Optical soliton propagation with different backgrounds for a variable coefficient optical fiber system

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A generalized optical fiber system with three variable coefficient functions is investigated, which is modelled as a nonlinear Schrödinger equation with variable gain coefficient, variable dispersion coefficient and variable nonlinearity coefficient. An analytic soliton solution and its compatible condition are derived via using the auxiliary equation method. Through appropriately setting the coefficient functions as specific functions, five types of backgrounds are studied, namely, plain, upper-arch, downward-arch, trapezium and periodic wave backgrounds. The results reveal the new characteristics for the optical fiber system and contribute to a better understanding to the optical propagation background.

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1. Introduction

Optical soliton and soliton theory take a critical part optical information and optical telecommunication [1-3]. A lot of attention and effort have been drawn to the study on optical solitons, and the results have been becoming more and more abundant [4-18].

In general, optical solitons may be generated because of a delicate balance between group velocity dispersion (GVD) and nonlinear effects [19]. Some studies on solitons have been carried out both theoretically and experimentally [20-25]. Analytic soliton solutions have been investigated [26], and nonlinear optics models giving rise to the appearance of solitons in a narrow sense have been considered [27].

The ubiquitous nonlinear Schrödinger equation (NLSE) is one of the most fundamental mathematical models to describe many nonlinear phenomena, especially in optical pulse propagation in nonlinear optical fiber system [1-3]. In Ref. [28], soliton evolution was considered which is driven by random polarization mode dispersion. Under some parametric conditions, solitons were obtained for a higher-order NLSE with non-Kerr nonlinearity in Ref. [29], and the interaction dynamics of solitons was reconsidered in Ref. [30].

Due to the practical applications, the factors that affect the system are very complicated. These real factors often produce fiber gain/loss, phase modulation and variable dispersion. The variable coefficient nonlinear Schrödinger equation (vcNLSEs) is becoming a class of effective models to describe the inhomogeneous effects of optical pulse propagations in nonlinear media. The vcNLSEs permit to reveal more abundant optical pulse propagation characteristics under various conditions and complex environments, such as varying GVD, Kerr nonlinearity and system gain/loss [1-3, 31-37].

Unlike constant coefficient nonlinear Schrödinger equations (ccNLSEs), the studies on vcNLSEs show that one can excite and control the soliton structures and propagation backgrounds through their inhomogeneity parameters. Even though identifying and controlling the solitons in vcNLSE systems have been investigated by several authors [35-37]. There are still a great amount of unknown and valuable problems to be explored for the vcNLSEs.

In this paper, we discuss the following vcNLSE system

$$u_z - i\frac{D(z)}{2}u_{\tau\tau} + i\rho(z)|u|^2 u = g(z)u, \qquad (1)$$

where $u(z,\tau)$ is the envelope of optical pulse waveguides, z is the longitudinal coordinate, τ is the time in the moving coordinate system, i represents the unit complex number. $D(z)$ is the GVD coefficient, $\rho(z)$ is the nonlinearity coefficient, and $g(z)$ is the system gain/loss coefficient. The system (1) can well describe the propagation of the picosecond optical pulse in inhomogeneous optical fiber systems [38].

The optical soliton amplification and its condition are studied in Refs. [39, 40]. The single-soliton and double-soliton were obtained by bilinear method, and some controls on the soliton dynamics have explored using GVD and gain coefficients [41,42]. Gaussian rogue waves are studied through similarity transformation method [43].

In this paper, we will seek for the generalized soliton solution to the sytem (1) by the auxiliary equation method, namely, the generalized (*G'/G*)-expansion method. Through choosing different gain coefficients, we discuss six types of soliton propagation backgrounds.

2. Analytic soliton solution to the system (1)

The solution comprising free functions is called as generalized solution, which is the basis to construct rich solitons [44]. The (*G'/G*)-expansion method was proposed to solve nonlinear evolution equation (NLEE) by Wang [45]. It is an effective tool to obtain soliton solution for NLEE [46], and has been expanded to solve generalized soliton solution [47, 48]. Here we will apply the (*G'/G*)-expansion method to construct the general analytic soliton solution to the system (1).

