Optical solitons in DWDM system by G'/G-expansion scheme

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This paper obtains the soliton solutions to DWDM systems with Kerr and parabolic laws of nonlinearity. The governing equation is integrated by the aid of G'/G-expansion method. Dark and singular soliton solutions are obtained by the application of this method. As a byproduct, singular periodic solutions are also available through this integration scheme.

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1. Introduction

There is an ever-increasing interest in the study of soliton propagation through optical fibers. There are lots of results reported in various published papers, books and monographs. However, most of these results are in one-dimensional cases. While in realistic situation, it is necessary to propagate solitons in bulk, it is therefore necessary to address the issue of mass propagation of these pulses through optical fibers. It is only then efficiency can be achieved. This is only possible with dense wavelength multiplexed system (DWDM). This paper is going to study the dynamics of optical soliton propagation through DWDM system for Kerr and parabolic law nonlinearity.

The integrability aspect of the governing equation will be the focus of research in this paper. It must be noted that there are several integration tools that are applicable to study these governing equations [1-15]. These are Lie symmetry analysis, F-expansion scheme, ansatz approach, Kudryashov's method and several others. This paper will be devoted to one such modern method of integrability. It is G'/G-expansion scheme. This algorithm will integrate the governing equation and will recover dark and singular solitons for the model with Kerr and parabolic laws of nonlinearity. There are a couple of other solutions that will be obtained which are of no interest in the optical fibers regime.

2. Overview of G'/G-expansion scheme

In this section, we describe the G'/G-expansion method [9-13] for finding traveling wave solutions of nonlinear partial differential equations (NLPDE).

We assume that the given NLPDE for $u(x,t)$ is in the form

$$P(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt} ...) = 0 \tag{1}$$

where P is a polynomial. The essence of G'/G-expansion method can be presented in the following steps:

Step-1: To find the traveling wave solutions of Eq. (1), we introduce the wave variable

$$u(x,t) = U(\xi),\ \xi = x - ct \tag{2}$$

Substituting Eq. (2) into Eq. (1), we obtain the following ODE

$$Q(U, U', U'', ...) = 0 \tag{3}$$

Step-2: Eq. (3) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

Step-3: Introduce the solution $U(\xi)$ of Eq. (3) in the finite series form

$$U(\xi) = \sum_{j=0}^{M} S_j \left(\frac{G'(\xi)}{G(\xi)} \right)^j \tag{4}$$

where s_i are real constants with $s_M \neq 0$ and M is a positive integer to be determined. The function $G(\xi)$ is the solution of the auxiliary linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \tag{5}$$

where λ and μ are real constants to be determined.

Step-4: Determining M, can be accomplished by balancing the linear term of highest order derivatives with the highest order nonlinear term in Eq. (3).

Step-5: Substituting the general solution of (5) together with (4) into Eq. (3) yields an algebraic equation involving powers of G'/G. Equating the coefficients of each power of G'/G to zero gives a system of algebraic equations for s_j, λ, μ and c. Then, we solve the system with the aid of a computer algebra system, such as Maple, to determine these constants. Next, depending on the sign of the discriminant $\Delta = \lambda^2 - 4\mu$, we get solutions of Eq. (3). So, we can obtain exact solutions of the given Eq. (1).

2.1 Application to NLSE

This section will apply the integration scheme to integrate the model equation for DWDM systems. There are two forms of nonlinearity that will be considered. These are Kerr law and parabolic laws of nonlinear media. The study will therefore be split into the next two subsections based on the type of nonlinear medium.

2.1.1 Kerr law nonlinearity

This type of nonlinearity is alternatively referred as cubic nonlinearity. It originates when a light wave in an optical fiber is subjected to nonlinear responses. The dynamics of soliton propagation through Kerr law nonlinear fibers in DWDM system is given by [1-8]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + \left\{ b_l \left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N} \alpha_{l\,n} \left|q^{(n)}\right|^2 \right\} q^{(l)} = 0 \tag{6}$$

