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \quad (55)$$

$$r(x,t) = A_2\operatorname{csch}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \quad (56)$$

This singular soliton will exist provided the constraints conditions given in this subsection.

4. Power law

For the power law nonlinearity, $F(s) = s^n$, where n represents the power law nonlinearity parameter. Here the stability issue dictates $0 < n < 2$ and also $n \neq 2$ for avoiding self-focusing singularity. Thus, the system (14) - (15) is rewritten as

$$\begin{aligned} & iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 |q|^{2n} + \eta_1 |r|^{2n}\right\} q = \\ & = Q_1 r + i\left\{\alpha_1 q_x + \lambda_1 \left(|q|^{2n} q\right)_x + \nu_1 \left(|q|^{2n}\right)_x q + \theta_1 |q|^{2n} q_x\right\} \end{aligned} \quad (57)$$

$$\begin{aligned} & ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 |r|^{2n} + \eta_2 |q|^{2n}\right\} r = \\ & = Q_2 q + i\left\{\alpha_2 r_x + \lambda_2 \left(|r|^{2n} r\right)_x + \nu_2 \left(|r|^{2n}\right)_x r + \theta_2 |r|^{2n} r_x\right\} \end{aligned} \quad (58)$$

Upon substituting (3) and (4) into (57) and (58) the resulting real part obtained is

$$\begin{aligned} & a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + (b_l \kappa\omega - \omega - a_l \kappa^2 - \alpha_l \kappa) P_l + \\ & + (\xi_l P_l^{2n} + \eta_l P_{\bar{l}}^{2n}) P_l = Q_l P_{\bar{l}} + \kappa(\lambda_l + \theta_l) P_l^{2n+1} \end{aligned} \quad (59)$$

and for the imaginary part

$$\begin{aligned} & (1 - \kappa b_l) \frac{\partial P_l}{\partial t} + (\omega b_l - 2 a_l \kappa - \alpha_l) \frac{\partial P_l}{\partial x} = \\ & = \{(2n+1)\lambda_l + 2n\nu_l + \theta_l\} P_l^{2n} \frac{\partial P_l}{\partial x} \end{aligned} \quad (60)$$

From (60) is possible to retrieve the solitons speed (8) as long as the constraints (9) and

$$(2n+1)\lambda_l + 2n\nu_l + \theta_l = 0 \quad (61)$$

Consequently (11) and (12) are also satisfied in this case, and the real part (59) becomes

$$\begin{aligned} & a \frac{\partial^2 P_l}{\partial x^2} + b \frac{\partial^2 P_l}{\partial x \partial t} + (b\kappa\omega - \omega - a\kappa^2 - \alpha\kappa) P_l + \\ & + (\xi_l P_l^{2n} + \eta_l P_{\bar{l}}^{2n}) P_l = Q_l P_{\bar{l}} + \kappa(\lambda_l + \theta_l) P_l^{2n+1} \end{aligned} \quad (62)$$

The ansatz approach will be applied to equation (62) in order to retrieve bright, dark and singular solitons.

4.1. Bright solitons

For bright solitons, the starting hypothesis is the same as that of Kerr law nonlinearity given by (20) along with (21). After substitution, (59) reduces to

$$\begin{aligned} & A_l \left\{(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) + p_l^2 (a - bv) B^2\right\} \operatorname{sech}^{p_l} \tau + \\ & + \left(\xi_l - \kappa(\lambda_l + \theta_l)\right) A_l^{2n+1} \operatorname{sech}^{(2n+1)p_l} \tau + \\ & + p_l (1 + p_l)(a - bv) A_l B^2 \operatorname{sech}^{p_l + 2} \tau + \\ & + \eta_l A_l A_{\bar{l}}^{2n+1} \operatorname{sech}^{p_l + 2n p_{\bar{l}}} \tau - Q_l A_{\bar{l}} \operatorname{sech}^{p_{\bar{l}}} \tau = 0 \end{aligned} \quad (63)$$

Balancing principle yields

$$p_l + 2 = (2n+1) p_{\bar{l}} \quad (64)$$

so that

$$p_l = \frac{1}{n} \quad (65)$$

for $l = 1, 2$. Next, from (63), setting the coefficients of the linearly independent functions $\operatorname{sech}^{p_l + j}\tau$ to zero, for $j = 0, 2$ leads to the speed and wave number of the bright soliton as

$$v = \frac{1}{(n+1) b B^2} \left[\begin{aligned} & (n+1) a B^2 - \\ & - n^2 \left(\xi_l + 2n\kappa(\lambda_l + \nu_l)\right) A_l^{2n} - n^2 \eta_l A_{\bar{l}}^{2n} \end{aligned} \right] \quad (66)$$

