Optical solitons in cascaded system with three integration schemes

M. MIRZAZADEH[a], Q. ZHOU[b], A. H. BHRAWY[c,d], A. BISWAS[e,c,*], M. BELIC[f]
[a]*Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157, Rudsar-Vajargah, Iran*
[b]*School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan, 430212, P.R. China*
[c]*Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah-21589, Saudi Arabia*
[d]*Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef 62511, Egypt*
[e]*Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA*
[f]*Science Program, Texas A & M University at Qatar, Doha, Qatar*

This paper obtains soliton solutions in cascaded system with Kerr law nonlinearity. There are three integration tools adopted in this paper. These are Q-function approach, Riccati equation method and G'/G-expansion scheme. These lead to topological and singular optical soliton solutions to the model. Additionally, there are singular periodic solutions that are also revealed. Finally constraint conditions are given that needs to hold for these solitons to exist.

(Received December 12, 2014; accepted January 21, 2015)

Keywords: Cascaded system; Solitons; Integrability; Q-function; Ricatti equation expansion; G'/G-expansion.

1. Introduction

Optical solitons is an essential concept in today's Internet dominated technological world [1-15]. These soliton molecules are visible in all walks of life on a daily basis. These solitons are needed for fiber-optic communication for long distances across the globe. Modern day technologies, such as twitter, face-book, electronic mail and others are only possible because of soliton transmission through optical fibers. This paper therefore studies the dynamics of solitons in cascaded system. It must be noted that in the past bright and dark soliton solutions to cascaded system was obtained by ansatz method [2, 7, 12].

The governing equation for the propagation of solitons through optical fibers is the nonlinear Schrödinger's equation (NLSE). Typically, this equation is studied with Kerr law nonlinearity so that NLSE is alternatively referred to as cubic Schrödinger's equation. For cascaded system, it is the vector NLSE that is studied. This paper will address the coupled NLSE from its integration standpoint. There are three integration algorithms that are utilized for analyzing the vector NLSE. These are Q-function approach, Riccati equation method and G'/G-expansion scheme. The result will be singular soliton solution, topological soliton solution and finally, as a by-product, singular periodic solutions are also obtained. From the analysis several constraint conditions naturally emerge that are needed for the existence of the solitons and other solutions.

2. Governing equation

The dynamics of solitons in cascaded system is governed by coupled NLSE which in dimensionless form is given by

$$ia_1 q_t + b_1 q_{xx} + c_1 |r|^2 q = 0 \tag{1}$$

$$ia_2 r_t + b_2 r_{xx} + (c_2 |q|^2 + d_2 |r|^2) r = 0 \tag{2}$$

In (1) and (2), $q(x,t)$ and $r(x,t)$ represent complex-valued wave profile. The independent variables are the x and the temporal variable t . Also, a_j , b_j and c_j for $j = 1, 2$ are the coefficients of the temporal evolution of the solitons, group velocity dispersion and the cross-phase modulation of the two components. Then, d_2 is the self-phase modulation of solitons.

3. Soliton solutions

In order to solve Eqs. (1) and (2), the following wave transformations are chosen [2]

$$q(x,t) = U_1(\xi) e^{i\Phi_1(x,t)} \tag{3}$$

$$r(x,t) = U_2(\xi) e^{i\Phi_2(x,t)} \tag{4}$$

where $U_l(\xi)$ represent the shape of the pulse and

$$\xi = B(x - vt) \tag{5}$$

$$\Phi_l(x,t) = -\kappa_l x + \omega_l t + \theta_l\,,\ l = 1, 2 \tag{6}$$

In Eqs. (3) and (4), the functions $\Phi_l(x,t)$ represent phase components of the soliton. From the phase, κ_l are the soliton frequency ω_l are the wave numbers and θ_l are the phase constants. Finally in Eq. (5), v is the velocity of the soliton. Substituting Eqs. (3) and (4) into Eqs. (1) and (2) and then decomposing into real and imaginary parts leads to a pair of relations. The imaginary parts give

$$v = -\frac{2b_1\kappa_1}{a_1} \tag{7}$$

and

$$v = -\frac{2b_2\kappa_2}{a_2} \tag{8}$$

Next, equating the two velocities with each other leads to a constraint relation between the soliton parameters as

$$a_2\kappa_1 b_1 = a_1\kappa_2 b_2 \tag{9}$$

which is a constraint condition for the solitons to exist. The real part equations are now written as

$$B^2 b_1 U_1'' - (a_1\omega_1 + b_1\kappa_1^2)U_1 + c_1 U_2^2 U_1 = 0 \tag{10}$$

$$\begin{aligned} &B^2 b_2 U_2'' - (a_2\omega_2 + b_2\kappa_2^2)U_2 \\ &+ c_2 U_1^2 U_2 + d_2 U_2^3 = 0 \end{aligned} \tag{11}$$

This pair of relations (10) and (11) will be now analyzed to retrieve soliton solutions in the following subsections.