Here, $1 \le l \le N$. The first term in (6) on left hand side is the evolution term, while a_l represents the coefficient of group velocity dispersion (GVD). Then, b_l is the coefficient of self-phase modulation (SPM) while $\alpha_{l\,n}$ are the coefficients of cross-phase modulation (XPM). The independent variables are x and t that represents the spatial and temporal variables respectively. The dependent variable is $q^{(l)}(x,t)$ that gives the soliton profile in every single channel. In (6), the first term is the temporal evolution, while a is the coefficient of group-velocity dispersion (GVD). Finally b and k are the coefficients of the nonlinear terms. We may choose the following traveling wave transformations

$$q^{(l)}(x,t) = U^{(l)}(\xi) e^{i\left(-\kappa^{(l)}x + \omega^{(l)}t + \theta^{(l)}\right)} \tag{7-1}$$

$$\xi = x - v^{(l)}t,\ 1 \le l \le N \tag{7-2}$$

where $\kappa^{(l)}$, $\omega^{(l)}$, $\theta^{(l)}$ and $v^{(l)}$ respectively represent the frequencies, wave numbers, phase constants and the speed of the waves.

Thus, from Eq. (7), we have

$$q_t^{(l)} = \left(-v^{(l)}\left(U^{(l)}\right)' + i\omega^{(l)} U^{(l)} \right) \times e^{i\left(-\kappa^{(l)}x + \omega^{(l)}t + \theta^{(l)}\right)} \tag{8}$$

$$q_{xx}^{(l)} = \left(\left(U^{(l)}\right)'' - 2i\kappa^{(l)}\left(U^{(l)}\right)' - \left(\kappa^{(l)}\right)^2 U^{(l)} \right) \times e^{i\left(-\kappa^{(l)}x + \omega^{(l)}t + \theta^{(l)}\right)} \tag{9}$$

Substituting Eqs. (7)-(9) into Eq. (6) and then decomposing into real and imaginary parts respectively yields

$$v^{(l)} = -2\kappa^{(l)} a_l \tag{10}$$

and

$$a_l \left(U^{(l)}\right)'' - \left(\omega^{(l)} + \left(\kappa^{(l)}\right)^2 a_l\right) U^{(l)} + b_l \left(U^{(l)}\right)^3 + \left\{ \sum_{n\neq l}^{N} \alpha_{l\,n} \left(U^{(n)}\right)^2 \right\} U^{(l)} = 0 \tag{11}$$

where $1 \le l \le N$.

Balancing $\left(U^{(l)}\right)''$ with $\left(U^{(n)}\right)^2 U^{(l)}$ in Eq. (11) give $M^{(l)} = 1$. Therefore, the solution of Eq. (11) can be written in the form

$$U^{(l)}(\xi) = s_0^{(l)} + s_1^{(l)} \left(\frac{G'(\xi)}{G(\xi)} \right),\ s_1^{(l)} \ne 0 \tag{12}$$

where $G(\xi)$ satisfies the second-order linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \tag{13}$$

where λ and μ are real constants to be determined.

Substituting Eq. (12) in Eq. (11) and equating all the coefficients of powers of G'/G to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we have

$$s_0^{(l)} = \frac{\lambda}{2} s_1^{(l)} \tag{14-1}$$

$$a_l = -\frac{1}{2}\left(\sum_{n\neq l}^{N} \alpha_{l\,n} \left(s_1^{(n)}\right)^2 + b_l \left(s_1^{(l)}\right)^2\right) \tag{14-2}$$

$$\omega_l = \left(\frac{\lambda^2 - 4\mu}{4} + \frac{1}{2}\left(\kappa^{(l)}\right)^2\right) \times \left(\sum_{n\neq l}^{N} \alpha_{l\,n} \left(s_1^{(n)}\right)^2 + b_l \left(s_1^{(l)}\right)^2\right) \tag{14-3}$$

where λ, $\kappa^{(l)}$, $s_1^{(l)}$, μ are arbitrary constants.

Substituting the solution set (14) into Eq. (12), the solution formulas of Eq. (11) can be written as

$$U^{(l)}(\xi) = s_1^{(l)} \left\{\frac{\lambda}{2} + \frac{G'(\xi)}{G(\xi)}\right\},\ 1 \le l \le N \tag{15}$$

Substituting the general solutions of second order linear ODE into Eq. (17) gives three types of traveling wave solutions.