and

$$\omega = \frac{1}{(n+1)(b\kappa - 1) A_l} \left[\begin{aligned} & \kappa(n+1)(\alpha + a\kappa) A_l - \\ & - n^2 \left(\xi_l + 2n\kappa(\lambda_l + \nu_l)\right) A_l^{2n+1} + \\ & + (n+1) Q_l A_{\bar{l}} - \eta_l A_l A_{\bar{l}}^{2n} \end{aligned} \right] \quad (67)$$

after using constraint (61). Next, equating the two expressions for the soliton speed v that arise for $l = 1, 2$ in (66) implies

$$\frac{A_1}{A_2} = \left[\frac{\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1}{\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2} \right]^{\frac{1}{2n}} \quad (68)$$

subject to

$$[\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1][\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2] > 0 \quad (69)$$

Similarly, by equating the two expressions for the soliton wave number from (67) for $l = 1, 2$ one obtains

$$\begin{aligned} & [\xi_2 + 2n\kappa(\lambda_2 + \theta_2) - \eta_1] A_2^{2n+1} A_1 - \\ & - [\xi_1 + 2n\kappa(\lambda_1 + \theta_1) - \eta_2] A_1^{2n+1} A_2 + (n+1)(A_2^2 Q_1 - A_1^2 Q_2) = 0 \end{aligned} \quad (70)$$

Next, by equating (12) and (66) for either $l = 1$ or $l = 2$ and considering (68), recovers the soliton amplitudes as

$$A_1 = B \times \left[\frac{(n+1)[\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1][a(b\kappa - 1) - b(b\omega - 2a\kappa - \alpha)]}{n^2 (b\kappa - 1)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1 \eta_2]} \right]^{\frac{1}{2n}} \quad (71)$$

subject to

$$[\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1][a(b\kappa - 1) - b(b\omega - 2a\kappa - \alpha)] \times$$
$$\times (b\kappa - 1)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2] > 0 \tag{72}$$

and

$$A_2 = B \times$$
$$\left[\frac{(n+1)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2][a(b\kappa - 1) + b(b\omega - 2a\kappa - \alpha)]}{n^2(b\kappa - 1)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2]}\right]^{\frac{1}{2n}} \tag{73}$$

Subject to

$$[\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2][a(b\kappa - 1) + b(b\omega - 2a\kappa - \alpha)] \times$$
$$(b\kappa - 1)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2] > 0 \tag{74}$$

Thus, the bright 1-soliton solution for magneto-optic waveguides with STD and power-law nonlinearity is

$$q(x,t) = A_1 \operatorname{sech}^{\frac{1}{n}}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{75}$$

$$r(x,t) = A_2 \operatorname{sech}^{\frac{1}{n}}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{76}$$

4.2. Dark solitons

For dark solitons, the starting hypothesis is as given by (40), thus the real part equation simplifies to

$$\begin{aligned}
&A_l\left\{(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) - 2p_l^2(a - bv)B^2\right\}\tanh^{p_l}\tau + \\
&+ p_l(1 + p_l)(a - bv)A_l B^2 \tanh^{p_l+2}\tau + \\
&+ p_l(p_l - 1)(a - bv)A_l B^2 \tanh^{p_l-2}\tau + \\
&+ \xi_l A_l^{2n+1}\tanh^{(2n+1)p_l}\tau + \\
&+ \eta_l A_l A_{\bar{l}}^{2n}\tanh^{p_l+2np_{\bar{l}}}\tau - Q_l A_{\bar{l}}\tanh^{p_{\bar{l}}}\tau = 0
\end{aligned} \tag{77}$$

Balancing principle leads to the same value of p_l as given in (65). Moreover, the standalone linearly independent function $\tanh^{p_l-2}\tau$ also yields the same value of p_l as given by (24). This means that dark solitons will exist in magneto-optic waveguides provided power law reduces to Kerr law. Therefore all the results from the subsection of dark solitons for Kerr law nonlinearity are valid for this subsection as well.

4.3. Singular solitons

For singular solitons, the starting hypotesis given by (51) reduce (59) into

$$\begin{aligned}
&A_l\left\{(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) + p_l^2(a - bv)B^2\right\}\operatorname{csch}^{p_l}\tau + \\
&+ \left(\xi_l + 2n\kappa(\lambda_l + \nu_l)\right)A_l^{2n+1}\operatorname{csch}^{(2n+1)p_l}\tau + \\
&+ p_l(1 + p_l)(a - bv)A_l B^2 \operatorname{csch}^{p_l+2}\tau + \\
&+ \eta_l A_l A_{\bar{l}}^{2n}\operatorname{csch}^{p_l+2p_{\bar{l}}}\tau - Q_l A_{\bar{l}}\operatorname{csch}^{p_{\bar{l}}}\tau = 0
\end{aligned} \tag{78}$$