3.1 Q-function method

By means of the Q function method, we can look for exact solutions of Eqs. (10) and (11) in the form of the following power series [8]

$$\begin{aligned} U_1(\xi) &= \sum_{l=0}^{M} A_l Q^l(\xi) \\ U_2(\xi) &= \sum_{l=0}^{M} B_l Q^l(\xi) \\ Q(\xi) &= \frac{1}{1 \pm e^{-\xi-\xi_0}} \end{aligned} \tag{12}$$

where M , N are positive integers, in most cases, that will be determined. Also, ξ_0 is an arbitrary constant. To determine the parameters M , N , we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms.

One can see that the function $Q(\xi)$ is solution of the equation

$$Q_\xi = Q - Q^2 \tag{13}$$

Equation (13) allows us to obtain U' and U'' using polynomials of Q . The balancing procedure yield $M = N = 1$. Thus, to search for solution of Eqs. (10) and (11) we can use following relations

$$U_1(\xi) = A_0 + A_1 Q(\xi) \tag{14}$$

$$U_2(\xi) = B_0 + B_1 Q(\xi) \tag{15}$$

Substituting (14) and (15) into Eqs. (10) and (11) and setting all the coefficients of powers $Q(\xi)$ to be zero, then we recover a system of nonlinear algebraic equations which, when solved, gives

$$\begin{aligned} A_0 &= \mp\sqrt{\frac{(b_1 d_2 - b_2 c_1)}{2c_1 c_2}}B \\ A_1 &= \pm\sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{c_1 c_2}}B \end{aligned} \tag{16}$$

$$\begin{aligned} B_0 &= \mp\sqrt{-\frac{b_1}{2c_1}}B \\ B_1 &= \pm\sqrt{-\frac{2b_1}{c_1}}B \end{aligned} \tag{17}$$

$$\begin{aligned} \omega_1 &= -\frac{b_1(B^2 + 2\kappa_1^2)}{2a_1} \\ \omega_2 &= -\frac{b_2(B^2 + 2\kappa_2^2)}{2a_2} \end{aligned} \tag{18}$$

where B, κ_1 and κ_2 are arbitrary constants. The width of the solitons given by (16) and (17) introduces the constraint

$$c_1c_2(b_2c_1 - b_1d_2) < 0\,,\ b_1c_1 < 0 \tag{19}$$

Finally, equating the two components of the soliton width B gives the ratio of the soliton amplitudes as

$$\frac{A_l}{B_l} = \sqrt{\frac{b_2c_1 - b_1d_2}{b_1c_2}}\,,\ l = 0, 1 \tag{20}$$

which naturally introduces the restriction

$$b_1c_2(b_2c_1 - b_1d_2) > 0 \tag{21}$$

Substituting (16)-(18) into Eqs. (14) and (15) and inserting the result into the transformations (3) and (4), we get the exact solutions of Eqs. (1) and (2) as follows:

Topological 1-soliton solutions:

$$\begin{aligned} q(x,t) &= \pm\sqrt{\frac{(b_1d_2 - b_2c_1)}{2c_1c_2}}B \\ &\times \tanh\left\{\frac{B}{2}\left(x + \frac{2b_1\kappa_1}{a_1}t\right)\right\} \\ &\times e^{i\left(-\kappa_1 x - \frac{b_1(B^2+2\kappa_1^2)}{2a_1}t + \theta_1\right)} \end{aligned} \tag{22}$$

$$\begin{aligned} r(x,t) &= \sqrt{-\frac{b_1}{2c_1}}B \\ &\times \tanh\left\{\frac{B}{2}\left(x + \frac{2b_2\kappa_2}{a_2}t\right)\right\} \\ &\times e^{i\left(-\kappa_2 x - \frac{b_2(B^2+2\kappa_2^2)}{2a_2}t + \theta_2\right)} \end{aligned} \tag{23}$$