Case-I: When $\Delta = \lambda^2 - 4\mu > 0$, we obtain the hyperbolic function traveling wave solution

$$q^{(l)}(x,t) = \frac{s_1^{(l)}\sqrt{\lambda^2 - 4\mu}}{2} \left\{\frac{C_1 \sinh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right] + C_2 \cosh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right]}{C_1 \cosh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right] + C_2 \sinh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right]}\right\} e^{i\left(-\kappa^{(l)} x + \left\{\left(\frac{\lambda^2-4\mu}{4} + \frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N} \alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\} t + \theta^{(l)}\right)} \tag{16}$$

where C_1 and C_2 are arbitrary constants and a_l are given by

$$\omega_l = -\frac{1}{2}\left(\sum_{n\neq l}^{N} \alpha_{l\,n} \left(s_1^{(n)}\right)^2 + b_l \left(s_1^{(l)}\right)^2\right) \tag{17}$$

On the other hand, assuming $C_1 \neq 0$ and $C_2 = 0$, the topological 1-soliton solution of the Eq. (6) can be written as.

$$q^{(l)}(x,t) = \frac{s_1^{(l)}\sqrt{\lambda^2 - 4\mu}}{2} \times \tanh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \left\{\kappa^{(l)}\left(\sum_{n\neq l}^{N} \alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\} t\right)\right] \times e^{i\left(-\kappa^{(l)} x + \left\{\left(\frac{\lambda^2-4\mu}{4} + \frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N} \alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\} t + \theta^{(l)}\right)} \tag{18}$$

Next, assuming $C_1 = 0$ and $C_2 \neq 0$, then we obtain the Eq. (6)

$$q^{(l)}(x,t) = \frac{s_1^{(l)}\sqrt{\lambda^2 - 4\mu}}{2} \times \coth\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x - \left\{\kappa^{(l)}\left(\sum_{n\neq l}^{N} \alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\} t\right)\right] \times e^{i\left(-\kappa^{(l)} x + \left\{\left(\frac{\lambda^2-4\mu}{4} + \frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N} \alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\} t + \theta^{(l)}\right)} \tag{19}$$

Equations (18) and (19) are dark and singular solitons respectively.

Case-II: When $\Delta = \lambda^2 - 4\mu < 0$, we obtain the hyperbolic function traveling wave solution

$$q^{(l)}(x,t) = \frac{s_1^{(l)}\sqrt{4\mu - \lambda^2}}{2} \left\{\frac{-C_1 \sin\left[\frac{\sqrt{4\mu-\lambda^2}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right] + C_2 \cos\left[\frac{\sqrt{4\mu-\lambda^2}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right]}{C_1 \cos\left[\frac{\sqrt{4\mu-\lambda^2}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right] + C_2 \sin\left[\frac{\sqrt{4\mu-\lambda^2}}{2}(x + \{2\kappa^{(l)} a_l\} t)\right]}\right\} e^{i\left(-\kappa^{(l)} x + \left\{\left(\frac{\lambda^2-4\mu}{4} + \frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N} \alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\} t + \theta^{(l)}\right)} \tag{20}$$

where C_1 and C_2 are arbitrary constants and a_l are given by

$$\omega_l = -\frac{1}{2}\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right) \quad (21)$$

Also, with the assumption $C_1 \neq 0$ and $C_2 = 0$,

$$q^{(l)}(x,t) = -\frac{s_1^{(l)}\sqrt{4\mu-\lambda^2}}{2} \times \tan\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\left(x - \left\{\kappa^{(l)}\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\}t\right)\right] \times e^{i\left(-\kappa^{(l)}x + \left\{\left(\frac{\lambda^2-4\mu}{4}+\frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\}t + \theta^{(l)}\right)} \quad (22)$$

and when $C_1 = 0$, $C_2 \neq 0$, the solution of Eq. (6) will be

$$q^{(l)}(x,t) = -\frac{s_1^{(l)}\sqrt{4\mu-\lambda^2}}{2} \times \cot\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\left(x - \left\{\kappa^{(l)}\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\}t\right)\right] \times e^{i\left(-\kappa^{(l)}x + \left\{\left(\frac{\lambda^2-4\mu}{4}+\frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\}t + \theta^{(l)}\right)} \quad (23)$$

Equations (22) and (23) are singular periodic solutions that are not studied in nonlinear fiber optics.