after adopting the constraint given in (61). Balancing principle gives the same value for p_l as in (65). Then, from (78), the linearly independent functions $\operatorname{csnh}^{p_l+j}\tau$ for j = 0, 2 leads to

$$v = \frac{1}{(n+1)bB^2}\left[\begin{aligned}&(n+1)aB^2 + \\ &+ n^2\left(\xi_l + 2n\kappa(\lambda_l + \nu_l)\right)A_l^{2n} + n^2\eta_l A_{\bar{l}}^{2n}\end{aligned}\right] \tag{79}$$

and

$$\omega = \frac{1}{(n+1)(b\kappa - 1)A_l}\left[\begin{aligned}&\kappa(n+1)(\alpha + a\kappa)A_l + \\ &+ \left(\xi_l + 2n\kappa(\lambda_l + \nu_l)\right)A_l^{2n+1} + \\ &+ (n+1)Q_l A_{\bar{l}} + \eta_l A_l A_{\bar{l}}^{2n}\end{aligned}\right] \tag{80}$$

Next, after equating the two expressions for the soliton speed v in (79), one gets (68) and consequently (69). In a similar manner, equating the wave numbers from (80) for l = 1, 2 prompts to

$$[\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1]A_2^{2n+1}A_1 -$$
$$-[\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2]A_1^{2n+1}A_2 + (n+1)(A_2^2 Q_1 - A_1^2 Q_2) = 0 \tag{81}$$

Next, by equating (12) and (79) for either l = 1 or l = 2 and considering (68), one generates

$$A_1 = B \times$$
$$\left[\frac{(n+1)[\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1][b(b\omega - 2a\kappa - \alpha) - a(1 - b\kappa)]}{n^2(1 - b\kappa)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2]}\right]^{\frac{1}{2n}} \tag{82}$$

subject to

$$\begin{aligned}&[\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1][b(b\omega - 2a\kappa - \alpha) - a(1 - b\kappa)] \times \\ &\times (1 - b\kappa)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2] > 0\end{aligned} \tag{83}$$

and

$$A_2 = B \times$$
$$\left[\frac{(n+1)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2][b(b\omega - 2a\kappa - \alpha) - a(1 - b\kappa)]}{n^2(1 - b\kappa)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2]}\right]^{\frac{1}{2n}} \tag{84}$$

Subject to

$$\begin{aligned}&[\xi_1 + 2n\kappa(\lambda_1 + \nu_1) - \eta_2][b(b\omega - 2a\kappa - \alpha) - a(1 - b\kappa)] \times \\ &(1 - b\kappa)[\xi_1 + 2n\kappa(\lambda_1 + \nu_1)][\xi_2 + 2n\kappa(\lambda_2 + \nu_2) - \eta_1\eta_2] > 0\end{aligned} \tag{85}$$

The singular 1-soliton solution for magneto-optic waveguides with STD is

$$q(x,t) = A_1 \operatorname{csch}^{\frac{1}{n}}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \quad (86)$$

$$r(x,t) = A_2 \operatorname{csch}^{\frac{1}{n}}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \quad (87)$$

These singular 1-soliton will exist provided necessary constraint conditions hold.

5. Log law nonlinearity

For log law nonlinear media, solitons in magneto-optic waveguides are modeled by

$$iq_t + aq_{xx} + bq_{xt} + \left\{\xi_1 \ln|q|^2 + \eta_1 \ln|r|^2\right\}q = = Q_1 r + i\alpha_1 q_x \quad (88)$$

$$ir_t + ar_{xx} + br_{xt} + \left\{\xi_2 \ln|r|^2 + \eta_2 \ln|q|^2\right\}r = = Q_2 q + i\alpha_2 r_x \quad (89)$$

For this kind of nonlinearity, the real part (16) is given by

$$a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + (b_l \kappa\omega - \omega - a_l \kappa^2 - \alpha_l \kappa)P_l + + 2\xi_l P_l \ln P_l + 2\eta_l P_l \ln P_{\bar{l}} = Q_l P_{\bar{l}} \quad (90)$$

For log-law nonlinearity, NLSE supports optical *Gaussons* that are given as

$$P_l = A_l e^{-\tau^2}, \quad \tau = B(x-vt) \quad (91)$$

where A_l represents the amplitudes of the Gaussons, while B is its inverse width. Inserting this hypothesis into (90) gives

$$2\tau^2 A_l \left\{2(a-bv)B^2 - (\xi_l + \eta_l) - 2A_l B^2 (a-bv)\right\} + + A_l (b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa) + 2A_l(\xi_l \ln\xi_l + \eta_l \ln\eta_l) - - Q_l \ln A_{\bar{l}} = 0 \quad (92)$$