Singular 1-soliton solutions:

$$\begin{aligned} q(x,t) &= \pm\sqrt{\frac{(b_1d_2 - b_2c_1)}{2c_1c_2}}B \\ &\times \coth\left\{\frac{B}{2}\left(x + \frac{2b_1\kappa_1}{a_1}t\right)\right\} \\ &\times e^{i\left(-\kappa_1 x - \frac{b_1(B^2+2\kappa_1^2)}{2a_1}t + \theta_1\right)} \end{aligned} \tag{24}$$

$$\begin{aligned} r(x,t) &= \sqrt{-\frac{b_1}{2c_1}}B \\ &\times \coth\left\{\frac{B}{2}\left(x + \frac{2b_2\kappa_2}{a_2}t\right)\right\} \\ &\times e^{i\left(-\kappa_2 x - \frac{b_2(B^2+2\kappa_2^2)}{2a_2}t + \theta_2\right)} \end{aligned} \tag{25}$$

3.2 Ricatti equation expansion approach

In this section, the Ricatti equation expansion approach will be shown in detail to obtain the singular solutions, singular and dark soliton solutions to Eqs. (1) and (2). According to the homogeneous balance method, Eqs. (10) and (11) has the solutions in the form

$$U_1(\xi) = A_0 + A_1\varphi(\xi) \tag{26}$$

$$U_2(\xi) = B_0 + B_1\varphi(\xi) \tag{27}$$

and $\varphi(\xi)$ satisfies the Riccati equation

$$\varphi'(\xi) = f + l\varphi^2(\xi) \tag{28}$$

where f and l are all non-zero real-valued constants that are independent on ξ. Eq. (28) is the well known Riccati equation, which admits the following explicit solutions:

$$\varphi(\xi) = \frac{\sqrt{fl}\ \tan(\sqrt{fl}\,\xi)}{l} \tag{29}$$

$$\varphi(\xi) = -\frac{\sqrt{fl}\ \cot(\sqrt{fl}\,\xi)}{l} \tag{30}$$

when $fl > 0$, and

$$\varphi(\xi) = -\frac{\sqrt{-fl}\ \tanh(\sqrt{-fl}\,\xi)}{l} \tag{31}$$

$$\varphi(\xi) = -\frac{\sqrt{-fl}\ \coth(\sqrt{-fl}\,\xi)}{l} \tag{32}$$

when $fl < 0$.

Substituting Eqs. (26)-(28) into Eqs. (10) and (11) leads to

$$
\begin{aligned}
&B^2 b_1 (2A_1 fl\varphi + 2A_1 l^2 \varphi^3) \\
&-(a_1\omega_1 + b_1\kappa_1^2)(A_0 + A_1\varphi) \\
&+ c_1 (B_0 + B_1\varphi)^2 (A_0 + A_1\varphi) = 0
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&B^2 b_2 (2B_1 fl\varphi + 2B_1 l^2 \varphi^3) \\
&-(a_2\omega_2 + b_2\kappa_2^2)(B_0 + B_1\varphi) \\
&+ c_2 (A_0 + A_1\varphi)^2 (B_0 + B_1\varphi) \\
&+ d_2 (B_0 + B_1\varphi)^3 = 0
\end{aligned}
\tag{34}
$$

Then, equating the coefficient of each power of $\varphi(\xi)$ to zero, we obtain a system of nonlinear algebraic equations which solve to

$$
A_0 = 0,\ A_1 = \pm\sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{c_1 c_2}} lB \tag{35}
$$

$$
B_0 = 0,\ B_1 = \pm\sqrt{-\frac{2b_1}{c_1}} lB \tag{36}
$$

$$
\begin{aligned}
\omega_1 &= -\frac{b_1(\kappa_1^2 - 2B^2 lf)}{a_1} \\
\omega_2 &= -\frac{b_2(\kappa_2^2 - 2B^2 lf)}{a_2}
\end{aligned}
\tag{37}
$$

where B, κ_1, κ_2, l and f are arbitrary constants.

The width of the solitons given by (35) and (36) introduces the constraint

$$
c_1 c_2 (b_2 c_1 - b_1 d_2) < 0,\ c_1 b_1 < 0 \tag{38}
$$

Finally, equating the two components of the soliton width B gives the ratio of the soliton amplitudes as

$$
\frac{A_1}{B_1} = \sqrt{\frac{b_2 c_1 - b_1 d_2}{b_1 c_2}} \tag{39}
$$

which naturally poses the restriction as given by (21).