Case III: When $\Delta = \lambda^2 - 4\mu = 0$, we obtain the rational function solution

$$q^{(l)}(x,t) = \frac{C_2 s_1^{(l)}}{C_1 + C_2\left(x - \left\{\kappa^{(l)}\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\}t\right)} \times e^{i\left(-\kappa^{(l)}x + \left\{\left(\frac{\lambda^2-4\mu}{4}+\frac{1}{2}(\kappa^{(l)})^2\right)\left(\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s_1^{(n)}\right)^2 + b_l\left(s_1^{(l)}\right)^2\right)\right\}t + \theta^{(l)}\right)} \quad (24)$$

This solution is sometimes refereed to as plane wave solution.

2.1.2 Parabolic law nonlinearity

The parabolic law nonlinearity appears when considerable $\chi^{(5)}$ nonlinearity is experienced, and is referred to as the fifth order susceptibility [3, 14, 15]. It is predominantly present in a transparent glass with intense femtosecond pulses at 620 nm. In this case NLSE is

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + \left[b_l\left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N}\alpha_{l\,n}\left|q^{(n)}\right|^2\right]q^{(l)} + \left[c_l\left|q^{(l)}\right|^4 + \sum_{n\neq l}^{N}\left\{\left|q^{(n)}\right|^2\left(\beta_{l\,n}\left|q^{(n)}\right|^2 + \gamma_{l\,n}\left|q^{(l)}\right|^2\right)\right\}\right]q^{(l)} = 0 \quad (25)$$

In (25), SPM terms are the coefficients of b_l and c_l, while XPM coefficients are $\alpha_{l\,n}$, $\beta_{l\,n}$ and $\gamma_{l\,n}$, while the remaining parameters have the same definition as in Kerr law nonlinear medium. Substituting Eqs. (7)-(9) into Eq. (25) and then decomposing into real and imaginary parts respectively, yields

$$v^{(l)} = -2\kappa^{(l)}a_l \quad (26)$$

and

$$a_l\left(U^{(l)}\right)'' - \left(\omega^{(l)} + \left(\kappa^{(l)}\right)^2 a_l\right)U^{(l)} + b_l\left(U^{(l)}\right)^3 + \left\{\sum_{n\neq l}^{N}\alpha_{l\,n}\left(U^{(n)}\right)^2\right\}U^{(l)} + c_l\left(U^{(l)}\right)^5 + \left\{\sum_{n\neq l}^{N}\beta_{l\,n}\left(U^{(n)}\right)^4\right\}U^{(l)} + \left\{\sum_{n\neq l}^{N}\gamma_{l\,n}\left(U^{(n)}\right)^2\right\}\left(U^{(l)}\right)^3 = 0 \quad (27)$$

Balancing $\left(U^{(l)}\right)''$ with $\left(U^{(n)}\right)^2\left(U^{(l)}\right)^3$ in Eq. (27) give $M^{(l)} = \frac{1}{2}$. Therefore, the solution of Eq. (27) can be written in the form [13]

$$U^{(l)}(\xi) = s^{(l)}\left(\frac{G'(\xi)}{G(\xi)}\right)^{\frac{1}{2}},\ s^{(l)} \neq 0 \quad (28)$$

where $G(\xi)$ satisfies the second-order linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \quad (29)$$

where λ and μ are real constants to be determined.

Therefore, we have

$$\left(U'(\xi)\right)' = -\frac{1}{2}s^{(l)}\left(\frac{G'}{G}\right)^{\frac{3}{2}} - \frac{1}{2}s^{(l)}\lambda\left(\frac{G'}{G}\right)^{\frac{1}{2}} - \frac{1}{2}s^{(l)}\mu\left(\frac{G'}{G}\right)^{-\frac{1}{2}} \tag{30}$$

$$\left(U'(\xi)\right)'' = \frac{3}{4}s^{(l)}\left(\frac{G'}{G}\right)^{\frac{5}{2}} + s^{(l)}\lambda\left(\frac{G'}{G}\right)^{\frac{3}{2}} + \left(\frac{1}{2}s^{(l)}\mu + \frac{1}{4}s^{(l)}\lambda^2\right)\left(\frac{G'}{G}\right)^{\frac{1}{2}} - \frac{1}{4}s^{(l)}\mu^2\left(\frac{G'}{G}\right)^{-\frac{3}{2}} \tag{31}$$

