Optical solitons in DWDM system with modified simple equation method

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This paper studies optical solitons in DWDM systems. There are two types of nonlinear media considered. They are Kerr law and parabolic law. The integration algorithm applied is the modified simple equation sheme. The constraint solutions hold the solutions in place.

(Received February 17, 2017; accepted February 12, 2018)

Keywords: Solitons, DWDM system; Modified simple equation method

1. Introduction

While optical solitons have made several advances in the field of nonlinear fiber optics, there is yet a lot to be explored [1-15]. Most of the results are visible for polarization preserving fibers. Therefore, it is important to focus attention to birefringent fibers and DWDM systems. This paper studies the latter for parallel propagation of solitons. There are several integration schemes proposed during the past few years to study the governing nonlinear Schrödinger's equation (NLSE) for optical fibers and DWDM systems. Two types of nonlinear media that are studied in this paper. They are Kerr (cubic) law and parabolic (cubic-quintic) law. This model was considered in the past using three integration schemes [4]. This paper focuses on modified simple equation method to retrieve soliton solutions to the model. After a quick revisitation to this algorithm, soliton extraction procedure procedure will be detailed for this model.

2. The modified simple equation method

Suppose we have a nonlinear evolution equation in the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \quad (1)$$

where P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [1, 2, 3].

Step-1: We use the transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct, \quad (2)$$

where c is a constant to be determined, to reduce Eq. (1) to the following ODE :

$$Q(u, u', u'', \ldots) = 0, \quad (3)$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}$.

Step-2: Assume Eq. (3) has the formal solution.

$$u(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi\prime(\xi)}{\psi(\xi)}\right)^l, \quad (4)$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3).

Step-4: We substitute (4) into (3), then we calculate all the necessary derivatives $u', u'', \ldots$ of the unknown function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi), j = 0,1,2, \ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi(\xi)$. Consequently, we can get the exact solutions of Eq. (1) .

3. Application to DWDM system

The soliton solution retrieval procedure will now be split into the following two subsections based on the type of nonlinearity.

3.1 Kerr law nonlinearity

For Kerr law nonlinearity, DWDM model is:

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left\{ c_l \left| q^{(l)} \right|^2 + \sum_{n\neq l}^{N} \alpha_{ln} \left| q^{(n)} \right|^2 \right\} q^{(l)} = 0, \tag{5}$$

where, $1 \leq l \leq N$. The first term in (5) on left-hand side is the evolution term, while a_l represents the coefficient of GVD; b_l represents the STD. Then, c_l is the coefficient of self-phase modulation (SPM) while α_{ln} are the coefficients of cross-phase modulation (XPM). The independent variables are x and t that represents the spatial and temporal variables respectively. The dependent variable is $q^{(l)}(x,t)$ that gives the soliton profile in every single channel.

In order to solve Eq. (5), we use the following wave transformation

$$q^{(l)}(x,t) = U^{(l)}(\xi) e^{i\Phi(x,t)} \tag{6}$$

where $U^{(l)}(\xi)$ represents the shape of the pulse in every channel and

$$\xi = k(x - vt), \tag{7}$$

$$\Phi(x,t) = -\kappa_l x + \omega_l t + \theta_l. \tag{8}$$

In Eq. (6), the function $\Phi(x,t)$ is the phase component of the soliton. Then, in Eq. (8), $\kappa_l, \omega_l, \theta$ and v are the frequencies, wave numbers, phase constants and the velocity of the soliton in every single channel. Substituting Eq. (6) into Eq. (5) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v = \frac{b_l \omega_l - 2a_l \kappa_l}{1 - b_l \kappa_l}, \tag{9}$$

while the real part gives

$$k^2 \left(a_l - b_l v\right)\left(U^{(l)}\right)'' - \left(\omega_l + a_l \kappa_l^2 - b_l \omega_l \kappa_l\right) U^{(l)} + c_l \left(U^{(l)}\right)^3 + \left\{ \sum_{n\neq l}^{N} \alpha_{ln} \left(U^{(n)}\right)^2 \right\} U^{(l)} = 0. \tag{10}$$