Finally, using solutions (29)-(32) of Eq. (28), we obtain the the following exact solutions to Eqs. (1) and (2):

Singular periodic solutions:

$$
\begin{aligned}
q(x,t) &= \pm\sqrt{\frac{2fl(b_1 d_2 - b_2 c_1)}{c_1 c_2}} B \\
&\times \tan\left\{\sqrt{fl} B\left(x + \frac{2b_1\kappa_1}{a_1} t\right)\right\} \\
&\times e^{i\left(-\kappa_1 x - \frac{b_1(\kappa_1^2 - 2B^2 lf)}{a_1} t + \theta_1\right)}
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
r(x,t) &= \pm\sqrt{-\frac{2flb_1}{c_1}} B \\
&\times \tan\left\{\sqrt{fl} B\left(x + \frac{2b_2\kappa_2}{a_2} t\right)\right\} \\
&\times e^{i\left(-\kappa_2 x - \frac{b_2(\kappa_2^2 - 2B^2 lf)}{a_2} t + \theta_2\right)}
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
q(x,t) &= \mp\sqrt{\frac{2fl(b_1 d_2 - b_2 c_1)}{c_1 c_2}} B \\
&\times \cot\left\{\sqrt{fl} B\left(x + \frac{2b_1\kappa_1}{a_1} t\right)\right\} \\
&\times e^{i\left(-\kappa_1 x - \frac{b_1(\kappa_1^2 - 2B^2 lf)}{a_1} t + \theta_1\right)}
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
r(x,t) &= \mp\sqrt{-\frac{2flb_1}{c_1}} B \\
&\times \cot\left\{\sqrt{fl} B\left(x + \frac{2b_2\kappa_2}{a_2} t\right)\right\} \\
&\times e^{i\left(-\kappa_2 x - \frac{b_2(\kappa_2^2 - 2B^2 lf)}{a_2} t + \theta_2\right)}
\end{aligned}
\tag{43}
$$

Topological 1-soliton solutions:

$$
\begin{aligned}
q(x,t) &= \mp\sqrt{-\frac{2fl(b_1 d_2 - b_2 c_1)}{c_1 c_2}} B \\
&\times \tanh\left\{\sqrt{-fl} B\left(x + \frac{2b_1\kappa_1}{a_1} t\right)\right\} \\
&\times e^{i\left(-\kappa_1 x - \frac{b_1(\kappa_1^2 - 2B^2 lf)}{a_1} t + \theta_1\right)}
\end{aligned}
\tag{44}
$$

$$r(x,t) = \mp\sqrt{\frac{2flb_1}{c_1}}B \times \tanh\left\{\sqrt{-fl}B\left(x + \frac{2b_2\kappa_2}{a_2}t\right)\right\} \times e^{i\left(-\kappa_2 x - \frac{b_2(\kappa_2^2 - 2B^2 lf)}{a_2}t + \theta_2\right)} \quad (45)$$

Singular 1-soliton solutions:

$$q(x,t) = \mp\sqrt{-\frac{2fl(b_1d_2 - b_2c_1)}{c_1c_2}}B \times \coth\left\{\sqrt{-fl}B\left(x + \frac{2b_1\kappa_1}{a_1}t\right)\right\} \times e^{i\left(-\kappa_1 x - \frac{b_1(\kappa_1^2 - 2B^2 lf)}{a_1}t + \theta_1\right)} \quad (46)$$

$$r(x,t) = \mp\sqrt{\frac{2flb_1}{c_1}}B \times \coth\left\{\sqrt{-fl}B\left(x + \frac{2b_2\kappa_2}{a_2}t\right)\right\} \times e^{i\left(-\kappa_2 x - \frac{b_2(\kappa_2^2 - 2B^2 lf)}{a_2}t + \theta_2\right)} \quad (47)$$

3.3 G'/G -expansion approach

In this section, the G'/G -expansion method [5] will be shown in detail to obtain the singular solutions, singular and dark soliton solutions to Eqs. (1) and (2). According to the homogeneous balance method, Eqs. (10) and (11) has the solutions in the form

$$U_1(\xi) = A_0 + A_1\left(\frac{G'(\xi)}{G(\xi)}\right) \quad (48)$$

$$U_2(\xi) = B_0 + B_1\left(\frac{G'(\xi)}{G(\xi)}\right) \quad (49)$$

where $G(\xi)$ satisfies the second-order linear ordinary diffierential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \quad (50)$$

where λ and μ are real constants to be determined.