We first separate $u(z,\tau)$ in the system (1) into real part and imaginary part, namely,

$$u(z,\tau) = p(z,\tau) + iq(z,\tau), \tag{2}$$

where $p(z,\tau)$ and $q(z,\tau)$ are real functions for variable x and τ. Then it follows

$$u_z = p_z + iq_z, u_\tau = p_\tau + iq_\tau, u_{\tau\tau} = p_{\tau\tau} + iq_{\tau\tau}, \tag{3}$$

$$|u|^2 = p^2 + q^2, |u|^2 u = (p^2+q^2)p + i(p^2+q^2)q. \tag{4}$$

Substituting (4) into the system (1) yields

$$\begin{cases} p_z + \dfrac{D(z)}{2} q_{\tau\tau} - \rho(z) q(p^2+q^2) = g(z)p, \\ q_z - \dfrac{D(z)}{2} p_{\tau\tau} + \rho(z) p(p^2+q^2) = g(z)q. \end{cases} \tag{5}$$

According to the generalized (*G'/G*)-expansion method, we are able to assume

$$\begin{cases} p(z,\tau) = \sum_{k=0}^{n} a_k(z)\left(G'/G\right)^k, a_n(z) \neq 0, \\ q(z,\tau) = \sum_{j=0}^{m} b_j(z)\left(G'/G\right)^j, b_m(z) \neq 0, \end{cases} \tag{6}$$

where $a_k(z), b_j(z)$ $(k = 0,1,2,\ldots,n, j = 0,1,2,\ldots,m)$ are the functions of z, $G = G(\xi)$ satisfies the following ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{7}$$

where $\xi = \xi(z,\tau)$, λ and μ are arbitrary constants.

Applying the homogeneous balance principle to Eqs. (5), we have $n = m = 1$ Thus the solutions of Eq. (5) can be written as

$$\begin{cases} p(z,\tau) = a_0(z) + a_1(z)\left(G'/G\right), \\ q(z,\tau) = b_0(z) + b_1(z)\left(G'/G\right), \end{cases} \tag{8}$$

where a_{0j}, a_{1j}, b_{0j}, b_{1j} (j = 1,2) are all functions to be determined later. To simplify computation, we consider $\xi(z,\tau) = f(z) + g(z)$. Substituting (8) into (6), collecting the term of (*G'/G*) with the same power, then letting each coefficient to be zero will yield a set of over-determined partial differential equations about the parameters a_{0j}, a_{1j}, b_{0j}, b_{1j} (j = 1,2) and $\xi(z,\tau)$.Solving the set of equations, and setting λ = 0, we get the modulus square soltion solutions to the system (1)

$$|u|^2 = l^2\left(d^2 + \left(G'/G\right)^2\right)e^{2\int g(z)dz}, \tag{9}$$

where

$$\frac{G'}{G} = \delta \frac{C_1 \sinh(\delta\xi) + C_2 \cosh(\delta\xi)}{C_1 \cosh(\delta\xi) + C_2 \sinh(\delta\xi)},$$

with $\xi = \xi(z,\tau) = dl^2 \int D(z)dz + l\tau + k$, $C_1, C_2, d, l, \delta, \mu$, are real constants, and $C_1 \neq C_2$, $d \neq 0$, $l \neq 0$, δ = sqrt($-\mu$), μ<0, and compatible condition

$$D(z) = \varepsilon\rho(z)e^{\int 2g(z)dz}, \tag{10}$$

where d, l and ε are non-zero real constants, k is an integral constant.

3. Soliton propagation backgrounds to the system (1)

In virtue of the free functions $D(z)$ and $g(z)$ in the soliton solutions (9), it is convenient to explore and observe the soliton propagation patterns. The groups of $D(z)$ and $g(z)$ are extremely rich, this admits ones to excite abundant soliton structures through setting these functions. A furthermore analysis illustrates that $g(z)$ can determines the soliton propagation backgrounds. In this work, our main task will focus on the gain/loss function $g(z)$. We discuss six types of backgrounds by choosing different $g(z)$ and fixing $D(z)$ as

$$D(z) = 3z. \tag{11}$$

3.1. The plain background

As a special case, we easily get a standard optical dark soliton with the plain background as $g(z) = 0$, where it means that the gain/loss function is not considered for the system. Different parameters can determine the direction and amplitude (see Fig. 1).

3.2. The upper arch background

If we set the gain/loss function $g(z)$ as a linear function

$$g(z) = cz, \tag{12}$$

where c is a constant, the system (1) holds the upper arch background where the optical soliton will separate the arch background into two parts. The parameter c in (12) can control the open width of the arch. The effect is demonstrated in Fig. 2.

3.3. The downward arch background

When setting the gain/loss function $g(z)$ as hyperbolic function

$$g(z) = 2c_1c_2\tanh(c_2z)\operatorname{sech}(c_2z), \tag{13}$$

where c_1 and c_2 are arbitrary constants, the system (1) will possess the downward arch background. Similar to the soliton with the upper arch background in the subsection 3.2, the downward arch background will separate the soliton into two parts. The parameters c_1 and c_2 in (13) can tune the open width of the arch. We illustrate the effect in Fig. 3.