Substituting (28)-(31) into (27) and equating all the coefficients of powers of G’/G to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we have

$$b_l = \frac{\left(\begin{array}{l} 4\lambda c_l\left(s^{(l)}\right)^4 + 4\lambda\sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + 4\lambda\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\left(s^{(l)}\right)^2 \\ - 3\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s^{(n)}\right)^4 \end{array}\right)}{3\left(s^{(l)}\right)^2} \tag{32-1}$$

$$\omega_l = \left(\frac{4\left(\kappa^{(l)}\right)^2 - \lambda^2}{3}\right) \times \left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right) \tag{32-2}$$

$$a_l = -\frac{4}{3}\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right) \tag{32-3}$$

$$\mu = 0 \tag{32-4}$$

where λ, $\kappa^{(l)}$, $s^{(l)}$ are arbitrary constants.

From Eqs. (7), (26), (28), (29) and Eq. (32), we obtain the exact traveling wave solution of the Eq. (25) as follows:

$$q^{(l)}(x,t) = \left[-\lambda\left(s^{(l)}\right)^2 \frac{A_2 e^{-\lambda\left(x - \left\{\frac{8\kappa^{(l)}}{3}\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t\right)}}{A_1 + A_2 e^{-\lambda\left(x - \left\{\frac{8\kappa^{(l)}}{3}\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t\right)}}\right]^{\frac{1}{2}} \times e^{i\left(-\kappa^{(l)}x + \left\{\left(\frac{4\kappa^{(l)} - \lambda^2}{3}\right)\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t + \theta^{(l)}\right)} \tag{33}$$

where A_1 and A_2 are arbitrary constants.

Eq. (33) is a new type of exact traveling wave solution to the Eq. (25). Especially, if we choose $A_1 = A_2$ in Eq. (33), we obtain the solitary wave solution of the Eq. (25), namely

$$q^{(l)}(x,t) = s^{(l)}\sqrt{\frac{-\lambda}{2}} \times \left\{1 - \tanh\left[\frac{\lambda}{2}\left(x - \left\{\frac{8\kappa^{(l)}}{3}\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t\right)\right]\right\}^{\frac{1}{2}} \times e^{i\left(-\kappa^{(l)}x + \left\{\left(\frac{4\kappa^{(l)} - \lambda^2}{3}\right)\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t + \theta^{(l)}\right)} \tag{34}$$

and

$$q^{(l)}(x,t) = s^{(l)}\sqrt{\frac{-\lambda}{2}} \times \left\{1 - \coth\left[\frac{\lambda}{2}\left(x - \left\{\frac{8\kappa^{(l)}}{3}\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t\right)\right]\right\}^{\frac{1}{2}} \times e^{i\left(-\kappa^{(l)}x + \left\{\left(\frac{4\kappa^{(l)} - \lambda^2}{3}\right)\left(\begin{array}{l} c_l\left(s^{(l)}\right)^4 + \sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + \left(\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\right)\left(s^{(l)}\right)^2 \end{array}\right)\right\}t + \theta^{(l)}\right)} \tag{35}$$

Remark: The exact solutions (33)-(35) are valid only if

$$b_l = \frac{\left(\begin{array}{l} 4\lambda c_l\left(s^{(l)}\right)^4 + 4\lambda\sum_{n\neq l}^{N}\beta_{l\,n}\left(s^{(n)}\right)^4 \\ + 4\lambda\sum_{n\neq l}^{N}\gamma_{l\,n}\left(s^{(n)}\right)^2\left(s^{(l)}\right)^2 \\ - 3\sum_{n\neq l}^{N}\alpha_{l\,n}\left(s^{(n)}\right)^4 \end{array}\right)}{3\left(s^{(l)}\right)^2}$$

$$\lambda < 0$$

3. Conclusion

This paper obtained singular and dark optical solitons in DWDM system. The G'/G-expansion scheme was applied for Kerr and parabolic laws of nonlinearity. Dark and singular optical soliton solutions were obtained. Additionally, singular periodic solutions were also listed, which are of no interest in the optical fibers regime. This scheme is unable to obtain bright soliton solution since this is an inherent drawback of this algorithm.

The results of this paper carry good hope for the future. Later, several other integration schemes will be applied where other types of soliton solutions could be available. Those results are awaited at this time and will be reported in future.

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