Substituting Eqs. (48)-(50) into Eqs. (10) and (11) leads to

$$B^2 b_1\left\{2A_1\left(\frac{G'}{G}\right)^3 + 3A_1\lambda\left(\frac{G'}{G}\right)^2 + (2A_1\mu + A_1\lambda^2)\left(\frac{G'}{G}\right) + \lambda\mu A_1)\right\} - (a_1\omega_1 + b_1\kappa_1^2)\left\{A_0 + A_1\left(\frac{G'}{G}\right)\right\} + c_1\left\{B_0 + B_1\left(\frac{G'}{G}\right)\right\}^2\left\{A_0 + A_1\left(\frac{G'}{G}\right)\right\} = 0 \quad (51)$$

$$B^2 b_2\left\{2B_1\left(\frac{G'}{G}\right)^3 + 3B_1\lambda\left(\frac{G'}{G}\right)^2 + (2B_1\mu + B_1\lambda^2)\left(\frac{G'}{G}\right) + \lambda\mu B_1)\right\} - (a_2\omega_2 + b_2\kappa_2^2)\left\{B_0 + B_1\left(\frac{G'}{G}\right)\right\} + c_2\left\{A_0 + A_1\left(\frac{G'}{G}\right)\right\}^2\left\{B_0 + B_1\left(\frac{G'}{G}\right)\right\} + d_2\left\{B_0 + B_1\left(\frac{G'}{G}\right)\right\}^3 = 0 \quad (52)$$

Then, equating the coefficient of each power of G'/G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$A_0 = \pm\sqrt{\frac{(b_1d_2 - b_2c_1)}{2c_1c_2}}\lambda B, \quad A_1 = \pm\sqrt{\frac{2(b_1d_2 - b_2c_1)}{c_1c_2}}B \quad (53)$$

$$B_0 = \pm\sqrt{-\frac{b_1}{2c_1}}\lambda B, \quad B_1 = \pm\sqrt{-\frac{2b_1}{c_1}}B \quad (54)$$

$$\omega_1 = -\frac{b_1(2\kappa_1^2 + B^2(\lambda^2 - 4\mu))}{2a_1}, \quad \omega_2 = -\frac{b_2(2\kappa_2^2 + B^2(\lambda^2 - 4\mu))}{2a_2} \quad (55)$$

where B, κ_1, κ_2, λ, μ are arbitrary constants.

The width of the solitons given by (53) and (54) introduces the constraint relations

$$c_1c_2(b_2c_1-b_1d_2)<0\,,\ b_1c_1<0 \tag{56}$$

Finally, equating the two components of the soliton width B gives the ratio of the soliton amplitudes as

$$\frac{A_l}{B_l}=\sqrt{\frac{b_2c_1-b_1d_2}{b_1c_2}}\,,\ l=0,1 \tag{57}$$

which naturally poses the restriction given by (21).

Substituting the solution set (53)-(55) into Eqs. (48) and (49), the solution formulae of Eqs. (10) and (11) can be written as

$$U_1(\xi)=\pm\sqrt{\frac{2(b_1d_2-b_2c_1)}{c_1c_2}}B\left\{\frac{\lambda}{2}+\frac{G'(\xi)}{G(\xi)}\right\} \tag{58}$$

$$U_2(\xi)=\pm\sqrt{-\frac{2b_1}{c_1}}B\left\{\frac{\lambda}{2}+\frac{G'(\xi)}{G(\xi)}\right\} \tag{59}$$

Substituting the general solutions of second order linear ODE into Eqs. (58) and (59) gives three types of traveling wave solutions.

Case-I: When $\Delta=\lambda^2-4\mu>0$, we obtain the hyperbolic function traveling wave solution

$$q(x,t)=\pm\sqrt{\frac{(\lambda^2-4\mu)(b_1d_2-b_2c_1)}{2c_1c_2}}B \times\left\{\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)}\right\} \times e^{i\left(-\kappa_1x-\frac{b_1\left(2\kappa_1^2+B^2\left(\lambda^2-4\mu\right)\right)}{2a_1}t+\theta_1\right)} \tag{60}$$

$$r(x,t)=\pm\sqrt{-\frac{(\lambda^2-4\mu)b_1}{2c_1}}B \times\left\{\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)}\right\} \times e^{i\left(-\kappa_2x-\frac{b_2\left(2\kappa_2^2+B^2\left(\lambda^2-4\mu\right)\right)}{2a_2}t+\theta_2\right)} \tag{61}$$

where C_1 and C_2 are arbitrary constants.