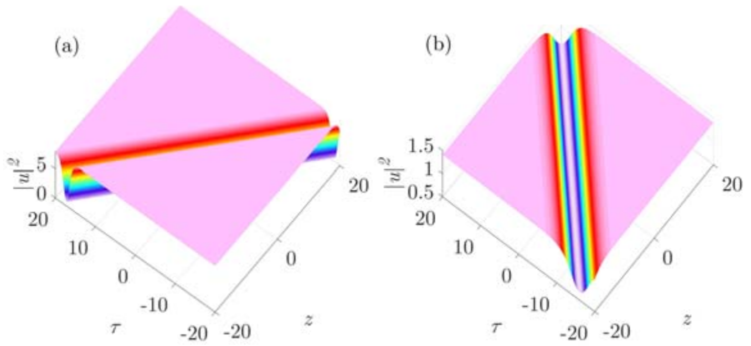

Fig. 1. The soliton propagation with the plain background. The settings are following (11), g(z) = 0, C_1 = 2, C_1 = 1, k = 0.2, d = 0.3, the indefinite integral constant in (9) is taken as 1, and (a) l = -1, μ = -1; (b) l = 1, μ = -0.1.

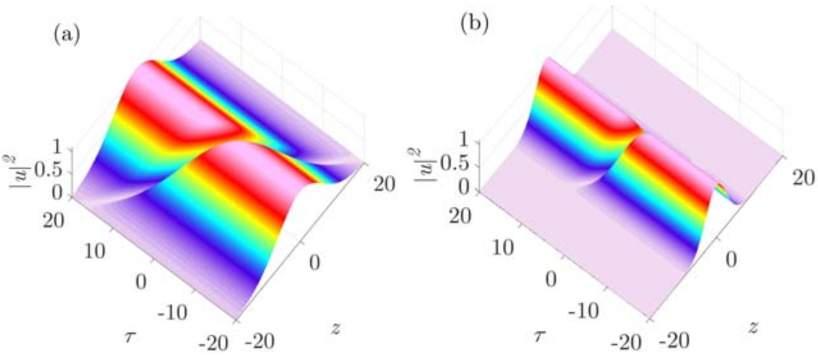

Fig. 2. The soliton propagation with the upper arch background. The settings are following (11), (12), C_1 = 2, C_1 = 1, k = 0.2, d = 0.3, l = 1, μ = -1, the indefinite integral constant in (9) is taken as 0, and (a) c = 0.01; (b) c = 0.05.

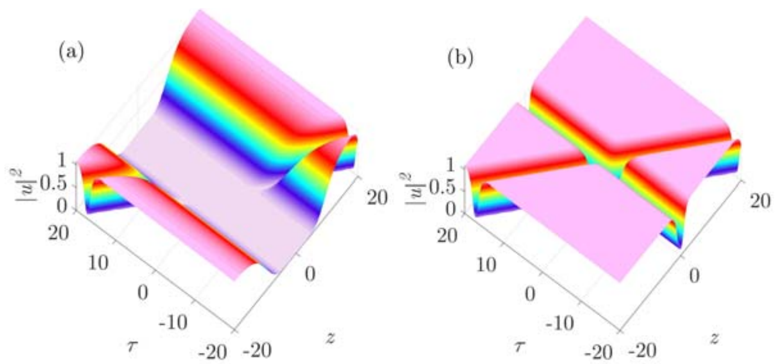

Fig. 3. The soliton propagation with the downward arch background. The settings are following (11), (13), C_1 = 2, C_1 = 1, k = 0.2, d = 0.3, l = 1, μ = -1, the indefinite integral constant in (10) is taken as 0, and (a) $c_1 = 5, c_2 = 0.2$; (b) $c_1 = 1, c_2 = 1$.

3.4. The trapezium background

When setting $g(z)$ as hyperbolic function

$$g(z) = c_1 \operatorname{sech}^2(c_2 z), \tag{14}$$

where c_1 and c_2 are arbitrary constants, the system (1) holds the trapezium background. The parameter c_1 can determine the trapezium direction, and c_2 can tune the slope rate and the amplitude of the trapezium (see Fig. 4.)