Setting the coefficients of the linearly independent functions τ^j, for j = 0, 2 leads to the velocity of the Gaussons being

$$v = \frac{2aB^2 - \eta_l - \xi_l}{2bB^2} \quad (93)$$

Thus, equating the two expressions for the v from (93) proposes the constraint

$$\xi_1 - \xi_2 = \eta_1 - \eta_2 \quad (94)$$

The second linearly independent function from (92) yields the wave number of Gaussons as

$$\omega = \frac{A_l\left\{\kappa(\alpha + a\kappa) - 2(\xi_l \ln A_l + \eta_l \ln A_{\bar{l}}) + \xi_l + \eta_l\right\} + Q_l \ln A_{\bar{l}}}{(b\kappa - 1)A_l} \quad (95)$$

Finally, equating the two values of ω from (95), for l = 1, 2, leads to the relation of the two amplitudes by

$$2A_1A_2\left\{(\eta_2 - \xi_1)\ln A_1 + (\xi_2 - \eta_1)\ln A_2\right\} + + A_1A_2\left\{(\xi_1 - \xi_2) + (\eta_1 - \eta_2)\ln A_2\right\} + A_2^2 Q_1 - A_1^2 Q_2 = 0 \quad (96)$$

Therefore, optical Gaussons in magneto-optic waveguides with log-law nonlinearity is

$$q(x,t) = A_1 e^{-B^2(x-vt)^2} e^{i(-\kappa x+\omega t+\theta)} \quad (97)$$

$$r(x,t) = A_2 e^{-B^2(x-vt)^2} e^{i(-\kappa x+\omega t+\theta)} \quad (98)$$

6. Conclusions

This paper obtained exact 1-soliton solutions to NLSE in magneto-optic waveguides with perturbation terms taken into consideration. The STD was included in addition to GVD which makes the problem well-posed. In fact, the inclusion of STD provides a means to control Internet bottleneck. The perturbation terms considered are inter-modal dispersion, self-steepening, nonlinear dispersions. Three types of nonlinearity are studied and they are Kerr law, power law and log law. Several constraint conditions are obtained. These relations must remain valid for the solitons to exist.

There are several avenues of extension to this project in future. There are additional laws of nonlinearity that will be touched upon in future. They are parabolic law, dual-power law, polynomial law, triple-power law, saturable law and many others. The computational aspects for these additional laws will require long and tedious algebraic maneuvering and simplification. Therefore study for those laws will be reported in future. This is just a tip of the iceberg.

Acknowledgements

This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University under grant number 64-130-35-HiCi. The authors therefore thankfully acknowledge this technical and financial support of KAU.

References

[1] L. D. S. Alcantara, F. L. Teixeira, A. C. Ceaser, B-H. V. Borges, Journal of Lightwave Technology. **23**, 2579 (2005).
[2] A. Biswas, Applied Mathematics and Computation. **153**, 387 (2004).
[3] A. Biswas, S. Konar, CRC Press, Boca Raton, FL. (2006).

[4] A. Biswas, K. R. Khan, A. Rahman, A. Yildirim, T. Hayat, O. M. Aldossary, J. Optoelectron. Adv. Mater. **14**, 571 (2012).
[5] A. D. Boardman, M. Xie, Journal of Optics B. **3**, S244 (2001).
[6] A. D. Boardman, M. Xie, Journal of the Optical Society of America B. **19**, 563 (2002).
[7] C. Dai, J. Zhang, Optics Letters. **35**, 2561 (2010).
[8] X. Geng, Y. Lv, Nonlinear Dynamics. **69**, 1621. (2012).
[9] L. Girgis, D. Milovic, S. Konar, A. Yildirim, H. Jafari, A. Biswas, Romanian Reports in Physics. **64**, 663 (2012).
[10] R. Gupta, A. Goyal, T. S. Raju, C. N. Kumar, Journal of Modern Optics. **60**, 1569 (2013).
[11] M. Khatir, N. Granpayeh, Optik. **122**, 2199 (2011).
[12] M. Khatir, N. Granpayeh, Optik **124**, 276 (2013).
[13] S. Kumar, K. Singh, R. K. Gupta, Pramana. **79**, 41 (2012).
[14] D. Milovic, A. Biswas, Serbian Journal of Electrical Engineering. **10**, 365 (2013).
[15] M. Savescu, A. H. Bhrawy, E. M. Hilal, A. A. Alshaery, A. Biswas, Frequenz. **68**, 445 (2014).

*Corresponding author: biswas.anjan@gmail.com