On the other hand, assuming $C_1\neq 0$ and $C_2=0$, the topological 1-soliton solutions of Eqs. (1) and (2) can be written as:

$$q(x,t)=\pm\sqrt{\frac{(\lambda^2-4\mu)(b_1d_2-b_2c_1)}{2c_1c_2}}B \times\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right) \times e^{i\left(-\kappa_1x-\frac{b_1\left(2\kappa_1^2+B^2\left(\lambda^2-4\mu\right)\right)}{2a_1}t+\theta_1\right)} \tag{62}$$

$$r(x,t)=\pm\sqrt{-\frac{(\lambda^2-4\mu)b_1}{2c_1}}B \times\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right) \times e^{i\left(-\kappa_2x-\frac{b_2\left(2\kappa_2^2+B^2\left(\lambda^2-4\mu\right)\right)}{2a_2}t+\theta_2\right)} \tag{63}$$

Next, assuming $C_1=0$ and $C_2\neq 0$, then we obtain singular 1-soliton solution for cascaded system (1) and (2) as

$$q(x,t)=\pm\sqrt{\frac{(\lambda^2-4\mu)(b_1d_2-b_2c_1)}{2c_1c_2}}B\times\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)\times e^{i\left(-\kappa_1x-\frac{b_1(2\kappa_1^2+B^2(\lambda^2-4\mu))}{2a_1}t+\theta_1\right)} \tag{64}$$

$$r(x,t)=\pm\sqrt{-\frac{(\lambda^2-4\mu)b_1}{2c_1}}B\times\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)\times e^{i\left(-\kappa_2x-\frac{b_2(2\kappa_2^2+B^2(\lambda^2-4\mu))}{2a_2}t+\theta_2\right)} \tag{65}$$

Case-II: When $\Delta=\lambda^2-4\mu<0$, we obtain the hyperbolic function traveling wave solution

$$q(x,t)=\pm\sqrt{\frac{(4\mu-\lambda^2)(b_1d_2-b_2c_1)}{2c_1c_2}}B\times\left\{\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)}\right\}\times e^{i\left(-\kappa_1x-\frac{b_1(2\kappa_1^2+B^2(\lambda^2-4\mu))}{2a_1}t+\theta_1\right)} \tag{66}$$

$$r(x,t)=\pm\sqrt{-\frac{(4\mu-\lambda^2)b_1}{2c_1}}B\times\left\{\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)}\right\}\times e^{i\left(-\kappa_2x-\frac{b_2(2\kappa_2^2+B^2(\lambda^2-4\mu))}{2a_2}t+\theta_2\right)} \tag{67}$$

where C_1 and C_2 are arbitrary constants.

Also, with the assumption $C_1\neq 0$ and $C_2=0$,

$$q(x,t)=\pm\sqrt{\frac{(4\mu-\lambda^2)(b_1d_2-b_2c_1)}{2c_1c_2}}B\times\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right)\times e^{i\left(-\kappa_1x-\frac{b_1(2\kappa_1^2+B^2(\lambda^2-4\mu))}{2a_1}t+\theta_1\right)} \tag{68}$$

$$r(x,t)=\pm\sqrt{-\frac{(4\mu-\lambda^2)b_1}{2c_1}}B\times\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right)\times e^{i\left(-\kappa_2x-\frac{b_2(2\kappa_2^2+B^2(\lambda^2-4\mu))}{2a_2}t+\theta_2\right)} \tag{69}$$

and when $C_1=0$, $C_2\neq 0$ the singular periodic solutions of Eqs. (1) and (2) will be

$$q(x,t)=\pm\sqrt{\frac{(4\mu-\lambda^2)(b_1d_2-b_2c_1)}{2c_1c_2}}B$$
$$\times\cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_1\kappa_1}{a_1}t\right)\right) \quad (70)$$
$$\times e^{i\left(-\kappa_1 x-\frac{b_1(2\kappa_1^2+B^2(\lambda^2-4\mu))}{2a_1}t+\theta_1\right)}$$

$$r(x,t)=\pm\sqrt{-\frac{(4\mu-\lambda^2)b_1}{2c_1}}B$$
$$\times\cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}B\left(x+\frac{2b_2\kappa_2}{a_2}t\right)\right) \quad (71)$$
$$\times e^{i\left(-\kappa_2 x-\frac{b_2(2\kappa_2^2+B^2(\lambda^2-4\mu))}{2a_2}t+\theta_2\right)}$$