Using the balancing principle leads to

$$U^{(l)} = U^{(n)}$$

Consequently, Eq. (10) reduces to

$$k^2 \left(a_l - b_l v\right)\left(U^{(l)}\right)'' - \left(\omega_l + a_l \kappa_l^2 - b_l \omega_l \kappa_l\right) U^{(l)} + \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right)\left(U^{(l)}\right)^3 = 0. \tag{11}$$

Balancing $U^{(l)''}$ with $U^{(l)^3}$ in Eq. (11) gives $M = 1$. Consequently we reach

$$U^{(l)}(\xi) = s_0^{(l)} + s_1^{(l)} \left(\frac{\psi'(\xi)}{\psi(\xi)}\right), s_1^{(l)} \neq 0. \tag{12}$$

Substituting Eq. (12) in Eq. (11) and then setting the coefficients of $\psi^{-j}(\xi), j = 0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving $s_0^{(l)}$, $s_1^{(l)}$, k, κ_l, v and ω_l as follows:

ψ^{-3} coeff.:

$$s_1^{(l)} \left(\psi'\right)^3 \left(2k^2 (a_l - b_l v) + s_1^{(l)2} \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) \right) = 0, \tag{13}$$

ψ^{-2} coeff.:

$$3s_1^{(l)} \psi' \left(k^2 \psi'' (b_l v - a_l) + s_0^{(l)} s_1^{(l)} \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) \psi' \right) = 0, \tag{14}$$

ψ^{-1} coeff.:

$$s_1^{(l)} \left(\begin{array}{l} \psi' \left(\begin{array}{l} -a_l \kappa_l^2 + 3s_0^{(l)2} \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) + \\ b_l \kappa_l \omega_l - \omega_l \end{array} \right) \\ + k^2 \psi''' (a_l - b_l v) \end{array} \right) = 0, \tag{15}$$

ψ^{0} coeff.:

$$s_0^{(l)} \left(-a_l \kappa_l^2 + s_0^{(l)2} \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) + \omega (b_l \kappa_l - 1) \right) = 0. \tag{16}$$

Solving this system, we obtain

$$s_0^{(l)} = \pm\sqrt{\frac{a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}},$$

$$s_1^{(l)} = \mp\sqrt{\frac{2k^2(b_l v - a_l)}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}}, \tag{17}$$

and

$$\psi'' = \sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\,\psi', \tag{18}$$

$$\psi''' = \frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}\psi' \tag{19}$$

From Eqs. (18) and (19), we can deduce that

$$\psi' = \sqrt{\frac{k^2(b_l v - a_l)}{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}}\,k_1 e^{\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\xi}, \tag{20}$$

and

$$\psi = \frac{k^2(b_l v - a_l)}{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}k_1 e^{\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\xi} + k_2, \tag{21}$$

where k_1 and k_2 are constants of integration. Substituting Eq. (20) and Eq. (21) into Eq. (12), we obtain following the following exact solution to Eq. (5)

$$q^{(l)}(x,t) = \left\{\sqrt{\frac{a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}} - \pm\frac{\frac{k^2(b_l v - a_l)}{\sqrt{\left(c_l + \sum_{n\neq l}^{N}\alpha_{ln}\right)\left(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l\right)}}k_1 e^{\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\xi}}{\frac{k^2(b_l v - a_l)}{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}k_1 e^{\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\xi} + k_2}\right\} \times e^{i(-\kappa_l x + \omega_l t + \theta_l)}, \tag{22}$$

If we set

$$k_1 = \frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}e^{\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\xi_0}, k_2 = \pm 1,$$

we obtain:

(i) When

$$(b_l v - a_l)(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l) > 0,$$

we have

$$q^{(l)}(x,t) = \pm\sqrt{\frac{a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}}\tanh\left[\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\left(k(x - vt) + \xi_0\right)\right] \times e^{i(-\kappa_l x + \omega_l t + \theta_l)}, \tag{23}$$

$$q^{(l)}(x,t) = \pm\sqrt{\frac{a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}}\coth\left[\sqrt{\frac{2(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l)}{k^2(b_l v - a_l)}}\left(k(x - vt) + \xi_0\right)\right] \times e^{i(-\kappa_l x + \omega_l t + \theta_l)}, \tag{24}$$

where (23) and (24) represent dark soliton and singular soliton solutions respecively.