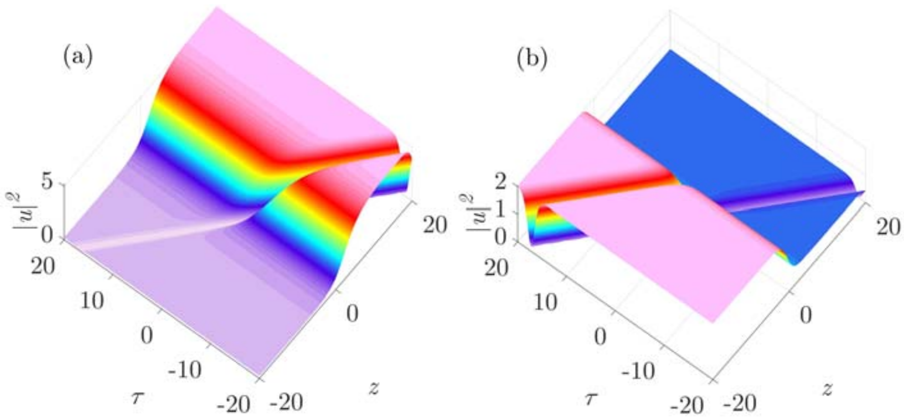

Fig. 4. The soliton propagation with the trapezium background. The settings are following (11), (14), $C_1 = 2$, $C_1 = 1$, $k = 0.2$, $d = 0.3$, $l = 1$, $\mu = -1$, the indefinite integral constant in (9) is taken as 0, and (a) $c_1 = 0.2$, $c_2 = 0.4$; (b) $c_1 = -0.2$, $c_2 = 1$.

3.5. The periodic wave background

Through setting $g(z)$ as trigonometric sine function

$$g(z) = c_1 \sin(c_2 z), \tag{15}$$

where c_1 and c_2 are arbitrary constants, the system (1) holds the periodic wave background. The parameter c_1 can determine the amplitude, and c_2 can tune the intensity of the periodic waves (see Fig. 5.)

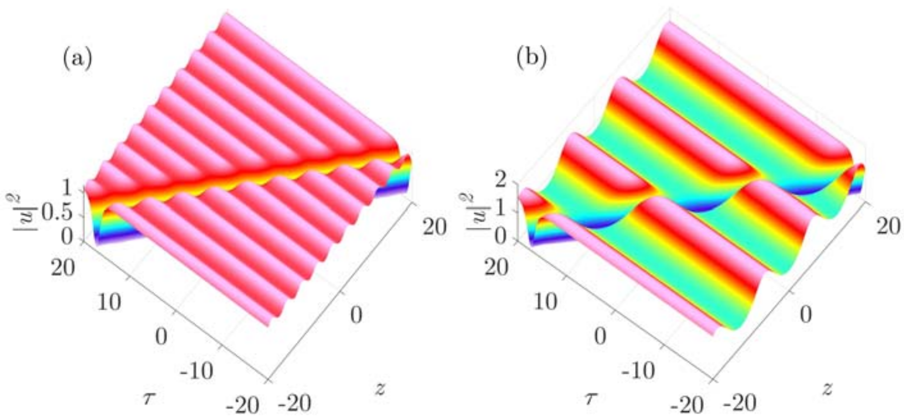

Fig. 5. The soliton propagation with the periodic wave background. The settings are following (11), (15), $C_1 = 2$, $C_1 = 1$, $k = 0.2$, $d = 0.3$, $l = 1$, $\mu = -1$, the indefinite integral constant in (9) is taken as 0, and (a) $c_1 = 0.05$, $c_2 = 1.5$; (b) $c_1 = 1.1$, $c_2 = 0.5$.

4. Conclusions

The system (1) is an important optical soliton propagation model with variable variable gain coefficient, variable dispersion coefficient and variable nonlinearity coefficient. The system can be used to describe the complex optical fiber environment. More exploration for the system can help us to deeply understand the characteristics of optical soliton in nonlinear fiber.

Applying a class of auxiliary equation method, namely, the generalized (G'/G)-expansion method, we obtain the generalized optical soliton solution and its compatible condition to the system (1). There are two free functions among the GVD, nonlinearity and gain/loss coefficient functions. It is fundamental to explore new optical soliton propagation properties.

We discuss the five types of soliton propagation backgrounds through choosing the gain/loss function in the system (1), namely, plain, upper arch, downward arch, trapezium and periodic wave backgrounds. All types are illustrated graphically. Specially, as gain/loss function is zero, we get a standard dark optical soliton with the plain background, which is often reported in various fiber system. The other four types of soliton in this work are novel and interesting.

In this article, a few typical functions are only chosen to discuss different backgrounds. In fact, it is easy to see that one can explore more abundant backgrounds by the way in this work. In addition, ones are able to choose arbitrary two of these three coefficient functions to investigate more soliton features.

We hope that the results here may be helpful to inspire new ideas in optical soliton theory, experiment, optical communication engineering in future.

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