Case-III: When $\Delta=\lambda^2-4\mu=0$, we obtain plane wave solutions

$$q(x,t)=\pm\sqrt{\frac{2(b_1d_2-b_2c_1)}{c_1c_2}}$$
$$\times\frac{BC_2}{C_1+C_2\left(x+\frac{2b_1\kappa_1}{a_1}t\right)} \quad (72)$$
$$\times e^{i\left(-\kappa_1 x-\frac{b_1\kappa_1^2}{a_1}t+\theta_1\right)}$$

$$r(x,t)=\pm\sqrt{-\frac{2b_1}{c_1}}$$
$$\times\frac{BC_2}{C_1+C_2\left(x+\frac{2b_2\kappa_2}{a_2}t\right)} \quad (73)$$
$$\times e^{i\left(-\kappa_2 x-\frac{b_2\kappa_2^2}{a_2}t+\theta_2\right)}$$

where C_1 and C_2 are arbitrary constants.

4. Conclusion

This paper studied optical soliton solutions by the aid of three forms of integration tools. These are Q-function approach, G'/G-expansion scheme and Riccati equation method. These algorithms lead to topological and singular soliton solutions to the governing coupled NLSE for cascaded system. It is interesting to observe that none of these integration techniques retrieved bright or dark soliton solutions. Instead, however, singular and topological soliton solutions are recovered. This shows the limitations of each of these three methods since bright solitons are the most important type of solitons that are handled on a daily basis in optical communication world. In future, the target will be to recover bright soliton solutions by resorting to additional integration schemes besides the ansatz approach [2, 7, 12]. The results of that research will be published later.

Acknowledgments

The work was supported by the Qatar National Research Fund (QNRF) under grant No. NPRP 6-021-1-005.

References

[1] M. Bache, F. W. Wise, Phys. Rev. A **81**, 053815 (2010).
[2] A. H. Bhrawy, A. A. Alshaery, E. M. Hilal, Z. Jovanoski, A. Biswas, Optik **125**, 6162 (2014).
[3] A. Biswas, K. Khan, A. Rahman, A. Yildirim, T. Hayat, O. M. Aldossary, J. Optoelectron. Adv. Mat. **14**, 571 (2012).
[4] A. Biswas, A.H. Bhrawy, A.A. Alshaeryd, E.M. Hilal, Optik **125**, 6162 (2014).
[5] G. Ebadi, A. Biswas, Commun. Nonlinear Sci. Numer. Simul. **16**, 2377 (2011).
[6] L. Girgis, D. Milovic, S. Konar, A. Yildirim, H. Jafari, A. Biswas, Rom. Rep. Phys. **64**, 663 (2012).
[7] J-V. Guzman, M. F. Mahmood, A. A. Alshaery, E. M. Hilal, A. H. Bhrawy, A. Biswas, Submitted.
[8] N. A. Kudryashov, A. S. Zakharchenko, Appl. Math. Lett. **32**, 53 (2014).
[9] H. Leblond, H. Triki, D. Mihalache, Rom. Rep. Phys. **65**, 925 (2013).
[10] D. Mihalache, Rom. J. Phys. **59**, 295 (2014).
[11] A. F. Mohammed, C. H. Telk, S. P. Majumdar. J. Opt. Commun. **21**, 165 (2000).
[12] A. K. Sarma, A. Biswas, Opt. Appl. **41**, 205 (2014).
[13] M. Savescu, A. H. Bhrawy, E. M. Hilal, A. A. Alshaery, A. Biswas, Rom. J. Phys. **59**, 582 (2014).
[14] Q. Zhou, Optik **125**, 3142 (2014).
[15] Q. Zhou, Q. Zhu, A. H. Bhrawy, L. Moraru, A. Biswas. Optoelectron. Adv. Mat. **8**, 800 (2014).

*Corresponding author: biswas.anjan@gmail.com