(ii) When

$$(b_l v - a_l)(a_l\kappa_l^2 - b_l\kappa_l\omega_l + \omega_l) < 0,$$

we have the following periodic singular solutions:

$$q^{(l)}(x,t) = \pm\sqrt{\frac{-a_l\kappa_l^2 + b_l\kappa_l\omega_l - \omega_l}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}}\tan\left[\sqrt{\frac{2(-a_l\kappa_l^2 + b_l\kappa_l\omega_l - \omega_l)}{k^2(b_l v - a_l)}}\left(k(x - vt) + \xi_0\right)\right] \times e^{i(-\kappa_l x + \omega_l t + \theta_l)}, \tag{25}$$

$$q^{(l)}(x,t) = \mp\sqrt{\frac{-a_l\kappa_l^2 + b_l\kappa_l\omega_l - \omega_l}{c_l + \sum_{n\neq l}^{N}\alpha_{ln}}}\cot\left[\sqrt{\frac{2(-a_l\kappa_l^2 + b_l\kappa_l\omega_l - \omega_l)}{k^2(b_l v - a_l)}}\left(k(x - vt) + \xi_0\right)\right] \times e^{i(-\kappa_l x + \omega_l t + \theta_l)}, \tag{26}$$

where v is given by Eq. (9) and ω is an arbitrary constant.

3.2 Parabolic law nonlinearity

For parabolic law nonlinearity, DWDM model extends to:

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left[c_l \left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N} \alpha_{ln} \left|q^{(n)}\right|^2 \right] q^{(l)} + \left[d_l \left|q^{(l)}\right|^4 + \sum_{n\neq l}^{N} \left\{ \left|q^{(n)}\right|^2 \left(\beta_{ln} \left|q^{(n)}\right|^2 + \gamma_{ln} \left|q^{(l)}\right|^2 \right) \right\} \right] q^{(l)} = 0, \quad (27)$$

For $1 \leq l \leq N$. In (27), SPM terms are the coefficients of c_l and d_l, while XPM coefficients are α_{ln}, β_{ln} and γ_{ln}, while the remaining parameters have the same definition as in Kerr law nonlinear medium. In mathematical physics equations (5) and (27) fall under the category of nonlinear evolution equation (NLEE).

In order to solve Eq. (27), we use the following wave transformation

$$q^{(l)}(x,t) = U^{(l)}(\xi) e^{i\Phi(x,t)} \quad (28)$$

where

$$\xi = k(x - vt), \quad (29)$$

and

$$\Phi(x,t) = -\kappa_l x + \omega_l t + \theta_l. \quad (30)$$

Substituting Eq. (28) into Eq. (27) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v = \frac{b_l \omega_l - 2a_l \kappa_l}{1 - b_l \kappa_l}, \quad (31)$$

while the real part gives

$$k^2 \left(a_l - b_l v\right) \left(U^{(l)}\right)'' - \left(\omega_l + a_l \kappa_l^2 - b_l \omega_l \kappa_l\right) U^{(l)} + c_l \left(U^{(l)}\right)^3 + \left\{ \sum_{n\neq l}^{N} \alpha_{ln} \left(U^{(n)}\right)^2 \right\} U^{(l)} + d_l \left(U^{(l)}\right)^5 + \left\{ \sum_{n\neq l}^{N} \beta_{ln} \left(U^{(n)}\right)^4 \right\} U^{(l)} + \left\{ \sum_{n\neq l}^{N} \gamma_{ln} \left(U^{(n)}\right)^2 \right\} U^{(l)3} = 0. \quad (32)$$

Using the balancing principle leads to

$$U^{(l)} = U^{(n)}$$

Consequently, Eq. (32) reduces to

$$k^2 \left(a_l - b_l v\right) \left(U^{(l)}\right)'' - \left(\omega_l + a_l \kappa_l^2 - b_l \omega_l \kappa_l\right) U^{(l)} + \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) \left(U^{(l)}\right)^3 + \left(d_l + \sum_{n\neq l}^{N} \left(\beta_{ln} + \gamma_{ln}\right) \right) \left(U^{(l)}\right)^5 = 0. \quad (33)$$

Set

$$U^{(l)} = V^{(l)\frac{1}{2}} \quad (34)$$

so that (33) transforms to

$$k^2 (a_l - b_l v) \left(2V^{(l)} V^{(l)''} - \left(V^{(l)'}\right)^2 \right) - 4\left(a_l \kappa_l^2 - b_l \kappa_l \omega_l + \omega_l \right) V^{(l)2} + 4\left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) V^{(l)3} + 4\left(d_l + \sum_{n\neq l}^{N} \left(\beta_{ln} + \gamma_{ln}\right) \right) V^{(l)4} = 0. \quad (35)$$

Balancing $V^{(l)} V^{(l)''}$ with $V^{(l)^4}$ in Eq. (35) gives $M = 1$. Consequently we reach

$$V^{(l)}(\xi) = s_0^{(l)} + s_1^{(l)} \left(\frac{\psi'(\xi)}{\psi(\xi)} \right), s_1^{(l)} \neq 0. \quad (36)$$

Substituting Eq. (36) in Eq. (35) and then setting the coefficients of $\psi^{-j}(\xi), j = 0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving $s_0^{(l)}$, $s_1^{(l)}$, k, κ_l, v and ω_l as follows:

ψ^{-4} coeff.:

$$s_1^{(l)2} \left(\psi'\right)^4 \left(3k^2 (a_l - b_l v) + 4s_1^{(l)2} \left(d_l + \sum_{n\neq l}^{N} \left(\beta_{ln} + \gamma_{ln}\right) \right) \right) = 0, \quad (37)$$

ψ^{-3} coeff.:

$$4s_1^{(l)2} \left(\psi'\right)^2 \left(s_1^{(l)} \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) \psi' - k^2 \psi'' (a_l - b_l v) \right) + 4s_0^{(l)} s_1^{(l)} \left(\psi'\right)^3 \left(k^2 (a_l - b_l v) + 4s_1^{(l)2} \left(d_l + \sum_{n\neq l}^{N} \left(\beta_{ln} + \gamma_{ln}\right) \right) \right) = 0, \quad (38)$$

ψ^{-2} coeff.:

$$-k^2 (a_l - b_l v) s_1^{(l)} \left(6s_0^{(l)} \psi' \psi'' + s_1^{(l)} \left(\psi''\right)^2 - 2s_1^{(l)} \psi' \psi''' \right) - 4s_1^{(l)2} \left(\psi'\right)^2 \times \left(a_l \kappa_l^2 - 3s_0^{(l)} \left(c_l + \sum_{n\neq l}^{N} \alpha_{ln} \right) - b_l \kappa_l \omega_l - 6s_0^{(l)2} \left(d_l + \sum_{n\neq l}^{N} \left(\beta_{ln} + \gamma_{ln}\right) \right) + \omega_l \right) = 0, \quad (39)$$

ψ^{-1} coeff.:

$$2s_0^{(l)}s_1^{(l)}k^2(a_l-b_l v)\psi''' + 4s_0^{(l)}s_1^{(l)} \times \begin{pmatrix} -2\left(a_l\kappa_l^2-b_l\kappa_l\omega_l+\omega_l\right)+3s_0^{(l)}\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)+ \\ 4s_0^{(l)2}\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right) \end{pmatrix}\psi' = 0, \tag{40}$$

ψ^{0} coeff.:

$$4s_0^{(l)2}\begin{pmatrix} -a_l\kappa_l^2+s_0^{(l)}\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)+\omega_l(b_l\kappa_l-1)+ \\ s_0^{(l)2}\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right) \end{pmatrix}=0. \tag{41}$$

Solving this system, we obtain

$$s_0^{(l)} = -\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)}{4\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)},$$

$$s_1^{(l)} = \pm\sqrt{\frac{3k^2(b_l v-a_l)}{4\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)}},$$

$$\omega_l = \frac{16a_l\kappa_l^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)+3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{16(b_l\kappa_l-1)\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)}, \tag{42}$$

and

$$\psi'' = \pm\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}}\,\psi' \tag{43}$$

$$\psi''' = \frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}\,\psi', \tag{44}$$

From Eqs. (43) and (44), it is possible to deduce

$$\psi' = \pm\sqrt{\frac{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}}\; k_1 e^{\pm\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}}\xi}, \tag{45}$$

and

$$\psi = \frac{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}\; k_1 e^{\pm\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}}\xi} + k_2, \tag{46}$$

where k_1 and k_2 are constants of integration. Substituting Eq. (45) and Eq. (46) into Eq. (36), we obtain the following exact solution to Eq. (27)

$$q^{(l)}(\xi) = \left\{\frac{-3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)}{4\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)} + \frac{\dfrac{k^2(b_l v-a_l)}{c_l+\sum_{n\neq l}^{N}\alpha_{ln}}k_1 e^{\pm\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}}\xi}}{\dfrac{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}k_1 e^{\pm\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}}\xi}+k_2}\right\} \times e^{i\left(-\kappa_l x+\left(\frac{16a_l\kappa_l^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)+3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{16(b_l\kappa_l-1)\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)}\right)t+\theta_l\right)}, \tag{47}$$

If we set

$$k_1 = \frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}\, e^{\pm\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{4k^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)(b_l v-a_l)}}\xi_0},\quad k_2=\pm 1,$$

we obtain:

$$q^{(l)}(x,t)=\left\{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)}{8\left(d_l+\sum_{n\neq l}^{N}\left(\beta_{ln}+\gamma_{ln}\right)\right)}\left(-1\pm\tanh\left[\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{16k^2\left(d_l+\sum_{n\neq l}^{N}\left(\beta_{ln}+\gamma_{ln}\right)\right)(b_l v-a_l)}}\,(\xi+\xi_0)\right]\right)\right\}^{\frac{1}{2}}$$

$$\times e^{i\left(-\kappa_l x+\left(\frac{16a_l\kappa_l^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)+3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{16(b_l\kappa_l-1)\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)}\right)t+\theta_l\right)}, \qquad (48)$$

$$q^{(l)}(x,t)=\left\{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)}{8\left(d_l+\sum_{n\neq l}^{N}\left(\beta_{ln}+\gamma_{ln}\right)\right)}\left(-1\pm\coth\left[\sqrt{\frac{3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{16k^2\left(d_l+\sum_{n\neq l}^{N}\left(\beta_{ln}+\gamma_{ln}\right)\right)(b_l v-a_l)}}\,(\xi+\xi_0)\right]\right)\right\}^{\frac{1}{2}}$$

$$\times e^{i\left(-\kappa_l x+\left(\frac{16a_l\kappa_l^2\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)+3\left(c_l+\sum_{n\neq l}^{N}\alpha_{ln}\right)^2}{16(b_l\kappa_l-1)\left(d_l+\sum_{n\neq l}^{N}(\beta_{ln}+\gamma_{ln})\right)}\right)t+\theta_l\right)}, \qquad (49)$$

where v is given by Eq. (31) and

$$\left\{d_l+\sum_{n\neq l}^{N}\left(\beta_{ln}+\gamma_{ln}\right)\right\}(b_l v-a_l)>0. \qquad (50)$$

Equations (48) and (49) represent dark and singular soliton solutions respectively.

4. Conclusions

This paper otained soliton solutions to DWDM system with Kerr and parabolic law nonlinearity. The modified simple equation method was the integration algorithm adopted in the paper. Both dark and singular soliton solutions are obtained with the corresponding constraint conditions for the existence of these solitons. The drawback of this scheme is that no bright soliton solutions are obtained for any of the two nonlinearities. Nevertheless, this scheme stands on a strong footing to study future projects such as dispersive solitons, metamaterials, metasurfaces and others. The results of those research will be disseminated elsewhere.

Acknowledgements

The first author (QZ) was funded by the National Science Foundation of Hubei Province in China under the grant number 2017dhzk001. The third author (SPM) would like to thank the research support provided by the Department of Mathematics and Statistics at Tshwane University of Technology and the support from the South African National Foundation under Grant Number 92052 IRF1202210126. The eighth author (AB) would like to thank Tshwane University of Technology during his academic visit during 2016. The research work of ninth author (MB) was supported by Qatar National Research Fund (QNRF) under the grant number NPRP 8-028-1-001.

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